On pulse propagation in porous visco-elastic felt-like material

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This paper presents a model of acoustic wave propagation through felt-like material. The question of theoretically possible band gap (BG) and negative group velocity (NGV) is discussed. It is shown theoretically and numerically that if material loading and unloading time-scale is much too great in comparison to the felt relaxation time, then any possible contribution of BG and NGV on the wave shape evolution will be negligibly small.

1 Modelling wave propagation in felt-like materials

Felt is created by matting wool or other fibres with heat, moisture, and pressure. It is employed, e.g., in automotive industry for tasks like noise, vibration and shock absorption.

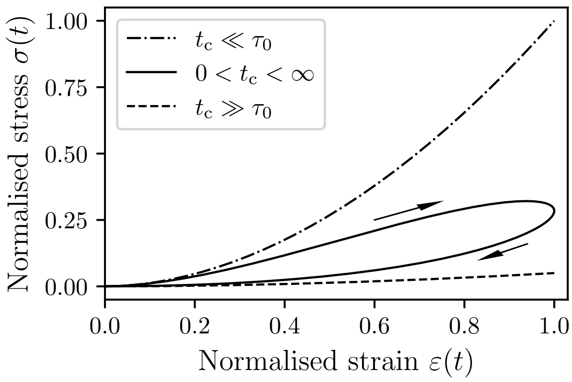

Figure 1: Typical idealised stress–strain curves corresponding to Equations 1 (continuous curve), 5 (dash-dotted curve), and 6 (dashed curve). Arrows indicate the direction of felt loading and unloading.

The constitutive equation of the felt that is valid for all material loading rates is given in the form

$$\sigma(\varepsilon) = E_{\rm d}\left[\varepsilon^p(t) - \mathcal{R}(t) * \varepsilon^p(t)\right], \tag{1}$$

where σ is the stress, $\varepsilon = \partial u/\partial x$ is the strain, u is the displacement, $E_{\rm d}$ is the dynamic Young's modulus, and p is the nonlinearity exponent. The operator $*$ denotes the convolution operation. Time-dependant relaxation function $\mathcal{R}(t)$ is selected as follows:

$$\mathcal{R}(t) = \frac{\gamma}{\tau_0}\exp\left(-\frac{t}{\tau_0}\right), \qquad 0 \leqslant \gamma < 1, \tag{2}$$

here γ is the hereditary amplitude and τ_0 is the relaxation time, *cf.* [1]. Figure 1 shows an example of a typical stress–strain hysteresis loop. Model parameters have been obtained experimentally [2, 3]. The 1-D equation of motion, that describes the wave propagation in the felt material, is derived using the classical equation of motion and Constitutive Equation 1:

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} \quad\Rightarrow\quad \rho\frac{\partial^2 u}{\partial t^2} + \rho\tau_0\frac{\partial^3 u}{\partial t^3} - E_{\rm d}\left\{(1-\gamma)\frac{\partial}{\partial x}\left[\left(\frac{\partial u}{\partial x}\right)^p\right] + \tau_0\frac{\partial^2}{\partial x\partial t}\left[\left(\frac{\partial u}{\partial x}\right)^p\right]\right\} = 0, \tag{3}$$

here ρ is the density and the equation is presented in terms of displacement variable u. One can derive a dimensionless form of Equation 3 by using the following dimensionless variables: $u \Rightarrow u/l_0$, $x \Rightarrow x/l_0$, and $t \Rightarrow t/\alpha_0$ where $\alpha_0 = \tau_0/\delta$, $l_0 = c_{\rm d}\alpha_0\sqrt{\delta}$, $\delta = 1-\gamma$, $c_{\rm d} = \sqrt{E_{\rm d}/\rho}$, and $c_{\rm s} = c_{\rm d}\sqrt{\delta}$. Thus, the dimensionless forms of Equation 3 in terms of displacement variable $u(x,t)$ and strain variable $\varepsilon(x,t)$ read as follows:

$$[(u_x)^p]_x - u_{tt} + [(u_x)^p]_{xt} - \delta u_{ttt} = 0, \qquad (\varepsilon^p)_{xx} - \varepsilon_{tt} + (\varepsilon^p)_{xxt} - \delta\varepsilon_{ttt} = 0, \tag{4}$$

