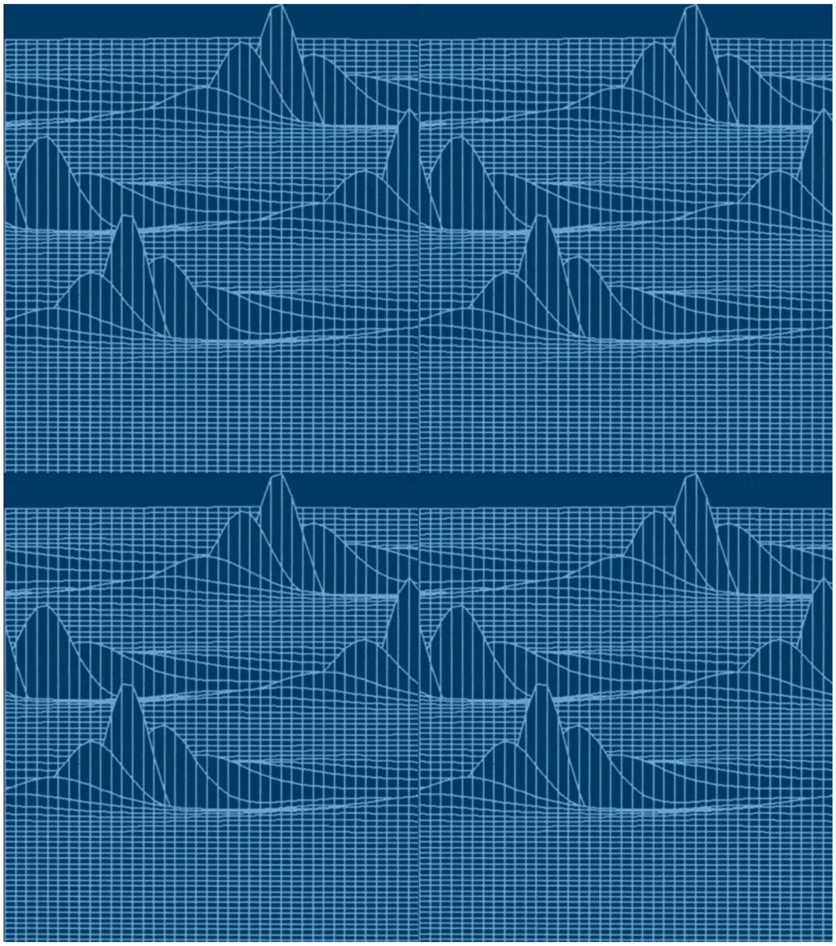
The Ninth IMACS International Conference on

Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory

April 1–4, 2015

Georgia Center for Continuing Education
University of Georgia, Athens, GA, USA



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Book of Abstracts

The Ninth IMACS International Conference On

Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory

Athens, Georgia
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Sponsored by

The International Association for Mathematics and Computers in Simulation (IMACS)
The Computer Science Department, University of Georgia

Edited by Gino Biondini and Thiab Taha

PROGRAM AT A GLANCE

Wednesday, April 01, 2015

	Mahler auditorium	*Room Q*	*Room R*	*Room Y/Z*	*Room F/G*	*Room T/U*	*Room V/W*
8:00 am – 8:30 am	Welcome						
8:30 am – 9:30 am	Keynote lecture 1: Irena Lasiecka						
9:30 am – 10:00 am	*Coffee Break*						
10:00 am – 10:50 am	S1 -- I/IV	PAPERS	S26 -- I/VI	S23 -- I/V	S4 -- I/III	S14 -- I/II	S6 -- I/II
10:55 am – 12:10 pm	PAPERS	S5 -- I/II	S26 -- II/VI	S23 -- II/V	S30 -- I/II	S19 -- I/II	S11 -- I/III
12:10 pm – 1:40 pm	*Lunch, Magnolia ballroom*						
1:40 am – 3:20 pm	S27 -- I/III	S5 -- II/II	S15 -- I/II	S12 -- I/II	S18 -- I/III	S22 -- I/III	PAPERS
3:20 am – 3:50 pm	*Coffee Break*						
3:50 pm – 5:55 pm	S1 -- II/IV	S10 -- I/III	S4 -- II/III	S12 -- II/II	S6 -- II/II	S22 -- II/III	S14 -- II/II

Thursday, April 02, 2015

	Mahler auditorium	*Room Q*	*Room R*	*Room Y/Z*	*Room F/G*	*Room T/U*	*Room V/W*	*Room J*
8:00 am – 9:00 am	Keynote lecture 2: Panos Kevrekidis							
9:10 am – 10:00 am	S1 -- III/IV	S25 -- I/II	S7 -- I/III	S23 -- III/V	S17 -- I/V	S22 -- III/III	PAPERS	S27 -- II/III
10:00 am – 10:30 am	*Coffee Break*							
10:30 am – 12:10 pm	S7 -- II/III	S30 -- II/II	S13 -- I/II	S31 -- I/V	S17 -- II/V	S29 -- I/II	S11 -- II/III	S16 -- I/I
12:10 pm – 1:40 pm	*Lunch (attendees on their own)*							
1:40 am – 3:20 pm	S15 -- II/II	S25 -- II/II	S13 -- II/II	S31 -- II/V	S17 -- III/V	S29 -- II/II	S11 -- III/III	S27 -- III/III
3:20 am – 3:50 pm	*Coffee Break*							
3:50 pm – 5:55 pm	S1 -- IV/IV	S10 -- II/III	S26 -- III/VI	S23 -- IV/V	S18 -- II/III	S19 -- II/II	S28 -- I/I	
5:00 – 7:00	*Posters, Atrium outside Mahler auditorium*							
7:00 – 9:00	*Conference banquet (including poster awards)*							

Friday, April 03, 2015

	Mahler auditorium	*Room Q*	*Room R*	*Room Y/Z*	*Room F/G*	*Room T/U*	*Room V/W*
8:00 am – 9:00 am	Keynote lecture 3: Sijue Wu						
9:10 am – 10:00 am	S2 -- I/II	S3 -- I/II	PAPERS	S26 -- IV/VI	S7 -- III/III	PAPERS	S24 -- I/III
10:00 am – 10:30 am	*Coffee Break*						
10:30 am – 12:10 pm	S10 -- III/III	S3 -- II/II	S31 -- III/V	S26 -- V/VI	S17 -- IV/V	S21 -- I/II	S9 -- I/I
12:10 pm – 1:40 pm	*Lunch (attendees on their own)*						
1:40 am – 3:20 pm	Tutorial	S4 -- III/III	S31 -- IV/V	S26 -- VI/VI	S17 -- V/V	S21 -- II/II	S24 -- II/III
3:20 am – 3:50 pm	*Coffee Break*						
3:50 pm – 5:55 pm	S2 -- II/II	PAPERS	S31 -- V/V	S20 -- I/I	S18 -- III/III	S23 -- V/V	S24 -- III/III

PROGRAM

TUESDAY, MARCH 31, 2015

5:00 – 6:00 REGISTRATION
5:00 – 7:00 RECEPTION

WEDNESDAY, APRIL 01, 2015

7:30 – 10:00 REGISTRATION
8:00 – 8:30 **WELCOME**, Mahler hall
Thiab Taha / Program Chair and Conference Coordinator
Alan Dorsey / Dean of the Franklin College of Arts and Sciences, UGA
Event Manager / Georgia Center

8:30 – 9:30 KEYNOTE LECTURE I, Mahler hall
Irena Lasiecka: Quasilinear dynamics arising in a 3-D fluid structure interaction with moving Frame
Chair: Thiab Taha

9:30 – 10:00 COFFEE BREAK

10:00 – 10:50 SESSION 1, Mahler hall: Nonlinear evolution equations and integrable systems – Part 1/IV
Chairs: Alex Himonas, Dionyssis Mantzavinos
10:00 – 10:25 *Gino Biondini:* Integrable nature of modulational instability
10:25 – 10:50 *Hongqiu Chen:* The BBM-BBM system on a quarter plane

10:00 – 10:50 PAPERS, Room Q -- *Chair: Wen-Xiu Ma*
10:00 – 10:25 *Will Cousins, Themistoklis Sapsis:* Prediction of extreme events in nonlinear dispersive wave equations
10:25 – 10:50 *Wen-Xiu Ma, Xiang Gu, Wen-Ying Zhang:* An integrable Hamiltonian hierarchy based on so(3,R) with three potentials

10:00 – 10:50 SESSION 26, Room R: Evolution equations in mathematical biology: Cellular and network processes -- Part I/VI; *Chairs: Gregor Kovacic, Remus Osan*
10:00 – 10:25 *Michael Caiola, Mark Holmes:* Basal ganglia dynamics during the transition to Parkinson's disease
10:25 – 10:50 *Jun Xia, Emily Su, Marius Osan, Emilia Titan, Remus Osan:* Determination of optimal targeting performances in a stochastic neural growth model with low branching probabilities

10:00 – 10:50 SESSION 23, Room Y/Z: Solitons and nonlinear dispersive waves -- Part I/V
Chairs: Zhen Wang, Taixi Xu, Zhenya Yan, Zhijun Qiao
10:00 – 10:25 *Stephen Anco:* Integrable multicomponent nonlinear wave equations from symplectic structures
10:25 – 10:50 *Qicheng Meng, Chongwei Zhang:* Fully nonlinear numerical simulation of large amplitude obliquely interacting solitary waves

10:00 – 10:50 SESSION 4, Room F/G: Discrete and ultra-discrete integrable systems and Painlevé equations -- Part I/III
Chairs: Anton Dzhamay, Sarah Lobb, Tomoyuki Takenawa
10:00 – 10:25 *Rei Inoue:* Generalization of discrete Toda lattice and toric network
10:25 – 10:50 *Tomoki Nakanishi:* On generalized cluster algebras

10:00 – 10:50 SESSION 14, Room T/U: Coupled systems with hyperbolic components – Part I/II
Chairs: Lorena Bociu, Daniel Toundykov
10:00 – 10:25 *Suncica Canic, Boris Muha, Martina Bukac:* Fluid-composite structure interaction
10:25 – 10:50 *Lorena Bociu, Lucas Castle, Kristina Martin, Daniel Toundykov:* Optimal control in a free boundary fluid-elasticity interaction

10:00 – 10:50 SESSION 6, Room V/W**:** Graphene lattices: phenomena and analysis -- Part I/II
Chairs: Christopher Curtis, Yiping Ma, Yi Zhu
10:00 – 10:25 *Laith Haddad, Lincoln Carr:* Relativistic topological defects and quantum Berezinskii-Kosterlitz-Thouless transition in the honeycomb lattice
10:25 – 10:50 *Charles Fefferman, James Lee-Thorp, Michael Weinstein:* Topologically protected states in one-dimensional systems

10:55 – 12:10 PAPERS, Mahler hall – *Chair: Vishal Vasan*
10:55 – 11:20 *Chang-Yeol Jung, Thien Binh Nguyen:* New WENO-theta scheme and its application to the 2D Riemann problem
11:20 – 11:45 *Ramaz Khomeriki, Jamal Berakdar, Levan Chotorlishvili, Boris Malomed:* Creation and amplification of electro-magnon solitons by electric field in nanostructured multiferroics
11:45 – 12:10 *Katie Oliveras, Patrick Sprenger, Vishal Vasan:* Stability of waves with vorticity

10:55 – 12:10 SESSION 5, Room Q: Applications of continuous and discrete integrable systems -- Part I/II
Chairs: Baofeng Feng, Annalisa Calini
10:55 – 11:20 *Baofeng Feng:* The physical, geometric and algebraic aspects of the complex short pulse equation
11:20 – 11:45 *Yannan Shen, John Zweck:* Numerical analysis of the spectrum of short pulse solutions of the cubic-quintic Ginzburg-Landau equation near zero dispersion
11:45 – 12:10 *Andras Balogh, J. Banda, K. Yagdjian:* Higgs boson equation in de Sitter spacetime: Numerical Investigation of bubbles using GPUs

10:55 – 12:10 SESSION 26, Room R: Evolution equations in mathematical biology: Cellular and network processes -- Part II/VI -- *Chairs: Gregor Kovacic, Remus Osan*
10:55 – 11:20 *Jay Newby:* Asymptotic and numerical methods for metastable events in stochastic gene networks
11:20 – 11:45 *Eli Shlizerman:* Functional connectomics from dynamic evolution data
11:45 – 12:10 *Francisco Javier Martínez Farías, Panayotis Panayotaros:* Elastic nonlineal model for a unidimensional chain with clustering zones

10:55 – 12:10 SESSION 23, Room Y/Z: Solitons and nonlinear dispersive waves – Part II/V
Chairs: Zhen Wang, Taixi Xu, Zhenya Yan, Zhijun Qiao
10:55 – 11:20 *Athanassios Fokas, Alex Himonas, Dionyssis Mantzavinos:* The unified transform method for nonlinear evolution equations
11:20 – 11:45 *Qiaoyi Hu:* Well-posedness and blowup phenomena for a new integrable system with peakon solutions
11:45 – 12:10 *Eric Tovar, Zhijun Qiao:* Peakon solutions for (2+1)-dimensional Camassa-Holm equation

10:55 – 12:10 SESSION 30, Room F/G: Wave phenomena in population biology and application to cancer – Part I/II
Chairs: Hassan Fathallah-Shaykh, Thierry Colin, Clair Poignard
10:55 – 11:20 *Pedro Lowenstein, Gregory Baker, Sebastien Motsch, Maria Castro:* Mechanisms of glioma formation: Exploring glioma growth through dialectic biological-computational approaches
11:20 – 11:45 *Hassan Fathallah-Shaykh, Thierry Colin, Olivier Saut, Elizabeth Scribner, Paula Province, Asim Bag:* Effects of anti-angiogenesis on glioblastoma growth and migration: Model to clinical predictions
11:45 – 12:10 *Dimah Dera, Nidhal Bouaynaya, Hassan Fathallah-Shaykh:* Level set segmentation using non-negative matrix factorization with application to MRI

10:55 – 12:10 SESSION 19, Room T/U: Modeling, Geometry, integrability, and analysis of nonlinear (dispersive) waves – Part I/II -- *Chairs: Stephen Anco, Robin Ming Chen, Yue Liu, Changzheng Qu*
10:55 – 11:20 *Peter J. Olver:* Dispersive quantization of linear and nonlinear waves
11:20 – 11:45 *Ohannes Karakashian, Yulong Xing:* Local discontinuous Galerkin methods for the generalized Korteweg-de Vries equations

10:55 – 12;10 SESSION 11, Room V/W: Nonlinear Waves – Part I/III
Chairs: Jerry Bona, Min Chen, Vassilios, Dimitrios Mitsotakis
10:55 – 11:20 *Min Chen:* Solutions of Boussinesq systems
11:20 – 11:45 *Dimitrios Mitsotakis, Angel Duran:* Solitary waves for some systems of internal wave theory
11:45 – 12:10 *Olivier Goubet, Min Chen, Youcef Mammeri:* Regularized long wave equation with white noise dispersion

12:10 – 1:40 LUNCH, Magnolia ballroom

1:40 – 3:20 SESSION 27, Mahler hall: Mechanisms for computations in neuronal networks – Part I/III
Chair: Andrea Barreiro
1:40 – 2:05 *Pamela Fuller, Gregor Kovacic, David Cai:* Integrate-and-fire model of insect olfaction
2:05 – 2:30 *Jennifer Kile, Gregor Kovacic, David Cai:* The role of gap junctions between excitatory neurons in synchronizing cortical dynamics
2:30 – 2:55 *Victor Barranca, Douglas Zhou, David Cai:* Efficient reconstruction of structural connectivity from neuronal dynamics
2:55 – 3:20 *Zachary Kilpatrick:* Stochastic synchronization of neural activity waves

1:40 – 3:20 SESSION 5, Room Q: Applications of continuous and discrete integrable systems- Part II/II
Chairs: Baofeng Feng, Annalisa Calini
1:40 – 2:05 *Sarah Raynor*, *Brian Pigott:* Asymptotic stability of KdV solitons in weighted Sobolev Spaces below the energy space
2:05 – 2:30 *Virgil Pierce:* Random triangulations and nonlinear differential equations
2:30 – 2:55 *Annalisa Calini*, *Philip Staley:* On generalisations of the integrable vortex filament evolution

1:40 – 3:20 SESSION 15, Room R: Geometric, algebraic and analytic approaches to integrable systems – Part I/II
Chairs: Yang Shi, Nobutaka Nakazono
1:40 – 2:05 *Nalini Joshi:* Delay Painlevé equations
2:05 – 2:30 *Vladimir Dragovic:* New algebro-geometric approach to the Schlesinger equations and Poncelet polygons
2:30 – 2:55 *Hajime Nagoya:* Irregular conformal field theory and Painleve tau functions
2:55 – 3:20 *Hayato Chiba:* Painleve equations and weight systems

