Strain wave propagation through felt

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This study presents the analytical and numerical analysis of acoustic wave propagation through porous felt-like material. A dispersion analysis is performed, examining the potential impact of theoretically found band gap (BG) and negative group velocities (NGV) on the wave evolution. A 1D wave equation is derived, based on the experimentally obtained constitutive relation. Pulses with characteristic frequencies corresponding to the wavenumbers located in the BG and the NGV regions are studied. No qualitative effect on the dispersion of wave pulses is identified. Theoretical explanation of this result is provided. The results obtained will be valuable for the use of felts in noise reduction and control applications.

1 Introduction

Dimensionless wave equation for modelling wave propagation through porous felt-like continuum was derived in [1, 2, 3, 4]:

$$\frac{\partial^2 (\varepsilon^p)}{\partial x^2} - \frac{\partial^2 \varepsilon}{\partial t^2} + \frac{\partial^3 (\varepsilon^p)}{\partial x^2 \partial t} - \delta \frac{\partial^3 \varepsilon}{\partial t^3} = 0, \tag{1}$$

where $\varepsilon(x,t)$ is the strain, $p \geq 1$ and $0 \leq \delta \leq 1$ are the material parameters. The variables x and t are space and time, respectively. Long-term dynamics of Equation 1 is analysed numerically and analytically. A dispersion analysis is performed on the linear form of Equation 1, which is obtained by selecting $p = 1$. [4]

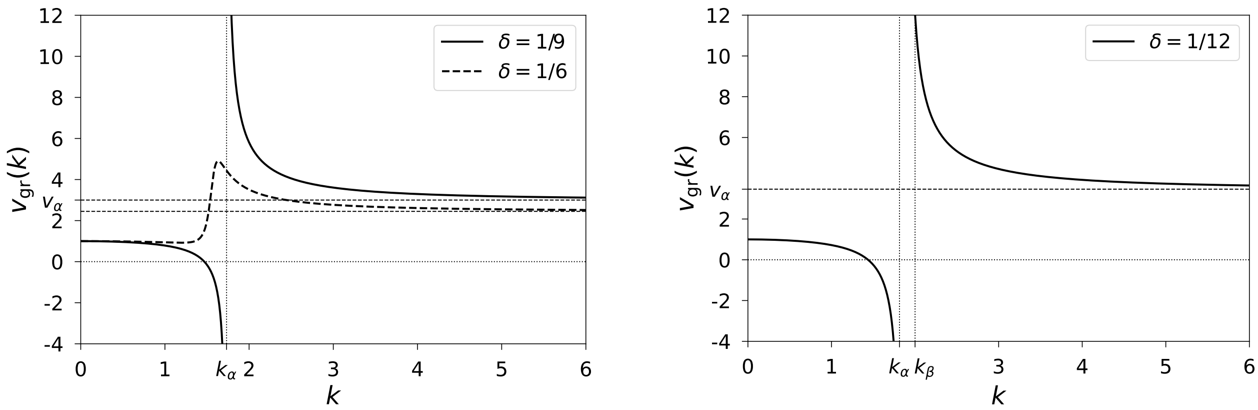

Figure 1: Group velocity curves calculated for the indicated values of δ. At the limit $k \to \infty$, $v_{\mathrm{gr}}(k) \to 1/\sqrt{\delta} = v_\alpha$.

2 Model analysis

The solution to Equation 1 can be assumed in the form

$$\varepsilon(x,t) \propto \exp(\sigma t + \mathrm{i}kx), \tag{2}$$

where $\sigma = \sigma(k)$ is the attenuation coefficient, k is the wavenumber. The characteristic equation of the linearised Equation 1 follows from Assumption 2

$$\delta\sigma^3 + \sigma^2 + k^2\sigma + k^2 = 0. \tag{3}$$

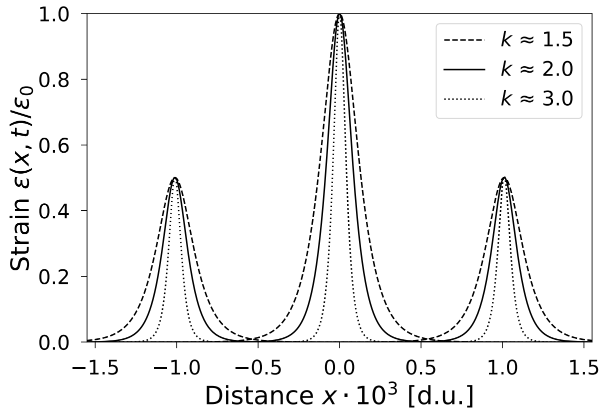

Figure 2: Comparison of solutions to the linear IVP. Parameter $\delta = 0.05$. Solutions are shown for the time moments $t = 0$ [d.u.] (initial condition) and $t = 4.50 \cdot 10^3$ [d.u.].

Equation 3 has complex valued solutions, where the imaginary part represents the attenuation of the wave components. Three different solution regimes are distinguishable depending on the parameter δ. First, a non-dispersive and non-dissipative regime, for $\delta = 1$. Second, a regime with a continuous dispersion curve, for $1/9 \leqslant \delta < 1$. Lastly, for $0 < \delta < 1/9$ a band gap (BG) region exists.
Additionally, a region of negative group velocity (NGV) appears for $0 \leq \delta \lessapprox 0.134$. When the δ values overlap, the NGV region precedes the BG region. This is shown in Figure 1 (right).
Figure 2 compares the evolution of three pulses in the context of an initial value problem (IVP): a pulse with a characteristic wavenumber laying inside the BG region, and two others that come before and after the region of interest. These results show that the BG and NGV do not have a qualitative effect on the pulse evolution and dispersion. The same conclusion holds for solutions to a boundary value problem.

3 Conclusions

A detailed dispersion analysis of the proposed model was conducted, including a theoretical possibility of the BG and NGV regions in the frequency domain. The effect of BG and NGV on pulse evolution was investigated by selecting appropriate characteristic pulse widths. The findings indicated that the BG and NGV regions did not impact the pulse dispersion, both in the linear and nonlinear cases. To gain a deeper understanding, further investigations and analyses can be conducted to explore the model's behaviour under different wave excitation conditions. In conclusion, the model and analysis results presented here will greatly contribute to the effective utilisation of felt and other felted materials for noise reduction and control and other applications.

Acknowledgements

This work was supported by the Estonian Ministry of Education and Research (Grant PRG1227), and co-funded by the Erasmus+ Programme of the European Union.

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