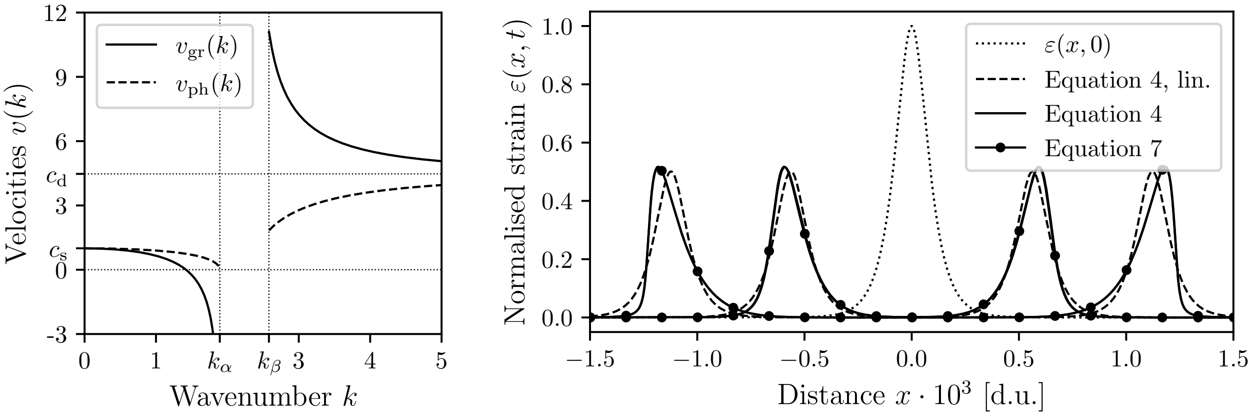

Figure 2: Left: Phase and group velocities corresponding to Equation 4. Right: Numerical solution to initial value problem showing two time moments. Comparison of solutions to Equation 4 where $p = 1.1$ and $\delta = 0.05$, its linear form where $p = 1$, and to Equation 7 where $p = 1.1$ and $\delta = 0.05$.

here the subscripted indices denote the differentiation with respect to the indicated variables [2, 3]. Analysis of the general model given by Equation 1 shows that for an extremely fast felt loading–unloading cycle lasting for t_{c} one can derive an additional model. This model is valid for high frequency waves or wave partials. If $t_{\mathrm{c}} \ll \tau_0$, then one obtains constitutive equation in the form

$$\sigma(\varepsilon) = E_{\mathrm{d}}\varepsilon^p(t), \tag{5}$$

where E_{d} is the dynamic Young's modulus. The hysteresis loop has collapsed, see Fig. 1. Similarly, a model for a slow loading–unloading cycle valid for low frequency waves is found for $t_{\mathrm{c}} \gg \tau_0$,

$$\sigma(\varepsilon) = E_{\mathrm{d}}(1-\gamma)\varepsilon^p(t) = E_{\mathrm{s}}\varepsilon^p(t), \tag{6}$$

here $E_{\mathrm{s}} = E_{\mathrm{d}}(1-\gamma)$ is the static Young's modulus, see Fig. 1. The wave equations, that follow from Constitutive Equations 5 and 6, presented in dimensionless strain variable $\varepsilon(x,t)$ are the following:

$$\varepsilon_{tt} = (\varepsilon^p)_{xx}, \qquad \varepsilon_{tt} = \delta^2(\varepsilon^p)_{xx}. \tag{7}$$

2 A bang gap and negative group velocity

Dispersion analysis of Equation 4 shows that felt features a band gap (BG) for wavenumbers $k \in (k_\alpha, k_\beta)$ and a small region with negative group velocity (NGV) for $k \in [\approx 1.5, k_\alpha]$, see Fig. 2. Both properties are surprising for the material that is comprised of randomly oriented visco-elastic fibres. The BG and NGV influence low frequency wave components. Wavelength corresponding to $k \approx 2$ happens to coincide with the case $t_{\mathrm{c}} \gg \tau_0$ for realistic physical parameter values [3]. This means that, e.g., a pulse with characteristic frequency that corresponds to $k \approx 2$ should propagate more like a pulse described by Equation 7 rather than 4 where the characteristic frequency of the pulse is high. Figure 2 demonstrates just that.

In conclusion, despite being predicted by the dispersion analysis the BG and NGV may not influence the actual wave evolution. If the material loading and unloading time-scales related to the initial value or boundary conditions are much too great in comparison to relaxation time τ_0, present in underlying Constitutive Relation 1, any imaginable effects will be negligibly small.

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References

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