1:40 – 3:20 SESSION 12, Room Y/Z: Mathematical progress on nonlineaer phenomena in parity-time-symmetric systems – Part I/II -- *Chairs: Jianke Yang, Panos Kevrekidis*
1:40 – 2:05 *Carl Bender:* Nonlinear eigenvalue problems and PT symmetry
2:05 – 2:30 *Vladimir Konotop, Dmitry Zezyulin:* Families of nonlinear modes in complex potentials
2:30 – 2:55 *Brenda Dana, Alon Bahabad, Boris Malomed:* CPT symmetry in optical systems
2:55 – 3:20 *Igor Barashenkov, Dmitry Pelinovsky, Philippe Dubard:* Nonlinear Schrodinger dimer with gain and loss: Integrability and PT symmetry restoration

1:40 – 3:20 SESSION 18, Room F/G: Advances in integrable systems and nonlinear wave theory – Part I/III
Chairs: Peter Miller, Robert Buckingham
1:40 – 2:05 *Otis Wright:* Effective integration of ultra-elliptic solutions of the cubic nonlinear Schrodinger equation
2:05 – 2:30 *Thomas Bothner:* On the zeros of large degree Vorobev-Yablonski polynomials
2:30 – 2:55 *Megan Stone:* Hurwitz numbers and random matrix theory
2:55 – 3:20 *Bob Jenkins:* Long-time asymptotics for Gross-Pitaevskii and asymptotic stability of N-soliton solutions

1:40 – 3:20 SESSION 22, Room T/U: Spectral methods in stability of traveling waves – Part I/III
Chairs: Yuri Latushkin, Alim Sukhtayev, Mat Johnson
1:40 – 2:05 *Alim Sukhtayev, Yuri Latushkin, Selim Sukhtaiev:* The Morse and Maslov indices for Schrodinger operators
2:05 – 2:30 *Graham Cox, Jeremy Marzuola, Christopher Jones:* Multidimensional instability criteria via the Maslov index
2:30 – 2:55 *Robby Marangell:* An Evans function for the Riccati equation on the Grassmannian
2:55 – 3:20 *Peter Howard:* Stability of transition front solutions in multidimensional Cahn-Hilliard systems

1:40 – 3:20 PAPERS, Room V/W – *Chair: Erwin Suazo*
1:40 – 2:05 *Pierre Garnier, Jean-Paul Chehab, Youcef Mammeri:* Long time behavior of dispersive PDEs with generalized damping
2:05 – 2:30 *Erwin Suazo, Alex Mahalov, Sergei K. Suslov, Primitivo B. Acosta-Humanez:* Closed solutions for the degenerate parametric oscillator and inhomogeneous nonlinear Schrodinger equation
2:30 – 2:55 *Sonia Boscolo, Sergei K Turitsyn, Christophe Finot:* New nonlinear regimes of pulse generation in mode-locked fiber lasers
2:55 – 2:30 *Matthew Harris:* Runup of nonlinear long waves in U-shaped bays of finite length: analytical theory and numerical

3:20 - 3:50 COFFEE BREAK

3:50 – 5:55 SESSION 1, Mahler hall: Nonlinear evolution equations and integrable systems --Part II/IV
Chairs: Alex Himonas, Dionyssis Mantzavinos
3:50 – 4:15 *Vladimir Novikov:* Integrable equations in 3D: deformations of dispersionless limits
4:15 – 4:40 *Zhijun Qiao:* A synthetical two-component model with peakon solutions
4:40 – 5:05 *Mats Ehrnstrom:* On Whitham's conjecture of a highest cusped wave for a nonlocal shallow water wave equation
5:05 – 5:30 *Curtis Holliman:* Ill-posedness of weakly dispersive equations
5:30 – 5:55 *Ryan Thompson:* Persistence properties and unique continuation for a generalized Camassa-Holm equation

5:30 – 5:55 SESSION 10, Room Q: Mathematical modeling and physical dynamics of solitary waves: From continuum mechanics to field theory – Part I/III -- *Chairs: Ivan Christov, Michail Todorov, Sanichiro Yoshida*
3:50 – 4:15 *Anthony Rosato, Denis Blackmore, Hao Wu, David Horntrop, Luo Zuo:* A simulation and dynamical model study of waves in 1D granular tapping flows
4:15 – 4:40 *Ronald Mickens:* Determination of traveling wave front asymptotic behavior using dominant balance
4:40 – 5:05 *Andrus Salupere, Mart Ratas:* On solitonic structures and discrete spectral analysis
5:05 – 5:30 *Yuanzhen Cheng, Alexander Kurganov, Zhuolin Qu, Tao Tang:* A fast and stable explicit operator splitting method for phase-field models
5:30 – 5:55 *Ivan Christov, Avadh Saxena, Avinash Khare:* Kinks and their statistical mechanics in higher-order scalar field theories

3:50 – 5:55 SESSION 4, Room R: Discrete and Ultra-discrete integrable systems and Painlevé equations -- Part II/III
Chairs: Anton Dzhamay, Sarah Lobb, Tomoyuki Takenawa
3:50 – 4:15 *James Atkinson:* Discrete integrability and the lattice-geometry of the Gosset-Elte figures
4:15 – 4:40 *Sarah Lobb:* Geometry of integrable lattice equations
4:40 – 5:05 *Yasuhiko Yamada:*Conserved curves for autonomous (ultra-) discrete Painlevé equations
5:05 – 5:30 *Claire Gilson:* Graphical techniques for analyzing solutions to ultra-discrete equations

3:50 – 5:55 SESSION 12, Room Y/Z: Mathematical progress on nonlineaer phenomena in parity-time-symmetric systems – Part II/II -- *Chairs: Jianke Yang, Panos Kevrekidis*
3:50 – 4:15 *Ziad Musslimani:* Integrable nonlocal nonlinear Schrodinger equation with PT symmetry
4:15 – 4:40 *Haitao Xu:* Some nonlinear Parity-Time-symmetric systems in NLS equations and dimer models
4:40 – 5:05 *Vassilis Rothos, Makrina Agaoglou, Hadi Susanto:* Gain-driven discrete breathers in PT-symmetric nonlinear metamaterials
5:05 – 5:30 *Jianke Yang:* Symmetry breaking of solitons in PT-symmetric potentials
5:30 – 5:55 *Demetrios Christodoulides, Mohammad Ali Miri:* PT symmetry in optics

3:50 – 5:55 SESSION 6, Room F/G: Graphene lattices: phenomena and analysis -- Part II/II
Chairs: Christopher Curtis, Yiping Ma, Yi Zhu
3:50 – 4:15 *Daohong Song, Vassilis Paltoglou, Sheng Liu, Yi Zhu, Daniel Gallardo, Liqin Tang, Jingjun Xu, Mark J. Ablowitz, Nikolaos K. Efremidis, Zhigang Chen:* Novel pseudospin-related phenomena in artificial "photonic graphene"
4:15 – 4:40 *Vassilis Paltoglou, Zhigang Chen, Nikolaos Efremidis:* Composite multi- vortex diffraction-free beams and van Hove singularities in honeycomb lattices
4:40 – 5:05 *Chris Curtis, Mark Ablowitz:* Linear dynamics in PT symmetric honeycomb lattices
5:05 – 5:30 *Yi Zhu, Christopher Curtis, Mark Ablowitz:* Nonlinear dynamics in deformed and PT symmetric honeycomb lattices
5:30 – 5:55 *Yi-Ping Ma, Mark Ablowitz, Christopher Curtis:* Traveling edge waves in photonic graphene

3:50 – 5:55 SESSION 22, Room T/U: Spectral methods in stability of traveling waves –Part II/III
Chairs: Yuri Latushkin, Alim Sukhtayev, Mat Johnson
3:50 – 4:15 *Zhiwu Lin:* Linear instability and invariant manifolds for Hamiltonian PDEs
4:15 – 4:40 *Jared Bronski, Lee Deville:* Phase locking and stability in the Kuramoto model
4:40 – 5:05 *Blake Barker:* Numerical stability analysis for thin film flow
5:05 – 5:05 *Alin Pogan, Jinghua Yao, Kevin Zumbrun:* O(2) Hopf bifurcation of viscous shock waves in a channel
5:30 – 5:55 *Richard Kollar:* Avoided Hamiltonian-Hopf bifurcations

3:50 – 5:55 SESSION 14, Room V/W: Coupled Systems with Hyperbolic Components – Part II/II
Chairs: Lorena Bociu, Daniel Toundykov
3:50 – 4:15 *Roberto Triggiani, Zhifei Zhang:* Global uniqueness and stability of an inverse problem for the Schrodinger equation on a Riemannian manifold via one boundary measurement
4:15 – 4:40 *Justin Webster:* "Hidden"' trace regularity of a tangentially degenerate wave equation and supersonic panel flutter
4:40 – 5:05 *Irena Lasiecka, Rodrigo Monteiro:* Global attractors for Full von Karman systems with thermal effects
5:05 – 5:30 *Marcelo Disconzi, David Ebin:* The free-boundary Euler equations: existence and convergence results
5:30 – 5:55 *Lorena Bociu, Jean-Paul Zolesio:* Shape differentiability and hidden boundary regularity for the wave equations with Neumann boundary conditions

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THURSDAY, APRIL 02, 2015

7:30 – 9:30 REGISTRATION
8:00 – 9:00 KEYNOTE LECTURE 2, Mahler hall
Panayotis Kevrekidis: Nonlinear waves in granular crystals: Theory, computation, experiments and applications
Chair: Gino Biondini

9:10 – 10:00 SESSION 1, Mahler hall: Nonlinear evolution equations and integrable systems – Part III/IV
Chairs: Alex Himonas, Dionyssis Mantzavinos
9:10 – 9:35 *Peter Miller:* Semiclassical initial-boundary value problems for the defocusing nonlinear Schrodinger equation
9:35 – 10:00 *Feride Tiglay:* The Cauchy problem for an integrable Euler-Poisson equation

9:10 – 10:00 SESSION 25, Room Q: Solitons, vortices, domain walls, their dynamics and their progenitors – Part I/II
Chairs: Stephen Gustafson, Israel Michael Sigal, Avy Soff
9:10 – 9:35 *Nicholas Ercolani, Patrick Waters:* Quantum Gravity and Quantum Groups
9:35 – 10:00 *Weiwei Ao, Monica Musso, Frank Pacard, Frodlin Ting, Juncheng Wei:* Finite energy bound states for the stationary Klein-Gordon, magnetic Ginzburg-Landau and Chern-Simons-Higgs system

9:10 – 10:00 SESSION 7, Room R: Applied nonlinear waves – Part I/III
Chairs: David Kaup, Constance Schober, Thomas Vogel
9:10 – 9:35 *Constance Schober:* The effects of wind and nonlinear damping on rogue waves and permanent downshifting
9:35 – 10:00 *Brenton J LeMesurier:* Approximately traveling wave pulses in binary exciton chain systems

9:10 – 10:00 SESSION 23, Room Y/Z: Solitons and nonlinear dispersive waves – Part III/V
Chairs: Zhen Wang, Taixi Xu, Zhenya Yan, Zhijun Qiao
9:10 – 9:35 *Zuo-Nong Zhu:* On non-integrable semi-discrete Hirota equations
9:35 – 10:00 *Matthew Russo, Roy Choudhury:* The extended Estabrook-Wahlquist method

9:10 – 10:00 SESSION 17, Room F/G: Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems – Part I/V -- *Chairs: Pavel Lushnikov, Alexander Korotkevich*
9:10 – 9:35 *Vladimir Zakharov, Dmitry Zakharov: C*noidal wave as a limit of N-solitonic solutions and a periodic one-gap potentials
9:35 – 10:00 *Gadi Fibich, Dima Shpigelman:* Necklace solitary waves on bounded domains

9:10 – 10:00 SESSION 22, Room T/U**:** Spectral methods in stability of traveling waves – Part III/III
Chairs: Yuri Latushkin, Alim Sukhtayev, Mat Johnson
9:10 – 9:35 *Stephane Lafortune, Annalisa Calini:* Orbital stability of waves traveling along vortex filaments
9:35 – 10:00 *Milena Stanislavova, Atanas Stefanov:* Instability index theory and applications

9:10 – 10:00 PAPERS, Room V/W – *Chair: Paul Christodoulides*
9:10 – 9:35 *Luiz Gustavo Farah:* On the existence of maximizers for Airy-Strichartz inequalities
9:35 – 1:00 *Paul Christodoulides, Lauranne Pellet, Frederic Dias, Lazaros Lazaris:* Interaction of waves, sea-bottom second-order pressure and microseisms

9:10 – 10:00 SESSION 27, Room J**:** Mechanisms for computations in neuronal networks – Part II/III
Chair: Andrea Barreiro
9:10 – 9:35 *Andrea Barreiro, J. Nathan Kutz, Eli Shlizerman:* Low-dimensional dynamics embedded in echo-state networks
9:35 – 10:00 *Julijana Gjorgjieva, Markus Meister, Haim Sompolinsky:* What does efficient coding theory tell us about the benefits of pathway splitting in sensory coding?

10:00 – 10:30 COFFEE BREAK

10:30 – 12:10 SESSION 7, Mahler hall: Applied nonlinear waves – Part II/III
Chairs: David Kaup, Constance Schober, Thomas Vogel
10:30 – 10:55 *Robert A. Van Gorder:* Non-local waves on quantum vortex filaments
10:55 – 11:20 *Andrei Ludu:* Nonlinear contour waves and hollow patterns, a signature of universality
11:20 – 11:45 *Thomas Vogel, D.J. Kaup:* Internally driven oceanic surface waves
11:45 – 12:10 *S. Roy Choudhury, Matthew Russo:* Building generalized Lax integrable KdV and MKdV equations with spatiotemporally varying coefficients

10:30 – 12:10 SESSION 30, Room Q: Wave phenomena in population biology and application to cancer – Part II/II
Chairs: Hassan Fathallah-Shaykh, Thierry Colin, Clair Poignard
10:30 – 10:55 *Thierry Colin, Frédéric Lagoutière, Thomas Michel, Clair Poignard:* A constructive existence proof for a coupled system of equations to model tumor growth
10:55 – 11:20 *Guillaume Lefebvre, Francois Cornelis, Patricio Cumsille, Thierry Colin, Clair Poignard, Olivier Saut:* Spatial modeling of tumor drug resistance: The case of GIST liver metastases
11:20 – 11:45 *Elizabeth Scribner, Hassan Fathallah, Thierry Colin, Olivier Saut:* Mathematical model of glioblastoma: Mechanistic insights of recurrence
11:45 – 12:10 *Thierry Colin, Olivier Gallinato, Clair Poignard, Takashi Suzuki:* Invadopodia modeling

10:30 – 12:10 SESSION 13, Room R: Recent developments in mathematical studies of water waves – Part I/II
Chair: John Carter
10:30 – 10:55 *John Carter:* Frequency downshift in a viscous fluid
10:55 – 11:20 *Diane Henderson:* Dissipation of narrow-banded, linear, deep-water waves
11:20 – 11:45 *Olga Trichtchenko:* Comparison of stability of solutions to Hamiltonian water wave models
11:45 – 12:10 *Henrik Kalisch:* On the Whitham equation

10:30 – 12:10 SESSION 31, Room Y/Z: Wave propagation in complex media – Part I/V
Chairs: D. N. Christodoulides, Z. H. Musslimani, K. G. Makris
10:30 – 10:55 *Mohammad-Ali Miri, Nicholas Nye, Demetrios Christodoulides, Hossein Hodaei:* PT-symmetric diffraction gratings
10:55 – 11:20 *Maksim Kozlov, Georgios Tsironis:* Beam dynamics in longitudinally modulated PT-symmetric lattices
11:20 – 11:45 *Tsampikos Kottos, Nicholas Bender, Mahboobeh Chitsazi, Huanan Li, Fred Ellis:* PT-symmetric Floquet lattices
11:45 – 12:10 *Mercedeh Khajavikhan, Demetrios Christodoulides, Hossein Hodaei:* PT-symmetric microring lasers

10:30 – 12:10 SESSION 17, Room F/G: Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems – Part II/V -- *Chairs: Pavel Lushnikov, Alexander Korotkevich*
10:30 – 10:55 *Gregor Kovacic, Victor Barranca, David Cai, Douglas Zhou:* Compressed coding in early sensory processing
10:55 – 11:20 *Ibrahim Fatkullin, Gleb Zhelezov:* Dynamics of blow-ups in chemotaxis-related particle system
11:20 – 11:45 *Katelyn Plaisier Leisman, Gregor Kovacic, David Cai:* Effective dispersion relation of the nonlinear Schrodinger equation
11:45 – 12:10 *Michael Schwarz, Gregor Kovacic, Peter Kramer, David Cai:* Waveaction spectrum for fully nonlinear MMT model

10:30 – 12:10 SESSION 29, Room T/U: Nonlinear Schrodinger models and applications – Part I/II
Chairs: Ricardo Carretero, Panos Kevrekidis
10:30 – 10:55 *Todd Kapitula:* Reformulating spectral problems with the Krein matrix
10:55 – 11:20 *Julia Rossi, Ricardo Carretero:* Nonconservative variational approximations for NLS: Application to symmetry - breaking bifurcation

11:20 – 11:45 *Efstathios Charalampidis, Panayotis Kevrekidis, Dimitri Frantzeskakis, Boris Malomed:* Dark-bright solitons in coupled nonlinear Schrodinger (NLS) equations with unequal dispersion coefficients

11:45 – 12:10 *Michael Jenkinson, Michael Weinstein:* On-site and off-site solitary waves of the discrete nonlinear Schrödinger equation in multiple dimensions

10:30 – 12:10 SESSION 11, Room V/W: Nonlinear waves – Part II/III
Chairs: Jerry Bona, Min Chen, Vassilios, Dimitrios Mitsotakis
10:30 – 10:55 *Ben Segal, Nghiem Nguyen, Bernard Deconinck:* Dynamics of short and long capillary-gravity waves
10:55 – 11:20 *Nghiem Nguyen, Bernard Deconinck, Ben Segal:* Some remarks on the NLS-KdV system
11:20 – 11:45 *Daulet Moldabayev, Henrik Kalisch, Olivier Verdier:* SpecTraVVave: A program package for computing traveling waves of nonlinear dispersive equations
11:45 – 12:10 *Shu-Ming Sun:* Existence of two-hump solutions for some model equations related to water waves

10:30 – 12:10 SESSION 16, Room J**:** Advances using the unified transform method – Part I/I
Chairs: Natalie Sheils, Dave Smith
10:30 – 10:55 *Darren Crowdy:* Transform methods, boundary integral methods, and applications
10:55 – 11:20 *Tom Trogdon:* Gibbs-like behavior of dispersive PDEs
11:20 – 11:45 *Natalie Sheils:* The time-dependent Schrodinger equation with piecewise constant potential
11:45 – 12:10 *David Smith:* Birkhoff regularity and well-posedness of linear initial-boundary value problems

12:10 – 1:40 LUNCH on your own

1:40 – 3:20 SESSION 15, Mahler hall: Geometric, algebraic and analytic approaches to integrable systems – Part II/II
Chair: Nalini Joshi
1:40 – 2:05 *Yang Shi:* Symmetry and combinatorics of discrete integrable system
2:05 – 2:30 *Nobutaka Nakazono:* Lax pairs of discrete Painlevé equations arising from the integer lattice
2:30 – 2:55 *Dmitry Korotkin:* Bergman tau-function and Witten classes
2:55 – 3:20 *Hiroshi Miki:* Exceptional orthogonal polynomials and extended Jacobi matrix

1:40 – 3:20 SESSION 25, Room Q: Solitons, vortices, domain walls, their dynamics and their progenitors – Part II/II
Chairs: Stephen Gustafson, Israel Michael Sigal, Avy Soff
1:40 – 2:05 *Jean Dolbeault, Maria Esteban, Ari Laptev, Michael Loss:* Flows related to one-dimensional Gagliardo-Nirenberg inequalities
2:05 – 2:30 *Sergey Cherkis:* Monopole wall dynamics
2:30 – 2:55 *Thomas Chen:* On the uniqueness and scattering for Gross-Pitaevskii hierarchies and quantum de Finetti
2:55 – 3:20 *Justin Holmer, Donlapark Pornnopparath:* Near amplitude crossing of mKdV double solitons and applications to effective dynamics of perturbed mKdV

1:40 – 3:20 SESSION 13, Room R: Recent developments in mathematical studies of water waves – Part II/II
Chair: John Carter
1:40 – 2:05 *Harvey Segur:* Oscillations in hyperasymptotic series
2:05 – 2:30 *Ruth Martin:* Toward a general solution of the three-wave resonant interaction equations
2:30 – 2:55 *Kristoffer Varholm:* Traveling water waves with point vortices
2:55 – 3:20 *Vishal Vasan:* A scalar time-dependent equation for water waves

1:40 – 3:20 SESSION 31, Room Y/Z: Wave propagation in complex media – Part II/V
Chairs: D. N. Christodoulides, Z. H. Musslimani, K. G. Makris
1:40 – 2:05 *Li Ge:* Selective excitation by active transformation optics in media with strong modal interaction
2:05 – 2:30 *Ramy El-Ganainy, Demetrios Christodoulides, Li Ge:* Singlet lasing in supersymmetric laser arrays
2:30 – 2:55 *Vassilios Kovanis:* Tunable photonic oscillators
2:55 – 3:20 *Tomas Donhal and Petr Siegl:* Nonlinear eigenvalues in a generalized PT-symmetric problem

1:40 – 3:20 SESSION 17, Room F/G: Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems – Part III/V -- *Chairs: Pavel Lushnikov, Alexander Korotkevich*
1:40 – 2:05 *Alexey Sukhinin, Alejandro Aceves, Edward Downes:* Two color filament beam interaction in air
2:05 – 2:30 *Justin Holmer, Xuwen Chen:* The derivation of 1D focusing NLS from 3D quantum many-body evolution

2:30 – 2:55 *Sergey Dyachenko, Pavel Lushnikov, Alexander Korotkevich:* Finding the Stokes wave: From low steepness to the highest wave
2:55 – 3:20 *Pavel Lushnikov, Sergey Dyachenko, Alexander Korotkevich:* Branch cut singularity of Stokes wave on deep water

1:40 – 3:20 SESSION 29, Room T/U: Nonlinear Schrodinger models and applications – Part II/II
Chairs: Ricardo Carretero, Panos Kevrekidis
1:40 – 2:05 *Daniel Spirn:* Vortex scattering across interfaces
2:05 – 2:30 *Ricardo Carretero-González, Panoyotis Kevrekidis, Theodore Kolokolnikov:* Vortex nucleation in a dissipative variant of the nonlinear Schrödinger equation under rotation
2:30 – 2:55 *Mark Edwards:* Finite temperature effects on vortex motion in ring Bose-Einstein condensates
2:55 – 3:20 *Peter Engels:* Engineering dispersion relations with a spin-orbit coupled Bose-Einstein condensate

1:40 – 3:20 SESSION 11, Room V/W: Nonlinear waves – Part III/III
Chairs: Jerry Bona, Min Chen, Vassilios, Dimitrios Mitsotakis
1:40 – 2:05 *Gennady El, Mark Hoefer, Michael Shearer:* Dispersive and diffusive-dispersive shock waves for nonconvex conservation laws
2:05 – 2:30 *Youcef Mammeri:* On vanishing nonlinear dissipative-dispersive perturbations of conservation laws
2:30 – 2:55 *Cristina Haidau:* A Study of well posedness for systems of coupled non-linear dispersive wave equations
2:55 – 3:20 *Francesco Fedele:* Hopf fibrations for turbulent pipe flows

1:40 – 2:55 SESSION 27, Room J**:** Mechanisms for computations in neuronal networks – Part III/III – *Chair: Andrea Barreiro*
1:40 – 2:05 *Michael Metzen, Maurice Chacron:* Coding natural stimuli through correlated neural activity
2:05 – 2:30 *Avinash Karamchandani, James Graham, Hermann Riecke:* Pulse-coupled mixed-mode oscillators: Rhythms and clusters in the olfactory system
2:30 – 2:55 *Woodrow Shew, Wesley Clawson, Nathaniel Wright, Yahya Karmimipanah, Jeff Pobst, Ralf Wessel:* Sensory adaptation tunes visual cortex to criticalit

3:20 - 3:50 COFFEE BREAK

3:50 – 5:55 SESSION 1, Mahler hall: Nonlinear evolution equations and integrable systems – Part IV/IV
Chairs: Alex Himonas,Dionyssis Mantzavinos
3:50 – 4:15 *Dionyssis Mantzavinos:* The “good” Boussinesq equation on the half-line
4:15 – 4:40 *Vassilis Rothos:* Adiabatic perturbation theory for vector NLS with nonvanishing boundary conditions
4:40 – 5:05 *Yusuke Shimabukuro:* Stability and well-posedness problems in the integrable systems
5:05 – 5:30 *John Holmes:* On the Cauchy problem for a Camassa-Holm type equation
5:30 – 5:55 *Alexei Rybkin:* On the Hankel operator approach to completely integrable systems

3:50 – 5:55 **SESSION 10**, Room Q**:** Mathematical modeling and physical dynamics of solitary waves: From continuum mechanics to field theory – Part II/III -- *Chairs: Ivan Christov, Michail Todorov, Sanichiro Yoshida*
3:50 – 4:15 *Philippe Trinh:* A steepest descents method for the study of water waves generated by a moving body
4:15 – 4:40 *Alina Chertock, Shumo Cui, Alexander Kurganov, Tong Wu:* The new semi-implicit Runge-Kutta methods and their applications in shallow water equations with friction terms
4:40 – 5:05 kert tamm, tanel peets, dima kartofelev: Boussinesq paradigm and negative group velocity in a material with double microstructure
5:05 – 5:30 *Sanichiro Yoshida, Masahiro Nishida, Yuki Kuroyanagi:* Deformation of solids as wave dynamics
5:30 – 5:55 *John Steinhoff, Subhashini Chitta:* Capturing diffraction using wave confinement

3:50 – 5:55 SESSION 26, Room R: Evolution equations in mathematical biology: Cellular and network processes -- Part III/VI -- *Chairs: Gregor Kovacic, Remus Osan*
3:50 – 4:15 *Remus Osan, Jie Zhang:* Effects of synaptic connectivity inhomogeneities on traveling waves of activity in integrate and fire neural networks
4:15 – 4:40 *Songting Li:* Theoretical modeling of nonlinear dendritic integration
4:40 – 5:05 *John Fricks:* Walking, sliding, and detaching: time series analysis for kinesin
5:05 – 5:30 *Ibrahim Fatkullin:* Bacterial chemotaxis and stochastic particle systems
5:30 – 5:55 *Rodica Curtu:* Perceptual alternation and neuronal dynamics in auditory streaming

3:50 – 5:55 SESSION 23, Room Y/Z: Solitons and nonlinear dispersive waves – Part IV/V
Chairs: Zhen Wang, Taixi Xu, Zhenya Yan, Zhijun Qiao
3:50 – 4:15 *Magdalena Toda:* Generalized Willmore surfaces, flow and applications
4:15 – 4:40 *Qilao Zha, Zhenchuan Zhou:* Multisolitons and rogue waves for the coupled Hirota equation
4:40 – 5:05 *Ping Liu, Bao-Qing Zeng:* Approximate similarity solutions of the modified (1+1)-dimensional displacement shallow water wave system
5:05 – 5:30 *Jiejian Mao, Kui Lv, Jianrong Yang, Xizhong Liu:* Periodic solitary waves of shear flow of barotropic atmosphere from the shallow water equations
5:30 – 5:55 *Shengqiang Tang, Lijing Qiao, Yining Liu:* Single peak soliton and periodic cusp wave of the generalized Schrodinger-Boussinesq equations

3:50 – 5:55 SESSION 18, Room F/G: Advances in integrable systems and nonlinear wave theory – Part II/III
Chairs: Peter Miller, Robert Buckingham
3:50 – 4:15 *Jonah Reeger:* Explorations of the solution space of the fourth Painleve equation
4:15 – 4:40 *Joe Gibney:* On the use of fast multipole methods in the numerical solution of Riemann Hilbert problems
4:40 – 5:05 *Alex Tovbis:* Formation of rogue waves in the small dispersion limit of the focusing NLS
5:05 – 5:30 *Anton Dzhamay:* Higher-rank Schlesinger transformations and difference Painleve equations

3:50 – 5:30 SESSION 19, Room T/U: Modeling, geometry, integrability, and analysis of nonlinear (dispersive) waves – Part II/II -- *Chairs: Stephen Anco, Robin Ming Chen, Yue Liu, Changzheng Qu*
3:50 – 4:15 *J. Douglas Wright:* Approximation of polyatomic FPU lattices by KdV equations
4:15 – 4:40 *Samuel Walsh, Robin Ming Chen:* Continuous dependence on the density for steady stratified water waves
4:40 – 5:05 *Jing Kang, Xiaochuan Liu, Peter Olver, Changzheng Qu:* Liouville correspondence between the modified KdV hierarchy and its dual integrable hierarchy
5:05 – 5:30 *Changzheng Qu, Feiyao Ma, Yue Liu:* Wave-breaking phenomena for the nonlocal Whitham-type equations

3:50 – 5:55 SESSION 28, Room V/W: Analytical and computational techniques for differential and difference equations – Part I/I -- *Chairs: Unal Goktas, Muhammad Usman,Willy Hereman*
3:50 – 4:15 *Barbara Prinari:* Inverse scattering transform for nonlinear Schrödinger equations with non-zero boundary conditions
4:15 – 4:40 *Ekaterina Shemyakova:* Darboux transformations of type I as generalization of Laplace-Darboux transformations method to operators of higher orders
4:40 – 5:05 *Willy Hereman:* Application of the Euler and homotopy operators to integrability testing
5:05 – 5:30 *Unal Goktas:* Application of the simplified Hirota's (homogenization) method to a (3+1)-dimensional evolution equation for deriving multiple soliton solutions
5:30 – 5:55 *Muhammad Usman:* A Computational study of the forced Korteweg-de Vries type equation

5:00 – 7:00 POSTERS: Atrium, outside Mahler hall

Pamela B. Fuller, Janet Best, Selenne Banuelos, Gemma Huguet, Alicia Prieto-Langarica, Markus H. Schmidt, Shelby Wilson: Effects of Thermoregulation on Human Sleep Patterns
Ioannis Mylonas, Vassilis Rothos, Panayotis Kevrekidis, Dimitris Frantzeskakis: Dynamics of soliton parameters in coupled NLS equations
Michelle Osborne, Tommy McDowell: Numerical studies of the KP line solitons
Alex Govan, John Carter: A higher-order generalization of the dissipative NLS equation
Matthew Russo: Traveling wave solutions for some generalized Camassa-Holm equations
Vladimir Gerdjikov, Michail Todorov, Assen Kyuldjiev: Modeling Manakov soliton trains: Effects of external potentials and inter-channel interactions

7:00- 9:00 BANQUET
Speaker: TBA
Thiab Taha: Presentation of best Student Paper Awards

FRIDAY, APRIL 03, 2015

7:30 – 9:30 REGISTRATION
8:00 – 9:00 KEYNOTE LECTURE 2, Mahler
Sijue Wu: On 2-d gravity water waves with angled crests
Chair: Jerry Bona

9:10 – 10:00 SESSION 2, Mahler hall: Nonlinear waves, patterns and vortices on compact fluid surfaces -- Part I/II
Chair: Andrei Ludu
9:10 – 9:35 *Natsuhiko Yoshinaga:* Self-propulsion of a droplet: motility and deformation
9:35 – 10:00 *Alexey Snezhko:* Self-assembled patterns and emerging dynamics in active magnetic suspensions at liquid interfaces

9:10 – 10:00 SESSION 3, Room Q: Geometric techniques in the analysis of traveling waves -- Part I/II
Chairs: Anna Ghazaryan, Stephane Lafortune
9:10 – 9:35 *Nicholas Ercolani, Nikola Kamburov, Joceline Lega:* How defects are born
9:35 – 10:00 *Mathew Johnson, Vera Hur:* Nondegeneracy and stability of periodic traveling waves in fractional KdV equations

9:10 – 10:00 PAPERS, Room R – *Chair: Nalini Joshi*
9:10 – 9:35 *Christopher Lustri, Nalini Joshi:* Stokes phenomena in discrete Painleve I
9:35 – 10:00 *Jose Vega-Guzman, Sergei Suslov, Sergey Kryuchkov:* The minimum uncertainty squeezed states for quantum harmonic oscillators

9:10 – 10:00 SESSION 26, Room Y/Z: Evolution equations in mathematical biology: Cellular and network processes -- Part IV/VI -- *Chairs: Gregor Kovacic, Remus Osan*
9:10 – 9:35 *Samuel Isaacson:* Particle-based stochastic reaction-diffusion equations for modeling cellular processes
9:35 – 10:00 *Kim Fessel:* The effect of radiation-induced dedifferentiation on cancer cell lineage

9:10 – 10:00 SESSION 7, Room F/G: Applied nonlinear waves – Part III/III
Chairs: David Kaup, Constance Schober, Thomas Vogel
9:10 – 9:35 *Wen-Xiu Ma:* Lump solutions to nonlinear integrable equations
9:35 – 10:00 *D.J. Kaup, G. T. Adamashvili, A. Knorr:* Maxwell-Bloch with multiple types of atoms

9:10 – 10:00 PAPERS, Room T/U – *Chair: Jerry Bona*
9:10 – 9:35 *David Ambrose, Jerry Bona, David Nicholls:* Ill-posedness of some water wave models
9:35 – 10:00 *Fabien Marche, David Lannes:* Numerical approximation of some new John Steinhoff classes of Green-Naghdi equations

9:10 – 10:00 SESSION 24, Room V/W: Long time dynamics of nonlinear dispersive waves – Part I/III
Chairs: Zhiwu Lin, Zaher Hani, Chongchun Zeng
9:10 – 9:35 *Stephen Gustafson:* The energy critical limit of ground state soliton
9:35 – 10:00 *Tuoc Phan:* Asymptotic stability of solitary waves in generalized Gross-Neveu model

10:00 – 10:30 COFFEE BREAK

10:30 – 12:10 SESSION 10, Mahler hall: Mathematical modeling and physical dynamics of solitary waves: From continuum mechanics to field theory – Part III/III -- *Chairs: Ivan Christov, Michail Todorov, Sanichiro Yoshida*
10:30 – 10:55 *Pedro Jordan:* Beyond Euler and Navier--Stokes: New theories of compressible flow
10:55 – 11:20 *Tanel Peets, Kert Tamm, Jüri Engelbrecht:* Mechanical waves in biomembranes
11:20 – 11:45 *Stephen Pankavich, Christian Parkinson:* Modeling the spatially heterogeneous dynamics of HIV in-vivo
11:45 – 12:10 *Michail Todorov, Vladimir Gerdjikov, Assen Kyuldjiev:* On N-soliton interactions of Gross-Pitaevski equation in two space-time dimensions

10:30 – 12:10 SESSION 3, Room Q: Geometric techniques in the analysis of traveling waves -- Part II/II
Chairs: Anna Ghazaryan, Stephane Lafortune
10:30 – 10:55 *Jeffrey Humpherys:* Balancing numerics and analysis in Evans function techniques

10:55 – 11:20 *Blake Barker, Jeffrey Humpherys, Gregory Lyng, Kevin Zumbrun:* Viscous hyperstabilization of detonation waves

11:20 – 11:45 *Anna Ghazaryan, Stephane Lafortune, Peter McLarnan:* Stability analysis for combustion fronts traveling in hydraulically resistant porous media

10:30 – 12:10 SESSION 31, Room R: Wave propagation in complex media – Part III/V
Chairs: D. N. Christodoulides, Z. H. Musslimani, K. G. Makris

10:30 – 10:55 *Liang Feng***:** Nanoscale light manipulation by parity-time symmetry

10:55 – 11:20 *Hossein Hodaei, Mohammad-Ali Miri, Demetrios Christodoulides, Mercedeh Khajavikhan:* On-chip PT-symmetric microring lasers

11:20 – 11:45 *Eva-Maria Graefe:* Wave packet evolution for non-Hermitian quantum systems in the semiclassical limit

11:45 – 12:10 *Petra Ruth Kapralova, Nimrod Moiseyev:* Time-asymmetric excitation of resonances by chirped laser pulses

10:30 – 12:10 SESSION 26, Room Y/Z: Evolution equations in mathematical biology: Cellular and network processes -- Part V/VI **–** *Chairs: Gregor Kovacic, Remus Osan*

10:30 – 10:55 *Monica Hurdal:* Pattern formation of folds in the brain

10:55 – 11:20 *Duane Nykamp:* A computational model of the influence of depolarization block on initiation of seizure-like activity

11:20 – 11:45 *Matthew Johnston:* Exploiting network structure in models of biochemical reaction systems

11:45 – 12:10 *Casian Pantea:* Multistationarity in chemical reaction networks: a parameter-free approach

10:30 – 12:10 SESSION 17, Room F/G, Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems -- Part VI/V -- *Chairs: Pavel Lushnikov, Alexander Korotkevich*

10:30 – 10:55 *Israel Michael Sigal:* Dynamics and singularity formation under the mean curvature flow

10:55 – 11:20 *Taras Lakoba, Yanzhi Zhang:* Long-time numerical integration of the generalized nonlinear Schrodinger equation with time steps exceeding the instability threshold

11:20 – 11:45 *David Cai:* New perspectives of wave turbulence

11:45 – 12:10 *Shaokang Wang, Curtis R. Menyuk:* Stability of modelocked lasers with slow saturable absorbers

10:30 – 12:10 SESSION 21, Room T/U: Functional Analysis and PDEs – Part I/II -- *Chair: Xu Runzhang*

10:30 – 10:55 *Hongtao Yang:* Global and blowup solutions for general Lotka-Volterra systems

10:55 – 11:20 *Weining Wu:* LSH-based active strategy for visual categories learning

11:20 – 11:45 *Weipeng Wu:* A method of image binarization for bubble detection

11:45 – 12:10 *Yue Wang:* The application of Kalman filter in dynamic obstacle perception technology for AUV

10:30 – 12:10 SESSION 9, Room V/W: Fully nonlinear Boussinesq models: Theory and practice -- Part I/I
Chairs: Henrik Kalisch, Mathieu Colin

10:30 – 10:55 *Stevan Bellec, Mathieu Colin:* Solitary waves for Boussinesq type systems

10:55 – 11:20 *Zahra Khorsand, Henrik Kalisch, Sergey Gavrilyuk, Dimitrios Mitsotakis:* Particle paths and conservation laws in the Serre system

11:20 – 11:45 *Maria Kazolea, Argiris Delis:* An unstructured finite volume numerical scheme for extended Bouusinesq-type equations for irregular wave propagation

11:45 – 12:10 *Volker Roeber, Jeremy D. Bricker:* Boussinesq-type modeling of tsunami-like bores generated from storm waves

12:10 – 1:40 LUNCH on your own

1:40 – 3:20 TUTORIAL, Mahler hall
Richard Moore, Tobias Schaefer: Using minimum action paths to compute rare event probabilities in wave equations

1:40 – 3:20 SESSION 4, Room Q: Discrete and ultra-discrete integrable systems and Painlevé equations- Part III/III
Chairs: Anton Dzhamay, Sarah Lobb, Tomoyuki Takenawa

1:40 – 2:05 *Tomoyuki Takenawa:* Ultra-discrete KdV equation with the periodic boundary condition

2:05 – 2:30 *Masataka Kanki:* Integrability criterion for discrete equations using the property of co-primeness

2:30 – 2:55 *Teruhisa Tsuda:* Hermite-Pade approximation, isomonodromic deformation and hypergeometric integral

2:55 – 3:20 *Robert Buckingham:* Large-degree asymptotics of rational Painlevé functions

1:40 – 3:20 SESSION 31, Room R: Wave propagation in complex media – Part IV/V
Chairs: D. N. Christodoulides, Z. H. Musslimani, K. G. Makris
1:40 – 2:05 *Manas Kulkarni, Herbert Spohn, David Huse:* Fluctuating hydrodynamics for a discrete nonlinear Schrodinger equation: mapping to Kardar-Parisi-Zhang universality class
2:05 – 2:30 *Absar Hassan, Nicholas Nye, Demetrios Christodoulides,* Ultra sensitivity using higher order exceptional points
2:30 – 2:55 *Konstantinos Makris:* Singular amplification and constant intensity waves in non-hermitian photonics
2:55 – 3:20 *Mohammad-Ali Miri, Hossein Hodaei:* Supersymmetry in optics

1:40 – 3:20 SESSION 26, Room Y/Z: Evolution equations in mathematical biology: Cellular and network processes -- Part VI/VI -- *Chairs: Gregor Kovacic, Remus Osan*
1:40 – 2:05 *Juan B. Gutierrez:* Hemodynamic model of malaria infection with detailed immune response
2:05 – 2:30 *Deena Schmidt:* Simultaneous node and edge coarsening for stochastic processes on graphs with applications to ion channel models
2:30 – 2:55 *Christina Lee:* Wave patterns in an excitable neuronal network
2:55 – 3:20 *Christel Hohenegger, Scott McKinley:* Fluctuating hydrodynamics of microparticles in biological fluids

1:40 – 3:20 SESSION 17, Room F/G, Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems -- Part V/V
Chairs: Pavel Lushnikov, Alexander Korotkevich
1:40 – 2:05 *Natalia Vladimirova, Gregory Falkovich:* Cascades in nonlocal turbulence
2:05 – 2:30 *Katie Newhall, Eric Vanden-Eijnden:* Low-damping transition times in a ferromagnetic model system
2:30 – 2:55 *Denis Silantyev, Pavel Lushnikov:* Subtracting complex singularity from Stokes wave
2:55 – 3:20 *Alexander Korotkevich, Sergei Lukaschuk:* Circular instability of a standing wave: numerical simulation and wave tank experiment

1:40 – 3:20 SESSION 21, Room T/U: Functional Analysis and PDEs – Part II/II – *Chair: Xu Runzhang*
1:40 – 2:05 *Joshua Du:* Kelvin Helmholtz instability and acoustic waves in twin-jet flow
2:05 – 2:30 *Shubin Wang:* The Cauchy problem for generalized Benney-Luke equation
2:30 – 2:55 *Runzhang Xu:* Finite time blow-up and global solutions for a semilinear parabolic equation with linear dynamical boundary conditions
2:55 – 3:20 *Zhengsheng Xu:* Global well-posedness of semilinear hyperbolic equations

1:40 – 3:20 SESSION 24, Room V/W: Long time dynamics of nonlinear dispersive waves – Part II/III
Chairs: Zhiwu Lin, Zaher Hani, Chongchun Zeng
1:40 – 2:05 *Shijun Zheng:* Recent progress on global regularity of rotating BE
2:05 – 2:30 *Geng Chen:* Uniqueness of conservative solution for Camassa-Holm and variational wave equations
2:30 – 2:55 *Timur Akhunov:* When is it possible to have wellposedness of the fully non-linear KdV equation without resorting to weighted spaces?
2:55 – 3:20 *Nathan Totz:* Global well-posedness for the hyperbolic nonlinear Schrodinger equation on subcritical Sobolev spaces

3:20 - 3:50 COFFEE BREAK

3:50 – 5:55 SESSION 2, Mahler hall: Nonlinear waves, patterns and vortices on compact fluid surfaces -- Part II/II
Chair: Andrei Ludu
3:50 – 4:15 *Timothy Smith:* A use of regression analysis to address the validity of the nonlinear stochastic differential equation solution of Black-Scholes model applied to the SandP 500 index
4:15 – 4:40 *Carlos Mejillones:* Nonlinear waves generated in water by a towing system
4:40 – 5:05 *Hannah Scott:* Nonlinear aerodynamics analysis of a flapping and bending flexible flag in air flow
5:05 – 5:30 *Eric Rickard:* Model for nonlinear energy resonant damping for a payload impact to the ground

3:50 – 5:55 SESSION 23, Room Q: Solitons and nonlinear dispersive waves – Part V/V
Chairs: Zhen Wang, Taixi Xu, Zhenya Yan, Zhijun Qiao
3:50 – 4:15 *Zhenya Yan:* Localized waves in nonlinear Schrodinger equations with external potentials and nonlinearities
4:15– 4:40 *Haicheng Gu:* An analysis of financial data with wavelets and fractal functions
4:40 – 5:05 *Jianrong Yang, Kui Lv, Jiejian Mao, Xizhong Liu:* Dust acoustic waves in magnetized dense plasmas with dust

neutral collisions
5:05 – 5:30 *Zhen Wang, Li Zou, Zhi Zong:* Nonlinear steady two layer interfacial flow about a submerged point vortex
5:30 – 5:55 *Christel Hohenegger, Scott McKinley:* Fluctuating hydrodynamics of microparticles in biological fluids

3:50 – 5:55 SESSION 31, Room R: Wave propagation in complex media – Part V/V
Chairs: D. N. Christodoulides, Z. H. Musslimani, K. G. Makris
3:50 – 4:15 *Stelios Tzortzakis:* Linear and nonlinear propagation effects in periodic and random photonic lattices
4:15 – 4:40 *Nikolaos Efremidis:* Accelerating and abruptly autofocusing waves
4:40 – 5:05 *Zhigang Chen, Demetrios Christodoulides:* Deep penetration of light needles through soft-matter nanosuspensions
5:05 – 5:30 *Ido Kaminer:* Controlling the radiation and lifetime of relativistic particles by shaping their wave packets
5:30 – 5:55 *Alexey Yamilov, Pauf Neupane, Raktim Sarma, Hui Cao:* Control of light transport via non-local wave interference effects in random media

3:50 – 5:55 SESSION 20, Room Y/Z: Numerical simulations for solving nonlinear wave equations – Part I/I
Chair: Thiab Taha
3:50 – 4:15 *Runchang Lin, Weijia Zhao:* A discontinuous Galerkin least-squares finite element-finite difference simulation of the Nagumo equation
4:15 – 4:40 *Wei Gao:* A method of 3-D seabed terrain generation based on data fusion theory
4:40 – 5:05 *Hossein Taheri, Ali Asghar Eftekhar, Kurt Wiesenfeld, Ali Adibi:* Creation and manipulation of temporal solitons in optical microresonators by pump phase modulation

3:50 – 5:55 SESSION 18, Room F/G**:** Advances in integrable systems and nonlinear wave theory – Part III/III
Chairs: Peter Miller, Robert Buckingham
3:50 – 4:15 *Al Osborne:* Soliton and breather turbulence in ocean surface waves
4:15 – 4:40 *Bingying (Luby) Lu:* A close-up look at the Semiclassical sine-Gordon equation
4:40 – 5:05 *Michael Music:* Global solutions for the zero-energy Novikov-Veselov equation
5:05 – 5:30 *Seung-Yeop Lee:* Orthogonal polynomials in normal random matrices: higher order corrections
5:30 – 5:55 *Alfredo Wetzel:* Direct scattering and small dispersion for the Benjamin-Ono equation with rational initial data

3:50 – 5:55 SESSION 24, Room V/W: Long Time Dynamics of Nonlinear Dispersive Waves – Part III/III
Chairs: Zhiwu Lin, Zaher Hani, Chongchun Zeng
3:50 – 4:15 *Mihai Tohaneanu:* Pointwise decay for the Maxwell system on black holes
4:15 – 4:40 *Gang Zhou:* A resonance problem in the setting of relaxation of ground states of nonlinear Schrodinger equations
4:40 – 5:05 *Shihui Zhu:* Stability and instability of standing waves for the nonlinear fractional Schrodinger equation
5:05 – 5:30 *Benoit Pausader:* On the mass-less limit in the Euler-Poisson system

KEYNOTE ABSTRACTS

Quasilinear dynamics arising in a 3-D fluid structure interaction with moving frame

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Abstract. Equations of fluid structure interactions are described by Navier Stokes equations coupled to a 3-d dynamic system of elasticity. The coupling is on a free boundary interface between the two regions: the fluid and oscillating structure. The interface is moving with the velocity of the flow. The resulting model is a quasilinear system with parabolic-hyperbolic coupling acting on a moving boundary. One of the main features and difficulty in handling the problem is a mismatch of regularity between parabolic and hyperbolic dynamics. The existence and uniqueness of smooth local solutions has been established by D. Coutand and S. Shkoller *Arch. Rational Mechanics and Analysis* in 2005. Other local wellposedness results with a decreased amount of necessary smoothness have been proved in a series of papers by I. Kukavica, A. Tuffaha and M. Ziane. The main contribution of the present paper is *global* existence of smooth solutions. This is accomplished by exploiting either internal or boundary viscous damping occurring in the model [1]. The mismatch of regularity between hyperbolocity and parabolicity is handled be exploiting recently established sharp regularity of the "Dirichlet-Neuman " map for hyperbolic solvers along with maximal parabolic regularity available for Stokes operators. Maximal parabolic regularity allows to determine the pressure from finite energy solutions of the Stokes problem, while sharp trace regularity allows to propagate the boundary traces through the interface without a loss of regularity (standard functional analysis trace theorems in L_p scales provide a loss of $1/p$ of derivative when restricting a function to the boundary). These two ingredients prevent a typical "loss" of derivatives and allow for the algebraic and topological closure of a suitably constructed fixed point.

This work is joint with M. Ignatova (Stanford University), I. Kukavica (University of Southern California, Los Angeles) and A. Tuffaha (The Petroleum Institute, Abu Dhabi, UAE).

References

"On well-posedness and small data global existence for an interface damped free boundary fluid-structure interaction model", with. M. Ignatova, I. Kukavica and A. Tuffaha – Nonlinearity, vol. 27, issue 3, pp. 467-499, 2014

Nonlinear waves in granular crystals: Theory, computation, experiments and applications

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Abstract. In this talk, we will provide an overview of results in the setting of granular crystals, consisting of beads interacting through Hertzian contacts. We will start from the simplest setting of one-dimensional, monoatomic chains where highly localized traveling waves exist and we will also examine states in the form of (dark) discrete breathers and shock waves therein. Wherever possible, we will corroborate these considerations with recent experimental results. We will then extend our considerations to the case of diatomic chains and examine how the properties of traveling waves and also of discrete breathers are modified in the latter setting. More highly heterogeneous chains will be briefly examined as well. In addition to considering the purely Hamiltonian case, select examples of the damped-driven variant of the system and its rich phenomenology, including chaotic response and bistability/hysteresis will also be shown. In the last part of the talk, time-permitting, a number of recent aspects will be touched upon including:

- formation of traveling waves with non-vanishing tails in elastic woodpile periodic structures;
- super-diffusive transport in disordered granular chains;
- applications of these lattices for the demonstration of switching and acoustic logic gates;
- prototypical examples of extensions to two dimensions in hexagonal, as well as square arrays.

On 2D gravity water waves with angled crests

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Abstract. In this talk, I will present our recent work on 2-D gravity water waves. We construct an energy functional E, that allows for angled crests in the interface, and we show that for any smooth initial data, the unique solution of the 2d water wave systems remains smooth so long as E remains finite, and we show that for any initial data satisfying $E(0) < \infty$, there is $T > 0$, depending only on $E(0)$, such that the water wave system is solvable for time $t \in [0, T]$.

TUTORIAL ABSTRACT

Using minimum action paths to compute rare event probabilities in wave equations

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Abstract. The importance of rare events has recently been demonstrated in a number of physical systems exhibiting wave behavior, including lasers, optical communication lines, rogue waves in the ocean, and models of turbulence. At the same time, newly developed numerical methods in combination with high-performance parallel computing platforms have made direct numerical simulations of these rare events accessible.

The goal of this tutorial is to present efficient computational methods to characterize the tails of probability distributions in such complex stochastic systems in the small-noise limit. We will discuss the mathematical foundations of the Freidlin-Wentzell theory of large deviations and its relationship to instantons of path integral representations. It will turn out that the probabilities of rare events are related to minimizers of the Freidlin-Wentzell action functional. We will discuss recently developed methods to compute these minimizers efficiently and show how to implement these methods in MATLAB. In particular, we will focus on two cases: (a) the transition between stable fixed points and (b) the computation of expectations related to exit problems. We will illustrate the application and implementation of the methods for concrete examples in the context of laser optics (using a stochastic complex Ginzburg-Landau equation) and turbulence (using a stochastic Burgers equation).

We will present our material as a tutorial to allow new researchers to get started in this area. No prior knowledge of the field is required, but familiarity with stochastic differential equations and basic knowledge of MATLAB are useful.

SESSION ABSTRACTS

SESSION 1

Nonlinear evolution equations and integrable systems

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Abstract. Nonlinear evolution equations have been at the forefront of advances in partial differential equations for a long time. They are involved in beautiful, yet extremely challenging problems, with a strong physical background, for which progress is achieved through a mixture of techniques lying at the interface between analysis and integrable systems. Topics studied for these equations include, among others, local and global well-posedness, inverse scattering, stability, integrability and travelling waves.

SESSION 2

Nonlinear waves, patterns and vortices on compact fluid surfaces

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Abstract. This session will focus on pattern formation and nonlinear waves in closed fluid contours or surfaces. Pattern formation in such lower dimensional systems occurs in many problems of physics because in this case the boundary induces strong effects in the dynamics of the inside by breaking the continuous symmetries of the unbounded system. In such systems the patterns generated by instabilities can develop in spontaneous motility and propulsion, rotating cnoidal waves, diagonal waves, nonlinear resonant energy damping, etc.

Among the topics to be discussed we mention self-propulsion of droplets, droplet deformation, parametric instabilities of rotating or vibrating flows, self-assembled patterns and emerging dynamics in active magnetic suspensions at liquid interfaces, flag waves. In addition, the session hosts talks related to waves generated in confined basins by ultra-fast towing systems, nonlinear stochastic differential equations and studies on Camassa-Holm types of equations. This session will host a combination of talks from of exact theoretical results, numerical computations and experimental work.

SESSION 3

Geometric techniques in the analysis of traveling waves

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Abstract. The general purpose of this special session is to bring together researchers working on various issues relating to traveling wave solutions of partial differential equations arising in physics, chemistry, and biology. More specifically, the problems of existence and stability of those solutions will be discussed. The session will place a special emphasis on the methods of geometric singular perturbation theory for the existence analysis and the Evans function for the stability analysis.

SESSION 4

Discrete and ultra-discrete integrable systems and Painlevé equations

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Abstract. Over the last twenty years there have been major advances in our understanding of the theory of discrete integrable systems and Painlevé equations. The goal of this special session is to bring together active researchers in these areas to discuss some recent advances in the field and connections to various other areas, such as cluster algebras, tropical geometry, nonlinear equations, as well as some diverse applications.

Among the main themes of the session are the geometric aspects of integrable lattice equations and their reductions, generalizations of discrete integrable systems, the theory and applications of continuous and discrete Painlevé equations, and the theory and applications of ultra-discrete integrable systems.

SESSION 5

Applications of continuous and discrete integrable systems

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Abstract. This session has two broad goals: discussing a number of diverse applications in which soliton equations, both continuous and discrete, arise naturally; and presenting theoretical and numerical approaches for studying key features of the ensuing models.

Applications range from fluid dynamics, including models of shallow water waves and vortex structures, to optical systems and cosmology, to graph enumeration and geometry. Theoretical approaches encompass analytical techniques for determining existence and stability of solitary waves, combinatorial tools, and algebraic methods. The numerical aspects include integrable discretisations leading to self-adaptive schemes and the use of Graphical Processing Units for studying cosmological bubbles.

SESSION 6

Graphene lattices: Phenomena and analysis

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Yiping Ma
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Yi Zhu
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Abstract. Over the last several years, physical problems involving graphene, or honeycomb, lattices have become increasingly important and exciting systems in many different contexts. In an optical context, this has allowed for the development of simple experimental environments which mimic complex behavior in condensed matter physics that is traditionally difficult to probe. This work has recently culminated in the discovery of topologically protected edge modes in the visible range, which is an optical analog of the famous Quantum Hall effect. Likewise, researchers in Bose-Einstein condensation have begun to explore the impact of honeycomb lattices on condensates. New theory predicts phenomena usually only seen in quantum field theory but at speeds two orders of magnitude slower than light. Preliminary experiments have begun to confirm these predictions. Thus in both subjects, the poten-

tial to study complex physical problems in relatively simple settings has been opened.

This session will present work on these new and exciting topics with focus on both experimental and modeling advances. Topics covered will be the experimental establishment of pseudo-spin, the analysis of edge states, and the role that nonlinearity plays in affecting their dynamics, the effect of lattice deformations, the existence and stability of nonlinear modes, and the impact of parity/time symmetric perturbations.

SESSION 7

Applied nonlinear waves

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Abstract. This session covers several applications of nonlinear waves and various types of nonlinear waves. Included are effects of wind and damping on rogue waves, surface waves driven by internal waves, nonlocal waves on quantum vortex filaments, and universality of nonlinear contour waves. In addition there are approximation methods for traveling wave pulses, self-induced transparency with multiple atoms, integrable generalizations of integrable equations as well as how to construct integrable lump solutions for several equations.

SESSION 9

Fully nonlinear Boussinesq models: Theory and practice

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Abstract. In coastal engineering, the description of the motion of the free surface and the evolution of the velocity field of an ideal, incompressible and irrotational fluid under the gravity is described by the Euler equations, especially in cases when surface tension and dissipative effects can be neglected. However, in many physical situations, the Euler equations appear too complex, and cannot be solved in a way efficient enough to be useful in practical modeling situations, and many features of the flow are well described by simplified model equations.

Consequently, many asymptotic models have been introduced, fitting restricted physical regimes. In this direction, significant effort has been made during the last two decades to develop systems of depth averaged equations which correctly reproduce the dispersion characteristic of wave propagation in the near-shore region such as for example Boussinesq models.

The aim of this session is to review state of the art on high-order and fully non-linear Boussinesq-type models from a theoretical point of view as well as for numerical purposes. We are particularly interested in the near-shore region, in which many interesting physical phenomena occur. The focus will be on the use of hybrid models, breaking criteria, and different approaches to switching between Boussinesq and shallow-water equations, as well as differences between hyperbolic and conservative approaches for numerical treatment, and on exact solutions such as solitary waves and corresponding stability criteria.

SESSION 10

Mathematical modeling and physical dynamics of solitary waves: From continuum mechanics to field theory

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Abstract. This session is of an interdisciplinary nature and the papers presented employ a diverse array of methods towards the analysis of nonlinear wave propagation in various physical contexts. A blend of analytical, numerical and experimental work will be showcased.

Through a breadth of talks covering topics ranging from fluid mechanics (e.g., compressible flows, acoustics, waves in fluids, etc.), solid mechanics (e.g., granular materials, elasticity and plasticity, microstructure, biomembranes, etc.), condensed matter physics (e.g., Bose–Einstein condensates), optics (e.g., mode-coupled pulses, polarization effects, etc.) and networks (e.g., localized waves at junctions), this session exemplifies how field theoretic mathematical modeling of physical phenomena leads to challenging problems that often give rise to nonlinear wave dynamics (e.g., solitary waves, shock waves, etc.), which can be interrogated through advanced mathematical techniques and careful numerical simulations taking into account separation of scales, long-time integration requirements, etc.

In addition, the session includes a number of exceptional student and young-career speakers.

SESSION 11

Nonlinear waves

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Abstract. This session is focused on the propagation of waves in water and other media where nonlinearity, dispersion and sometimes dissipation and capillarity are all acting. Featured in the session will be theoretical work about well-posedness, stability, existence of solitary waves, asymptotics and numerical simulations of solutions of various nonlinear, dispersive systems and applications to fluid mechanics and problems in geophysics.

SESSION 12

Mathematical progress on nonlinear phenomena in parity-time-symmetric systems

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Abstract. The study of parity-time (PT) symmetric systems is currently a research frontier in physics (such as nonlinear optics) and applied mathematics. PT-symmetric systems contain balanced gain and loss. Though being dissipative, they exhibit an array of novel phenomena which resemble those of conservative systems (such as all-real spectra). This renders PT-symmetric systems intriguing alternatives to standard quantum mechanical ones, defying the postulate of Hermiticity. The interplay of PT symmetry and nonlinearity gives rise to more surprising properties which are currently under intensive investigation. In this minisymposium, the latest important developments in this multidisciplinary research frontier will be highlighted, with an emphasis on the mathematical/theoretical results.

SESSION 13

Recent developments in mathematical studies of water waves

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Abstract. This minisymposium brings together mathematicians, engineers, and oceanographers. With a focus on nonlinear water waves, the speakers in this session will present experimental, analytical, and numerical results from mathematical models of waves on shallow and/or deep water.

SESSION 14

Coupled systems with hyperbolic components

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Abstract. This session will be devoted to the advances in the study of distributed parameter evolution equations arising in engineering and physical sciences. The focus of the discussion will be on coupled models, especially those wherein one of the dynamical components is either of a hyperbolic characteristic, or exhibits some hyperbolic-like properties. Possible topics for exploration include: fluid-structure interaction problems, free and moving boundaries, structural acoustics, thermo/magneto-elasticity. Questions concerning modeling, well-posedness, stability, control, optimization and numerical simulations will be of primary interest.

SESSION 15

Geometric, algebraic and analytic approaches to integrable systems

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Abstract. Integrable systems in mathematics appear in different contexts and theories, including the classical Hamiltonian dynamics, solitons, and discrete systems. Current developments in each of these branches show intriguing interplay with geometry, algebraic geometry, symplectic topology, quantum field theory and other areas of mathematics and physics. In this session, we are bringing together researchers working with various aspects of integrable systems with purpose of intensifying the exchange of experience, methods and ideas.

SESSION 16

Advances using the unified transform method

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Abstract. In the last decade the use of a new method for solving boundary value problems due to Fokas has been expanded by him, his collaborators, and others. This unified transform method contains the classical solution methods as special cases and allows for the explicit solution of problems which could not previously be solved. This session will bring together practitioners of the Unified Transform (Fokas) Method and expose interested parties to the many applications of this technique.

SESSION 17

Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems

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Abstract. Waves and formation of singularities are important problems in many physical, hydrodynamical and biological systems as well as these are important phenomena for the applied mathematics in general. We encounter waves in all areas of our everyday lives, from waves on the surface of a lake or a pool and sound waves to the electromagnetic waves propagation in ionosphere and plasma excitations on the sun. Waves of finite amplitude require solutions beyond linear approximation by taking into account nonlinear effects. Solutions of nonlinear equations usually result in the formation of singularities, coherent structures or solitary waves. Examples of the corresponding phenomena can be observed in filamentation of laser beams in nonlinear media, wave breaking in hydrodynamics and aggregation of bacterial colonies. Leading order nonlinear equations loose their applicability near formation of singularities resulting qualitative changes in the underlying nonlinear phenomena. Then new mechanisms become important such as dissipation, formation of plasma, excluded volume constrains and many other problem-dependent mechanisms of the regularization of singularities. The minisymposium is devoted to new advances in the theory of nonlinear waves and singularities demonstrating vividly the similarity of approaches in a broad spectrum of applications.

SESSION 18

Advances in integrable systems and nonlinear wave theory

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Abstract. The study of integrable wave equations has had a large impact on the analysis of topics ranging from random matrices to ordinary differential equations to particle systems and beyond. In return, progress made in these related areas has helped lead to new results for nonlinear wave equations. This session will present recent results from a variety of topics related to integrability with the goal of encouraging the further exchange of ideas between different areas.

SESSION 19

Modeling, geometry, integrability, and analysis of nonlinear (dispersive) waves

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Abstract. Nonlinear dispersive partial differential equations appear in many fields, including fluid mechanics, differential geometry, plasma physics, and optics, where they describe wave phenomenon such as solitons, wave collapse and turbulence. Despite all the important work done on these equations, our mathematical understanding of them is still incomplete. This session will bring together researchers at all career stages to share their recent results on various aspects in nonlinear dispersive wave equations. It will focus on (but not be restricted to) developments connected with physical modeling, integrability structure, well-posedness, stability analysis, and geometric aspects.

SESSION 20

Numerical simulations for solving nonlinear wave equations

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Abstract. In this session, different numerical methods for solving nonlinear wave equations will be presented. Furthermore, the properties of these numerical methods will be discussed.

SESSION 21

Functional analysis and PDEs

Xu Runzhang

Abstract. This session mainly focuses on the functional analysis especially the potential well methods or variational methods and its applications to the partial differential equations especially the nonlinear wave equations.

SESSION 22

Spectral methods in stability of traveling waves

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Abstract. The purpose of this special session is to bring together researchers working on various stability issues for such special solutions of partial differential equations as periodic and solitary waves. All aspects of stability/instability will be discussed, from spectral to nonlinear, with special emphasis on methods of spectral theory. It is expected that the speakers will spend some time of their talks to address possible perspectives in the field of their work as we believe that such a perspective would be not only interesting for the audience but it can also stimulate further discussion and further research in the field.

SESSION 23

Solitons and nonlinear dispersive waves

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Abstract. Integrable evolutionary differential equations were discovered at the end of the 60's in the study of dispersive waves. Mathematical progress of the theory of integrable systems was related to Riemann-Hilbert boundary value problems, to algebraic and differential geometry, to infinite dimensional Lie algebras, to algebraic geometry, and so on.

Recently, there has been growing interest in the algebraic, geometric, solitons, non-smooth solution (e.g. peakon) aspects of continuous and discrete integrable systems. Integrable systems turn up in a wide variety of areas and have an extensive range of applications: mathematical physics, numerical analysis, computer science, mathematical biology, statistical physics, the theory of special functions, asymptotic analysis, discrete geometry, mathematical design, quantum physics, quantum field theory, and so on. It is expected that the mathematical theory of integrable systems play crucial role for applications.

Our special session will report the latest development on integrable peakon systems including mathematical theory and applications in physics and fluid mechanics.

SESSION 24

Long time dynamics of nonlinear dispersive waves

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Abstract. This session will focus on long time behaviors of nonlinear dispersive waves models arising in water waves and other physical models such as Bose-Einstein condensates. The

topics include global well-posedness and blowup, stability and instability of coherent states (traveling waves, solitary waves, standing waves etc.).

SESSION 25

Solitons, vortices, domain walls, their dynamics and their progenitors

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Abstract. This session is dedicated to discussing geometrical and dynamical properties of solutions of nonlinear partial differential equations arising in physical sciences, in particular, in quantum physics. The goal is to understand the formation of localized structures, such as solitons, vortices, monopoles, domain walls (known also as interfaces or fronts), etc., their patterns, stability and dynamics. The derivation of such localized structures from microscopic models is also of great interest.

SESSION 26

Evolution equations in mathematical biology: Cellular and network processes

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Abstract. Due to the impressive recent progress in both acquiring new experimental data and in the computational power needed to analyze and model these data, research efforts in Mathematical Biology are becoming crucial for our understanding of the biological processes that take place at the cellular and network level. Recognizing the need for communicating advances in mathematical approaches to biological systems, our workshop brings together researchers working on various aspects of the biological systems such as chemotaxis, cellular processes, dynamics of biological fluids, cancer, infectious diseases, connectomics, neuron and brain population dynamics, to name a few. The exchange of ideas, results, and mathematical approaches to biological problems will foster a creative environment for better understanding of the structures studied. Furthermore, this exchange will likely also result in applying some of the insights from one area to other systems and in generating new exciting questions as a way to establish new research directions.

SESSION 27

Mechanisms for computations in neuronal networks

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Abstract. Different brain regions can be thought of and modeled as large networks of neurons interacting through synaptic connections. While these networks can often be analyzed with familiar techniques used to analyze differential equations in physics, novel properties arise from the unique ways in which its constituent particles — neurons — interact. Specifically, neurons are pulse-coupled, and networks of neurons exhibit both spatially structured and random connectivity. A further question is to consider how these mechanisms perform specific computational functions, which vary widely through the nervous system.

In this session, we address a range of neural network mechanisms in order to understand their impact on specific computations. Some topics include: the role of oscillations in creating rhythms and olfactory coding, the role of correlations and adaptation in coding and information transmission, and the function of low-dimensional dynamics in hippocampus and chaotic networks.

SESSION 28

Analytical and computational techniques for differential and difference equations

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Abstract. The session covers analytic, symbolic, and numerical aspects of the properties and solutions of ordinary and partial differential equations (ODEs and PDEs) and differential-difference equations (DDEs).

With respect to analytic properties, new results on the Inverse Scattering Transform for the focusing and defocusing nonlinear Schrödinger equations will be presented. Also covered are applications of a generalized Laplace-Darboux transformation method to solve nonlinear PDEs involving higher-order operators; a simplified Hirota (homogenization) method to find soliton solutions with *Mathematica*; and Euler and homotopy operators for testing exactness and the computation of conservation laws of nonlinear PDEs and DDEs. The Lie algebra, Noether operators, and first integrals of a generalized coupled Lane-Emden system will also be discussed.

With respect to the numerical aspects, the time periodicity of solutions to the Korteweg-de Vries type equation with periodic forcing at the boundary will be demonstrated numerically using the Sinc-Collocation method.

SESSION 29

Nonlinear Schrödinger models and applications

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Abstract. The nonlinear Schrödinger (NLS) equation describes a very large variety of physical systems since it is the lowest order *nonlinear* (cubic) partial differential equation that describes the propagation of modulated waves. For example, two of the most salient applications of NLS equations, that will be the main theme of this session, stem from the realm on nonlinear optics and Bose-Einstein condensates (BECs). In optics, the NLS stems from the nonlinear (Kerr) response of the refractive index of some nonlinear media. On the other hand, for BECs, the mean-field description of a condensed cloud of atoms is well approximate, at low enough temperatures, by the NLS with an external potential. In addition, in recent years, numerous variants of these themes both Hamiltonian ones such as spin-orbit coupling, Bose-Fermi mixtures, dipolar systems with long-range interactions, as well as ones bearing gain-loss such as PT-symmetric media, and exciton-polariton condensates have opened novel avenues of exploration of NLS themes.

The aim of this mini-symposium is to bring together experts, as well as young researchers working in the theory, the numerical simulation and the experimental study of nonlinear Schrödinger equations and its applications. The focus is to establish a fruitful discussion of the current state-of-the-art and an examination of future challenges and directions of interest. This should be a session appealing to theoretical physicists, experimental physicists and applied mathematicians alike and will be a vehicle for the exchange of ideas that could cross-fertilize different disciplines and spurt the initiation of new collaborations that could address some of the pertinent open problems.

SESSION 30

Wave phenomena in population biology and application to cancer

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Abstract. By spanning multiple scales from genes, to cell and organs, biological systems create the phenotypes of malignancy. The authors will present mathematical models, theory and numerical simulations of cancer systems pertaining to their dynamical behavior as well as to their biological and clinical implications. The models will span the molecular, cellular, and organ scales; the malignancies include brain cancer, melanomas, and GIST.

SESSION 31

Wave Propagation in complex media

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Abstract. This session focuses on theoretical and experimental aspects of linear and nonlinear wave propagation in complex photonic media. Typical topics of interest include linear and nonlinear phenomena in optical waveguides and free space, PT-symmetric and non-hermitian systems, supersymmetric optics, random lasers, nanolasers, and novel nonlinear materials.

PAPER ABSTRACTS

SESSION 1
Nonlinear evolution equations and integrable systems

The nonlinear stage of modulational instability

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Abstract. We use the inverse scattering transform for the focusing nonlinear Schrödinger on the whole line with non-zero boundary conditions at infinity to characterize the nonlinear stage of the modulational instability.

The BBM-BBM system on a quarter plane

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Abstract. Our purpose here is to investigate a class of initial-boundary-value problem of the form

$$\begin{gathered}u_t + u_x - u_{xxt} + \left(Au^2 + Buv + Cv^2\right)_x = 0,\\ v_t + v_x - v_{xxt} + \left(Du^2 + Euv + Fv^2\right)_x = 0,\\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x),\\ u(0,t) = g(t), \qquad v(0,t) = h(t)\end{gathered}$$

posed for $x, t \geq 0$, where $A, B, \cdots, F$ are real numbers. We show that the problem is locally well posed for rough initilal and boundary data for all $A, B, \cdots, F$. Moreover, if there is a triple (a, b, c) that is a non-trivial solution of the system of linear equations

$$\begin{gathered}2Ba + (E - 2A)b - 4Dc = 0,\\ 4Ca + (2F - B)b - 2Ec = 0\end{gathered}$$

which satisfies $4ac - b^2 \geq 0$, then the problem is well posed globally in time.

On the Cauchy problem for a Camassa-Holm type equation.

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Abstract. We explore the Cauchy problem for a Camassa-Holm type equation. These types of equations have been shown to be well-posed in Sobolev spaces, H^s for sufficiently large Sobolev indexes s. We build on these results as we explore properties of the data-to-solution map and related results.

Ill-posedness of weakly dispersive equations

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Abstract. We consider the Cauchy problem for a number of weakly dispersive equations related to the celebrated Camassa-Holm Equation. We focus our discussion on the phenomenon of norm inflation that appears in Sobolev spaces with exponents s lying between 1/2 and 3/2. This property of norm inflation entails that one can find arbitrarily small initial data which generate solutions that grow as large as one wants within as short a time as desired. This phenomenon prevents the data-to-solution map from being continuous, and hence demonstrates the ill-posedness of these equations in these spaces.

On Whitham's conjecture of a highest cusped wave for a nonlocal shallow water wave equation

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Abstract. We consider the Whitham equation $u_t + 2uu_x + Lu_x = 0$, where L is the non-local (Fourier multiplier) operator given by the symbol $m(\xi) = (\frac{\tanh(\xi)}{\xi})^{1/2}$. Whitham conjectured that for this equation there would be a highest, cusped, travelling wave solution. We find this wave as a limiting case at the end of the main bifurcation curve of periodic solutions, and prove that it belongs to the Hölder space C^α for all $\alpha < 1/2$, but to no Hölder space C^α with $\alpha > 1/2$. Further properties of the wave, and of traveling-wave solutions of the Whitham equation in general, are given. An essential part of the main proof consists in a precise analysis of the integral kernel corresponding to the symbol m.

The "good" Boussinesq equation on the half-line

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Abstract. The initial-boundary value problem for the "good" Boussinesq (GB) equation on the half-line with data in Sobolev spaces is analysed via Fokas' unified transform method and a contraction mapping approach. First, space and time estimates

for the linear problem are derived and hence, the Fokas solution formula for the linear GB on the half-line is shown to belong to appropriate Sobolev spaces. Next, well-posedness of the nonlinear initial-boundary value problem is established by showing that the iteration map defined via Fokas' formula for GB when the forcing is replaced with the Boussinesq nonlinearity is a contraction. This work places Fokas' unified transform method for evolution equations into the broader Sobolev spaces framework.

Semiclassical initial-boundary value problems for the defocusing nonlinear Schrödinger equation

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Abstract. Initial-boundary value problems for integrable nonlinear partial differential equations have become tractable in recent years due to the development of so-called unified transform techniques. The main obstruction to applying these methods in practice is that calculation of the spectral transforms of the initial and boundary data requires knowledge of too many boundary conditions, more than are required make the problem well-posed. The elimination of the unknown boundary values is frequently addressed in the spectral domain via the so-called global relation, and types of boundary conditions for which the global relation can be solved are called linearizable. For the defocusing nonlinear Schrödinger equation, the global relation is only known to be explicitly solvable in rather restrictive situations, namely homogeneous boundary conditions of Dirichlet, Neumann, and Robin (mixed) type. General nonhomogeneous boundary conditions are not known to be linearizable. In this talk, we propose an explicit approximation for the nonlinear Dirichlet-to-Neumann map supplied by the defocusing nonlinear Schrödinger equation and use it to provide approximate solutions of general nonhomogeneous boundary value problems for this equation posed as an initial-boundary value problem on the half-line. Our method sidesteps entirely the solution of the global relation. The accuracy of our method is proven in the semiclassical limit, and we provide explicit asymptotics for the solution in the interior of the quarter-plane space-time domain.

Integrable equations in 3D: Deformations of dispersionless limits

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Abstract. Classification of integrable systems remains as a topic of active research from the beginning of soliton theory. Numerous classification results are obtained in 1 + 1 dimensions by means of the symmetry approach. Although the symmetry approach is also applicable to 2 + 1-dimensional systems, one encounters additional difficulties due to the appearance of nonlocal variables. There are several techniques to tackle the problem (e.g. the perturbative symmetry approach). In the perturbative symmetry approach one starts with a linear equation having a degenerate dispersion law and reconstructs the allowed nonlinearity.

In this talk we present a novel perturbative approach to the classification problem. Based on the method hydrodynamic reductions, we first classify integrable quasilinear systems which may potentially occur as dispersionless limits of integrable 2 + 1-dimensional soliton equations. Subsequently we construct dispersive deformations preserving integrability deforming the hyrdrodynamic reductions by dispersive deformations and requiring that all hydrodynamic reductions of the dispersionless limit will be inherited by the corresponding dispersive counterpart. The method also allows to effectively reconstruct Lax representations of the deformed systems. We present various classification results obtained in the frame of the new approach, e.g. the classification of scalar 2+1-dimensional equations generalizing KP, BKP/CKP, the classification of Davey-Stewartson type systems as well as various classifications of 2 + 1-dimensional differential-difference equations.

The talk is based on joint work with E. Ferapontov, A. Moro, B. Huard and I. Roustemoglou.

Stability and well-posedness problems in the integrable systems

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Abstract. Our talk will concern with the recent development of integrability technique, used in the context of orbital stability of solitons as well as well-posedness. In particular, we focus on the integrable system on the class of nonlinear Dirac equations, known as the massive Thirring model. A standard way of proving orbital stability relies on an energy functional that attains a local convexity at a family of solitons. A difficulty with the nonlinear Dirac equations comes from a sign-indefinite Dirac energy that cannot serve as a Lyapunov functional. By exploiting integrability of the MTM, we prove orbital stability of one solitons in L^2 by using the Bäcklund transform [1] and in H^1 by using the higher Hamiltonian [3]. The MTM is L^2-critical and known to be globally well-posed with any L^2 initial data due to Candy [2]. The Bäcklund transformation method allows for the orbital stability proof in the function space of lower regularity than the energy space.

In the end, we aim to discuss our recent investigation on global well-posedness of the derivative NLS equation by using the inverse scattering transform. This equation is related to the Kaup-Newell spectral problem.

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A synthetical two-component model with peakon solutions

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Abstract. A generalized two-component model [1] with peakon solutions is proposed in this paper. It allows an arbitrary function to be involved in as well as including some existing integrable peakon equations as special reductions. The generalized two-component system is shown to possess Lax pair and infinitely many conservation laws. Bi-Hamiltonian structures and peakon interactions are discussed in detail for typical representative equations of the generalized system.In particular, a new type of N-peakon solution, which is not in the traveling wave type, is obtained from the generalized system.

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Adiabatic perturbation theory for vector NLS with nonvanishing boundary conditions

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Abstract. In this talk, we develop an adiabatic perturbation theory for vector NLS equations with nonvanishing boundary conditions. We employ the methods of Inverse Scattering and RH-problem to derive the pertubred equations of motion of soliton parameters in dark-bright and dark-dark solitons. Adopting a mean-field description for a two-component atomic Bose-Einstein condensate, we study the statics and dynamics of dark-bright solitons in the presence of localized impurities. We use adiabatic perturbation theory to derive an equation of motion for the dark-bright soliton center. We show that, counter intuitively, an attractive (repulsive) delta-like impurity, acting solely on the bright soliton component, induces an effective localized barrier (well) in the effective potential felt by the soliton; this way, dark-bright solitons are reflected from (transmitted through) attractive (repulsive) impurities. Our analytical results for the small-amplitude oscillations of solitons are found to be in good agreement with results obtained via a Bogoliubov-de Gennes analysis and direct numerical simulations.

Persistence properties and unique continuation for a generalized Camassa-Holm equation

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Abstract. Persistence properties of solutions are investigated for a generalized Camassa-Holm equation (g-kbCH) having $(k+1)$-degree nonlinearities and containing as its integrable members the Camassa-Holm, the Degasperi-Procesi and the Novikov equations. These properties will imply that strong solutions of the g-kbCH equation will decay at infinity in the spatial variable provided that the initial data does. Furthermore, it is shown that the equation exhibits unique continuation for appropriate values of the parameters b and k. Finally, existence of global solutions is established when $b = k + 1$.

The Cauchy problem for an integrable Euler-Poisson equation

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Abstract. We consider a nonlinear system that describes the fluctuations in the ion density of a two-component plasma of positively charged ions and negatively charged electrons. This system admits an approximation which preserves dispersion and leads to the Korteweg-de Vries equation. Furthermore the system is integrable in the sense that it admits two Hamiltonian structures that are compatible. We will discuss several results about the Cauchy problem for this integrable Euler-Poisson equation.

SESSION 2
Nonlinear waves, patterns and vortices on compact fluid surfaces

Spontaneous motion and deformation of a self-propelled droplet

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Abstract. In this talk, I will discuss recent theoretical developments of active soft materials. One of the striking phenomena in biology is the ability of a cell to move without external force. This is distinct from passive systems. Active matters are assemblages of moving soft materials individually fueled by energy source. The studies in this field fall into two categories; one is mechanism of self-propulsion of an individual object such as cell motility and a chemically-driven droplet, and the second is the collective motion of these actively moving objects. The central question of the former studies is how spontaneous motion appears under nonequilibrium states. This is captured by a bifurcation of a moving domain.

Spontaneous motion is not driven by external force, but is sustained under a force-free condition. This requires breaking translational invariance in space. The broken symmetry is either extrinsic that is imposed externally by boundary conditions or by material properties or intrinsic namely nonlinear coupling makes an isotropic state unstable. The latter mechanism makes the system going to lower symmetry. When there is relative distance between a propelled object and another component, the translational symmetry is broken and a steady velocity emerges. I will show how various motion, translational and rotational motion, and deformation appear from the coupling between fluid flow and concentration field with chemical reaction. I will also discuss spontaneous motion arising from waves created by nonlinear reaction-diffusion equations.

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Self-assembled patterns and emerging dynamics in active magnetic suspensions at liquid interfaces

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Abstract. Magnetic colloids suspended at a liquid interface and driven out of equilibrium by an alternating magnetic field develop nontrivial dynamic self-assembled patterns not generally available through thermodynamic processes. Experiments reveal new types of nontrivially ordered patterns (“asters”, “magnetic snakes”) emerging in such systems in a certain range of excitation parameters [1, 2]. These remarkable magnetic non-equilibrium structures emerge as a result of the competition between magnetic and hydrodynamic forces and have complex magnetic ordering facilitated by a self-induced interfacial waves [2, 3]. Above certain frequency threshold some of the dynamic patterns spontaneously break the symmetry of self-induced interfacial flows and turn into swimmers [4]. Induced self-propulsion of robust aster-like structures in a presence of small in-plane DC perturbations has been discovered [2]. Observed phenomena can be understood in the framework of an amplitude equation for parametric waves coupled to the conservation law equation describing the evolution of the magnetic particle density. Molecular dynamic simulations capturing microscopic mechanisms of the non-equilibrium self-assembly in these out-of-equilibrium systems are developed [5].

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Nonlinear aerodynamics analysis of a flapping and bending flexible flag in air flow

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Abstract. We investigate the flapping and bending of a flexible ultralight 3D in laminar air flow (wind tunnel), which includes the flapping of flags and the self-streamlining of flexible bodies like drone and insect wings . The modeling of such a complex problem of fluid-body interactions raises a lot of difficulties, and we tried to prove the existence of stable and

meta-stable patterns in the bending, flapping and rotating of samples of ultra soft and ultra light various fabrics. Experiments in wind tunnel with rapid photography and stereo photogrametry surface reconstruction were performed, and a hydrodynamic theoretical model was built. Many new nonlinear phenomena can be substantiated in these systems, [1], as well as advances in theoretical understanding from the techniques used to mutually couple fluid and flexible solids dynamics.

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Model for nonlinear energy resonant damping for a payload impact to the ground

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Abstract. We study the nonlinear dynamics and optimization of the shock deceleration supported by a payload when the airborne carrier impacts the ground. A nonlinear visco-elastic, [1], model is designed for the container and a nonlinear elastic shock absorber suspension system for the payload. We model the dynamics of this system and extract information on maximum deceleration, and energy transfer between the container and the payload, and about the energy resonant damping [2]. We performed experiments for various terminal velocities and types of grounds compare with the model and make predictions.

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Nonlinear waves in a water tunnel + wave tank

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Abstract. We designed and built at ERAU a wave lab hosting in an acrylic wave tank of sizes 32' x 4' x 4' which can sustain 4,000 gl water*. The filling and level control of the water is computerized and remotely controllable on line. The water volume can be recirculated with a system of 6 electric pumps totaling 50 HP with a laminar flow region of 16' length with water velocity of maximum 1 m/s at a cross section of 4' x 3'. The wave maker consist in a towing system suspended on friction-less rails powered by two computer controlled drivers and a chain system with maximum velocity in water of 10-15 m/s and time/space precision of 0.1 mm/0.05s. The towing system is rigidly connected to a paddle which generates waves by its oscillatory motion. Other systems, including coloration of water with solvable ink, artificial beaches, extra linear actuators, pneumatic piston for tsunami simulations, air bubble generator are also used. The diagnostic systems consists in laser in visible and IR, a rapid camera with 1,000 FPS and a new stereo-photogrametry computer software reconstruction of the waves vectors on water surface. Various types of 1D and 2D of waves have been created and described bit theoretical and experimental. The contribution reviews also the possibilities of further exploiting of this new nonlinear hydrodynamics lab.

* http://daytonabeach.erau.edu/about/labs/nonlinear-waves-lab/index.html

A use of regression analysis to address the validity of the nonlinear stochastic differential equation solution of Black-Scholes model applied to the $S\&P$ 500 index.

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Abstract. As it is well know the assumptions of many differential equation models require restrictions on certain values being held constant. In most applications these assumptions are reasonable, however in the Black-Scholes model, [1], for predicting the value of options some of the assumptions are not reasonable doing rapidly changing market times. In this study we study we create a multiple regression model to predict the $S\&P500$ in terms of solid macroeconomic predictors such as gross domestic product (GDP) and total money aggregate (M1). We then compare these results to the output of the Black-Scholes model and develop a scheme to analytically address when the Black-Scholes model is moving out of volatility range. In the financial sector one may view this as a new approach to evaluating the so called "implied volatility.

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Traveling wave solutions for some generalized Camassa-Holm equations

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In this paper we employ two recent analytical approaches to investigate the possible classes of traveling wave solutions of some members of a recently-derived integrable family of generalized Camassa-Holm (GCH) equations. A recent, novel application of phase-plane analysis is employed to analyze the singular traveling wave equations of three of the GCH NLPDEs, i.e. the possible non-smooth peakon and cuspon solutions.

Moreover, smooth traveling waves of the three GCH equations are considered. Here, we use a recent technique to derive convergent multi-infinite series solutions for the homoclinic orbits of their traveling-wave equations, corresponding to pulse (kink or shock) solutions respectively of the original PDEs. We perform many numerical tests in different parameter regime to pinpoint real saddle equilibrium points of the corresponding GCH equations, as well as ensure simultaneous convergence and continuity of the multi-infinite series solutions for the homoclinic orbits anchored by these saddle points. Unlike the majority of unaccelerated convergent series, high accuracy is attained with relatively few terms. We also show the traveling wave nature of these pulse and front solutions to the GCH NLPDEs.

SESSION 3
Geometric techniques in the analysis of traveling waves

How defects are born

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Abstract. Pattern-forming systems typically exhibit defects, whose nature is associated with the symmetries of the pattern in which they appear; examples include dislocations of stripe patterns in systems invariant under translations, disclinations in stripe-forming systems invariant under rotations, and spiral defects of oscillatory patterns in systems invariant under time translations. Numerical simulations suggest that pairs of defects are created when the phase of the pattern ceases to be slaved to its amplitude. Such an event is typically mediated by the build up of large, localized, phase gradients.

This talk will describe recent advances on a long-term project whose goal is to follow such a defect-forming mechanism in a system that is amenable to analysis. Specifically, we focus on the appearance of pairs of dislocations at the core of a grain boundary of the Swift-Hohenberg equation. Taking advantage of the variational nature of this system, we show that as the angle between the two stripe patterns on each side of the grain boundary is reduced, the phase of each pattern, as described by the Cross-Newell equation, develops large derivatives in a region of diminishing size.

Nondegeneracy and stability of periodic traveling waves in fractional KdV equations

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Abstract. In the stability and blowup analysis for traveling waves in nonlinear Hamiltonian dispersive equations, the nondegenearcy of the linearization about such a traveling wave is of paramount importance. That is, one must verify that the kernel of the second variation of the Hamiltonian is spanned by modes associated to the translational invariance of the PDE. The proof of this property is often far from trivial, especially in cases where the dispersion admits a nonlocal description, the main obstruction often being that shooting arguments and other ODE methods may not be applicable. In this talk, we discuss the nondegeneracy of the linearization associated to constrained energy minimizers in a KdV equation with fractional dispersion. By obtaining a periodic Sturm-Liouville type theory for fractional Schrodinger operators on torus, we are able to analytically verify the nondegeneracy of the linearization in this setting. Our proof here follows the ideas of Frank and Lenzmann for obtaining non-degeneracy for fractional Schrodinger operators on the line, but with appropriate modifications to accommodate the periodic nature of our problem. As an application, we obtain the nonlinear stability of such constrained energy minimizers to period preserving perturbations.

Viscous hyperstabilization of detonation waves

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Abstract. It has long been a standard practice to neglect diffusive effects in stability analyses of detonation waves. In this talk, I will describe recent work aimed at quantifying the impact of these oft-neglected effects on the stability characteristics of such waves. In particular, I will discuss the use of numerical Evans-function techniques to study the (spectral) stability of viscous strong detonation waves—particular traveling-wave solutions of the Navier–Stokes equations modeling a mixture of reacting gases. Remarkably, the results show a surprising synergy between the high-activation-energy limit typically studied in stability analyses of detonation waves and the presence of small but nonzero diffusive effects. While the calculations do show a modest delay in the onset of instability in agreement with recently reported calculations by direct numerical simulation of the physical equations, the Evans-function approach also provides additional spectral information. In particular, for each of the families of detonation waves in our computational domain, we find an unexpected kind of hysteresis in the limit of increasing activation energy; that is, our calculations suggest that, whenever diffusive effects are present, there is a return to stability as activation energy is further increased, with unstable eigenvalues returning to the stable complex half plane.

Stability analysis for combustion fronts traveling in hydraulically resistant porous media.

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Abstract. We study front solutions of a system that models combustion in highly hydraulically resistant porous media. The spectral stability of the fronts is tackled by a combination of energy estimates and numerical Evans function computations. Our results suggest that there is a parameter regime for which there are no unstable eigenvalues. We use recent results about partially parabolic systems to prove that in the absence of unstable eigenvalues the fronts are convectively unstable.

Balancing numerics and analysis in Evans function techniques

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Abstract. In this talk, we present an overview of analytical and numerical techniques that can be used in concert to demonstrate traveling wave stability. Our primary interest is in the stability analysis of viscous shock layers in compressible fluid flow, but the techniques can be applied to a broad class of models. We have recently completed two major projects, one dealing with the stability of planar shocks in multi-D Navier Stokes and the other dealing with one-D detonations in reactive Navier Stokes. Both of these problems have myriad technical difficulties and our analytical and computational toolboxes have evolved significantly as a result of having to overcome these challenges.

SESSION 4
Discrete and ultra-discrete integrable systems and Painlevé equations

Discrete integrability and the lattice-geometry of the Gosset-Elte figures

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Abstract. The Gosset-Elte figures are a family of semi-regular polytopes and tessellations which may be generated from a reflection group by the Wythoff construction. Examples include the multi-dimensional simplex and the multi-dimensional demi-hypercube, but the family also contains exceptional cases of polytopes with symmetry groups E_6, E_7 and E_8. A unified description of this class of figures was given by Coxeter in terms of a T-shaped graph: by distinguishing a node at the extrema of one of the three branches, each polytope within the family corresponds to a unique graph. Only if the Coxeter-graph defines a finite or affine group does it lead to a Gosset-Elte figure, otherwise the associated combinatorial object cannot be realised in a Euclidean space.

This combinatorial structure is the natural domain for a particular discrete equation, namely the six-point multi-ratio equation, which is already well-known for its algebraic and geometric significance. It extends and unifies three other domains, the best known of which is a lattice with A_∞ symmetry where the equation is the Schwarzian form of discrete KP. Less commonly known is that the same equation defines a version of discrete KdV on a lattice with D_∞ symmetry. Finally it also defines dynamics on a lattice with E_{10} symmetry, which we have established defines a version of the elliptic-Painlevé system. This is known as the multidimensional embedding in the first two cases, and in all three cases the embedding encodes directly the Bäcklund-transformations of the respective system. Therefore integrability.

The generalised lattice-geometry shows that the three systems are sub-systems of a larger whole, which is complementary to the usual hierarchical relation via reduction. Most interestingly, we find two local consistency properties are essential, one of which is known already from the discrete-KP setting, and has been connected in recent years with Desargues' Theorem of projective geometry, for the other kind of consistency we give a simple algebraic interpretation in the Möbius group. Besides the local consistency connected with integrability, this kind of unification raises possibilities, especially to give a simultaneous description also of the geometric meaning of these

systems, somewhere between the incidence-geometry and the algebraic geometry.

Large-degree asymptotics of rational Painlevé functions

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Abstract. Rational solutions of Painlevé equations have been extensively studied numerically due to the remarkably regular geometric patterns formed by their zeros and poles. Furthermore, these functions arise in the study of a wide variety of nonlinear wave equations, the point-vortex equations, exceptional orthogonal polynomials, and Coulomb gases. Using Riemann-Hilbert analysis, we obtain rigorous asymptotic expressions for the large-degree behavior of rational Painlevé functions in the entire complex plane. Along the way we confirm for the first time the Kametaka-Noda-Fukui-Hirano conjecture from 1986 concerning the pattern of zeros and poles. We also answer certain analogous questions for rational solutions of Painlevé IV. Part of this work is joint with Peter Miller.

Graphical techniques for analysing solutions to ultra discrete equations

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Abstract. The ultradiscrete KdV equation is a discrete version of the KdV equation where not only are the independent variables x and t discrete but the dependent variable is also discrete and takes two possible values, 0 or 1. Amongst the interesting properties of this equation, is the solitonic behaviour of the solutions. If you take some arbitrary initial profile, in this context this means a row of 0's and 1's and evolve in time, this profile will split into a set of solitons, the larger ones moving faster than the smaller ones. It has been observed that, if the values of the dependent variable u at the different lattice points are generalised to integers [1, 2] rather than just the binary 0 and 1, the solutions still exhibit solitonic behaviour. Indeed if the dependent variable takes real values then the solitonic behaviour is still preserved. In this paper we present at some simple graphical techniques for identifying the soliton content of solutions and their evolution in the case where the dependent variable is generalised to the reals.

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Toric networks and generalizations of discrete Toda lattice

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Abstract. We define generalizations of discrete Toda lattice parametrized by a triple of positive integers (n, m, k), by using the networks on a torus. Each such system has a family of $m + \gcd(n, k)$ commuting rational maps, which comes from the extended symmetric group action on the network. The case of $(n, 2, n-1)$ is the (original) discrete Toda lattice introduced in [1], which is written as a system of difference equations,

$$q_j^{t+1} w_j^{t+1} = w_j^t q_{j+1}^t, \qquad q_{j+1}^{t+1} + w_j^{t+1} = w_{j+1}^t + q_{j+1}^t.$$

Here we assume n-periodicity in the 'space' coordinate, $q_{j+n}^t = q_j^t$ and $w_{j+n}^t = w_j^t$. By utilizing combinatorial properties of the network, we describe the maps in terms of algebro-geometrical data, and study the initial value problem. It is based on a joint work with Thomas Lam and Pavlo Pylyavskyy [2].

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Integrability criterion for discrete equations using the property of 'co-primeness'

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Abstract. We explain our recent results on the theory of integrability criteria for discrete equations. Our focus is the so-called 'co-primeness condition' for discrete dynamical systems. The system is said to satisfy the co-primeness condition, if every pair of two terms is co-prime on condition that these two terms are not too close to each other. For example, one type of QRT mappings and the discrete KdV equation are proved to have the co-primeness theorems [1]. The QRT mapping treated in [1] is an autonomization of the discrete Painlevé I equation and is also related to the Somos-4 sequence. Through these investigations, we conjecture that the

co-primeness condition is a global version of the singularity confinement test (one of the most famous integrability criteria), and therefore is quite hopeful as an integrability criterion for discrete equations. In this talk, let us mainly investigate the discrete Toda equation with various boundary conditions [2]. We study the equation under the three types of boundaries: semi-infinite, molecule (Dirichlet) and the periodic, and prove that the co-primeness theorems hold for all the cases. In particular, the result under the periodic boundary condition is explained in detail. Although the singularity confinement test has been a powerful tool, we have not been able to judge the integrability of discrete equations with periodic boundary condition with the test. We believe that co-primeness condition can be seen as an extension of the singularity confinement test to the periodic case. Our another advantage is that it is easy to check the co-primeness of partial difference equations (we can at least have a conjecture, although rigorous proof can be complicated). Finally let us note on the development of the singularity confinement test itself in relation to the deautonomization technique [3]. This talk is based on a joint work with Prof. Jun Mada (Nihon University, Japan), Prof. Tetsuji Tokihiro (University of Tokyo, Japan) and Dr. Takafumi Mase (University of Tokyo, Japan).

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Geometry of integrable lattice equations

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Abstract. Lattice equations are integrable in the same way as one-dimensional discrete equations: for example, they possess Lax pairs, they have vanishing entropy, and they obey the singularity confinement property. One-dimensional discrete integrable systems, the Painlevé equations, have a characteristic geometry of the space of their initial conditions, detailed in [1]. This geometric space is the regularized space S of initial conditions; it can be obtained from $\mathbb{P}^1 \times \mathbb{P}^1$ after blowing up 8 base points. The equation can be considered as a mapping from the initial value space S to itself.

It is natural then to examine the space of initial values of two-dimensional discrete equations, or lattice equations. This space is now infinite-dimensional, but in the case of equations specified on a single plaquette, it can be obtained from copies of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ after blowing up the base varieties.

We examine the construction of the regularized space of initial conditions for examples in the Adler-Bobenko-Suris classification, and consider higher-order systems such as Boussinesq-type equations.

This is joint work with Nalini Joshi at the University of Sydney.

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On generalized cluster algebras

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Abstract. Generalized cluster algebras (GCAs) was introduced by Chekhov and Shapiro in 2011. They are generalizations of the ordinary cluster algebras by replacing the "binomial" in the right hand side of the exchange relation of cluster variables with "polynomial" so that the Laurent property still holds. Recently GCAs appear in several different contexts such as Teichmüller theory, quantum groups, exact WKB analysis. In this talk I explain that GCAs preserve (perhaps) all basic and important properties of the ordinary cluster algebras, including the structure theorems of seeds and quantization.

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Ultra-discrete KdV equation with periodic boundary condition

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Abstract. Ultra-discrete (UD) integrable systems are combinatorial versions of integrable systems obtained by special limiting procedure called the UD limit from discrete integrable systems. As both the UD Toda lattice and the UD Lotka-Volterra equation are essentially equivalent to the integrable Box and Ball system, the ultra-discrete KdV equation (UD-KdV) is also essentially equivalent to it. While the UD-Toda with periodic boundary condition was solved by using the tropical algebraic geometry or the limit procedure from the discrete

Toda lattice, little is known about the UD-KdV with periodic boundary condition. In this talk, we propose the general solution of the UD-KdV with periodic boundary condition, by using the tropical theta function and the bilinear form.

Hermite-Padé approximation, isomonodromic deformation and hypergeometric integral

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Abstract. We develop an underlying relationship between the theory of rational approximations and that of isomonodromic deformations. We show that a certain duality in Hermite's two approximation problems for functions leads to the Schlesinger transformations, i.e. transformations of a linear differential equation shifting its characteristic exponents by integers while keeping its monodromy invariant. Since approximants and remainders are described by block-Toeplitzs determinants, one can clearly understand the determinantal structure in isomonodromic deformations. We demonstrate our method in a certain family of Hamiltonian systems of isomonodromy type including the sixth Painlevé equation and Garnier systems; particularly, we present their solutions written in terms of iterated hypergeometric integrals. An algorithm for constructing the Schlesinger transformations is also discussed through vector continued fractions.

This talk is based on the collaborated work [1] with Toshiyuki Mano.

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Conserved curves for autonomous (ultra-)discrete Painlevé equations

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Abstract. It is known [1][2] that all of the q-Painlevé equations have realizations as subtraction-free birational mappings from which one can obtain ultra-discrete Painlevé equations. In this talk, using the geometric picture given in [3], we study the autonomous cases of such discrete and ultra-discrete Painlevé equations. We give their conserved curves explicitly. A relation to the gauge/string theory will be briefly discussed.

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SESSION 5
Applications of continuous and discrete integrable systems

Higgs boson equation in de sitter spacetime: Numerical investigation of bubbles using GPUs

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Abstract. It has been shown [1] that under certain conditions the Higgs Boson Equation in de Sitter Spacetime has global solution with zeros of odd multiplicity. These zeros, where the solution changes sign, are called bubbles and are of great interest to particle physics and inflationary cosmology. We use various explicit numerical schemes with OpenCL on Graphical Processing Units (GPUs) to approximate the creation, growth, and interactions of these bubbles.

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On a generalisation of the vortex filament equation

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Abstract. I will report on a moving frames approach to studying the evolution of a surface in 4-dimensional space under the skew-mean-curvature flow, a generalisation of the binormal flow for vortex filaments. This is work in progress with graduate student Phillip Staley.

The physical, geometric and algebraic aspects of the complex short pulse equation

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Abstract. In this talk, we will discuss the complex short pulse equation from physical, geometric and algebraic aspects. First, we will derive the complex short pulse equation from physics context, and show the connection with the motion of space curves. Then we will show how the complex short pulse equation can be reduced from the two-component KP hierarchy based on the Sato theory, and thus provide its multi-soliton solution in determinant. If time permits, we will show the integrable discretization of the complex short pulse equation and use it as an integrable self-adaptive moving mesh method for numerical simulations.
This is a joint work with Dr. Ohta at Kobe University and Dr. Maruno at Waseda University of Japan.

Random triangulations and nonlinear differential equations

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Abstract. The generating function of polygonizations of oriented surfaces is a tau-function of the Toda lattice hierarchy, a fact which we have exploited to derive solutions to the enumeration problem of polygonizations partitioned by the genus of the surface for some special cases. Another approach to this combinatoric problem is to randomly sample polygonizations, existing results give limit laws for the expected genus of the result. An interesting question is then what the governing nonlinear differential equations tell us about the structure of random polygonizations.

Asymptotic stability of KdV solitons in weighted sobolev spaces below the energy space

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In this work, we consider the stability of solitons for the KdV equation below the energy space, using spatially-exponentially-weighted norms. Using a combination of the I-method and spectral analysis following Pego and Weinstein, we are able to show that, in the exponentially weighted space, the perturbation of a soliton decays exponentially for arbitrarily long times. The finite time restriction is due to a lack of global control of the unweighted perturbation.

Numerical analysis of the spectrum of short pulse solutions of the cubic-quintic Ginzburg-Landau equation near zero dispersion

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Abstract. The cubic-quintic complex Ginzburg Landau (CQCGL) equation

$$u_z = \delta u + (\beta + \frac{i}{2}D)u_{tt} + (\epsilon + i\gamma)|u|^2 u + (\mu + i\nu)|u|^4 u,$$

provides a simple model for studying the short pulses generated by mode-locked fiber lasers. We numerically compute stationary solutions of the CQCGL equation near zero dispersion and determine both their stability and the spectrum of the linearized operator. We show that as the chromatic dispersion parameter, D, varies continuously from positive to negative, there is a continuous family of stable stationary solutions. However, the number of discrete eigenvalues and the configuration of the discrete and continuous spectrum in the complex plane undergoes a qualitative change as the sign of the dispersion changes from the anomalous (classical soliton) regime to the normal regime.
These results are obtained using an adaptation of numerical methods developed by Wang *et al.* [1], in which stationary solutions are found by solving a root-finding problem and the stability region is computed by solving a linear eigenvalue problem. The stability results are consistent with the stability region about zero dispersion that was first discovered by Akhmediev *et al.* [2] using a numerical solver for the CQCGL propagation equation.

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SESSION 6
Graphene lattices: Phenomena and analysis

Relativistic topological defects and quantum Berezinskii-Kosterlitz-Thouless transition in the honeycomb lattice

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Abstract. We present a new type of two-dimensional zero-temperature Berezinskii-Kosterlitz-Thouless (BKT) transition in the superfluid phase of a Bose gas near a Dirac point of a honeycomb optical lattice. Using functional integral methods we demonstrate that continuous tuning from small to large mass gap drives a quantum phase transition separating nematic

and magnetic orders. In particular, at the critical point the system responds to the increasing gap size by suddenly shifting from a pseudo-spin balanced condensate to an asymmetric state characterized by disorder in one sublattice. The microscopic physics at criticality is dominated by proliferation of topological defects which solve the massive nonlinear Dirac equation. We show how these defects combine hedgehogs and skyrmions into a single object, analogous to Dyons which carry electric and magnetic charge predicted by grand unified theories. In the lattice, hedgehogs are quantum nucleations of inter-sublattice superfluid flow, signaling a normal-to-superfluid transition. Skyrmions are fluctuations in the relative sublattice phase associated with loss of phase coherence in one sublattice. The free energy of our system is symmetric upon exchange of these two types of defects, thus realizing a superfluid analogue of electromagnetic duality. We derive the scaling law due to unbinding of defects by exact mapping of the free energy to two copies of the Coulomb gas. Computing the renormalization-group flow of the defect binding energy with the mass parameter reveals exponential BKT scaling near a critical value of the mass parameter, at zero temperature.

Topologically protected states in one-dimensional systems

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Abstract. We rigorously study a class of one-dimensional periodic Schrödinger operators, which we prove have Dirac points. We show that the introduction of an "edge" via adiabatic modulation of a periodic potential by a domain wall results in the bifurcation of spatially localized "edge states", associated with the topologically protected zero-energy mode of an asymptotic one-dimensional Dirac operator. As an application, we formulate and computationally investigate a photonic realization of these highly robust bound states in a class of photonic crystal waveguide structures. Our model also captures many aspects of the phenomenon of topologically protected edge states for two-dimensional bulk structures such as the honeycomb structure of graphene.

Linear dynamics in PT symmetric honeycomb lattices

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Abstract. Optical graphene, or an optical honeycomb waveguide, has become a material of much interest and excitement in the optics community. This is due to the presence of Dirac points in the dispersion relationship which are a result of the symmetry of the lattice. Also of interest in optics are so called parity-time (PT) symmetric perturbations, representing the careful introduction of gain and loss terms. In this talk, we examine the impact of introducing PT symmetritc perturbations into honeycomb lattices and their impact on Dirac points. We categorize two types of PT perturbations one of which we rigorously show prevents the formation of complex dispersion relationships. We then track how Dirac points separate, and thus we show how PT perturbations can be used to introduce band gaps. We also present numerical results which show how PT perturbations could be used to engineer dispersion relationships which induce wave motion in particular directions.

Nonlinear dynamics in deformed and PT symmetric honeycomb lattices

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Abstract. In this talk, we investigate the impacts of deformations and PT-symmetric perturbations on nonlinear wave packets in honeycomb lattices. Deformations can result in the merging and disappearance of the Dirac points, and the originally intersecting dispersion relation branches separate. Correspondingly, nonlinear envelope equations, such as nonlinear Schrödinger-Kadomtsev- Petviashvili (NLSKP)-type equation, nonlinear Dirac equation with nonzero mass and so on, are obtained. On the other hand, when a PT-symmetric perturbation is added to the honeycomb lattice, nontrivial phase dynamics arise. When the PT-symmetric perturbation is added to deformed honeycomb lattices, we show new nonlinear wave equations describe the effective dynamics. We also find the existence of embedded solitons in this case.

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Traveling edge waves in photonic graphene

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Abstract. Recently in [1] it was shown that introducing edges and suitable waveguides in the direction of propagation, unidirectional edge wave propagation at optical frequencies occurs in photonic graphene. The system is described analytically by the lattice nonlinear Schrödinger (NLS) equation with a honeycomb potential and a pseudo-magnetic field. In certain parameter regimes, these edge waves were found to be nearly immune to backscattering, and thus exhibit the hallmarks of (Floquet) topological insulators.

This talk addresses the linear and nonlinear dynamics of traveling edge waves in this system, using a tight-binding model derived from the lattice NLS equation. Two different asymptotic regimes are discussed, in which the pseudo-magnetic field is respectively assumed to vary rapidly [2] and slowly [3]. In both cases, by using Floquet theory, an asymptotic theory is developed which leads to explicit formulae for the dispersion relations of traveling edge modes. For different choices of the pseudo-field, both topologically nontrivial and topologically trivial dispersion relations are found. In the presence of weak nonlinearity, the envelope dynamics of edge waves are found to be described by 1D NLS equations along the edge. When the NLS equation is focusing, the nonlinearity balances dispersion to produce nonlinear edge solitons. These edge solitons appear to be immune to backscattering precisely when the dispersion relation is topologically nontrivial. The mechanisms of topological protection will also be discussed in different asymptotic regimes.

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Composite multi-vortex diffraction-free beams and van Hove singularities in honeycomb lattices

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Abstract. Over the last years there has been a large amount of effort in studying graphene, a 2D material with extraordinary properties [1]. Optical wave propagation in honeycomb lattices has been recently used as an artificial graphene system. In particular phenomena studied include conical diffraction [2], Zitterbewegung [3], the Klein tunneling [4], and the relation between pseudospin and orbital angular momentum [5]. In this work, we find diffraction-free beams (DFBs) for graphene and MoS_2-type honeycomb optical lattices. DFBs not only constitute a fundamental class of solutions and building blocks for other classes of beams but are also important in unveiling the fundamental properties of the system. We introduce a physically relevant spinor field decomposition method to analyze the underlying structure of these waves. The resulting composite solutions have the form of multi-vortices or semi-vortices, in the sense that the two spinor components are associated with different topological charges $(n,\ n \pm 1)$. Asymptotically, close to the Dirac point, the system is described by the Dirac-Weyl equation (massless or massive). In this limit, we analytically obtain diffraction-free composite multi-vortex solutions which are in agreement with our numerical results. Furthermore, the effect of the valley inequivalence of the K and K' points leads to vorticies with different topological charges. As the propagation constant traverses the band structure (BS), the diffraction-free solutions undergo a transition exactly at the van-Hove singularity. As a result, after this transition, the spinor components of the DFBs are no longer associated with multi-integer vorticities. Exactly at the van-Hove singularity three different solutions are obtained which have the form of an array of infinite extend in one direction but strongly localized in the orthogonal direction.

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Novel pseudospin-related phenomena in artificial "photonic graphene"

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Abstract. Graphene, a single layer of carbon atoms, has attracted tremendous attention in condensed matter physics and other related fields. Recently, artificial "photonic graphene" - optical analog of graphene structured by periodic waveguide arrays arranged in a honeycomb lattice, has been demonstrated as a useful platform to simulate the fundamental graphene physics. In this talk, we will discuss some novel pseudospin-related phenomena in photonic graphene. In particular, in a honeycomb photonic lattice optically induced in a nonlinear crystal, the pseudospin-mediated vortex generation and topological charge flipping were observed in both real and momentum space. Our experimental results were confirmed by directly solving the Dirac-like equations. These results advocate that the concept of pseudospin in graphene is not just a mathematic formality, but also has real angular momentum, and our work represent another successful demonstration of the feasibility using artificial photonic graphene to study intriguing fundamental physics.

SESSION 7
Applied nonlinear waves

Non-local waves on quantum vortex filaments

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Abstract. The motion of a quantum vortex filament can be approximated by the Schwarz approach [1], which gives a phenomenological extension of the classical local induction approximation (LIA) that takes into account mutual friction and normal fluid effects. A variety of vortex filament configurations can be obtained under the quantum form of the LIA, and recently solitons [2], Kelvin waves [3], and deformed planar filaments [4] we constructed analytically under this framework. As is well-known for the classical setting, the LIA is a local approximation to the non-local Biot-Savart dynamics governing the filament dynamics, and this too is true in the quantum setting. What we shall do here is discuss the non-local Biot-Savart dynamics governing the quantum filament, obtaining a non-local form of the Schwarz phenomenological model. Mathematically, this model is a nonlinear integro-partial differential equation. We construct analogues of some well-known filament configurations for these non-local quantum dynamics, highlighting aspects of the problem for which the non-locality greatly complicates the analysis.

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Internally driven oceanic surface waves

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Abstract. Intermediate long-wave equations arise when considering the evolution of water waves in between the limiting regimes of the KdV (shallow water) and the Benjamin Ono equation (deep water). Such equations have been studied throughout the last couple of decades, some of which are integrable [2]. This talk will consider a two-layer stratified fluid 2D+1 model [1] in order to investigate the connection between the geometry of the thermocline and its corresponding effect on the evolution of the free surface. The evolution of the thermocline is induced by the horizontally propagating internal gravity waves in the lower layer. Using a nonlinear perturbation scheme, a time-dependent equation is established which governs the effect of the internal gravity waves on the thermocline. The resulting nonlinear intermediate long-wave equation is then used to study the resulting evolution of the free surface.

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Lump solutions to nonlinear integrable equations

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Abstract. We will talk about lump solutions to nonlinear integrable equations described by bilinear equations in terms of bilinear differential operators. A few examples of lump solutions will be shown by Maple symbolic computations, together with their dynamical properties.

Nonlinear contour waves and hollow patterns, a signature of universality

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Abstract. There is experimental evidence of formation of regular polygonal rotating patterns in rotating liquids, Leidenfrost drops and hurricane eye wall [1, 2]. We will describe this behavior at such different space scales, and also present our recent experimental results concerning liquid nitrogen rotating shapes. We present a nonlinear 2-dimensional contour wave model which explains the formation of the rotating stable patterns, predicts the formation or limitation of other such patterns, and arguments in favor of the universality and scale-independence of this behavior.

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Maxwell-Bloch with multiple types of atoms

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Abstract. A theory of the propagation of dispersive solitons in a medium consisting of atoms or semiconductor quantum dots, of two different types, is considered. The two-component medium is modeled by a set of two-level atoms of two different types, embedded in a conductive host material. The two types of atoms are passive atoms (attenuator atoms, in the ground state) and active atoms (amplifier atoms, in an excited state). The inverse scattering transform (IST) is applied to the Maxwell-Bloch equations for such an ensemble of two-type atoms, where each can have their own different parameter values as well as even different initial conditions. The IST solution of the Maxwell-Bloch equations for many-component atomic systems, will also be discussed in general. It is to be noted that when the dipole moments of the different transitions are equal, then these Maxwell-Bloch equations do become fully integrable. But when the dipole moments are not exactly equal, one can still use soliton perturbation theory to obtain appropriate corrections to soliton motion. In addition to dipole moment differences, we shall also describe first-order effects of longitudinal and transverse relaxation times, pumping and conductivity effects on a soliton's evolution.

Approximately traveling wave pulses in binary exciton chain systems

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Abstract. Models of coupled chains of nonlinear oscillators arise in models of optical waveguide arrays, polymers and elsewhere. There is recent interest in optical waveguides with an alternation in the coefficients along the chain, leading to systems of ODEs that generalize the discrete nonlinear Schrödinger equation.

Solutions of such systems are often seen to develop regions of slow variation even when the initial data are impulsive, in particular the emergence of a slowly varying leading pulse that propagates in an approximately traveling wave form, and also in stationary oscillations near the endpoints. This has motivated the search for long-wave approximations by PDEs.

In this talk it is shown that the patterns of slow variation are substantially different from those assumed in most previously-considered long-wave approximations, and several new PDE approximations are presented: third order systems describing leading pulses of an approximately traveling wave form, related to the Airy PDE in the simplest case of a linear system with no binary alternation. There is also another quite different system describing stationary oscillations near the endpoint where a pulse originates, which involves a splitting into four slowly varying subchains.

Numerical solutions of these PDE approximations and linear analysis confirm that they provide a good agreement with the phenomena observed in the ODE systems.

The effects of wind and nonlinear damping on permanent downshifting and rogue waves

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Abstract. In this talk we consider the evolution of surface waves in deep water governed by a higher order nonlinear Schrödinger (HONLS) equation and subjected to wind and a nonlinear damping of the mean flow. Wind effects are incorporated by adding a uniform linear damping/forcing term to the model. The NLS equation possesses N-dimensional homoclinic solutions which are widely used to model rogue waves and which include the Akhmediev breathers (N = 1). We examine the formation of rogue waves and the conditions for irreversible downshifting in the presence of wind and damping for lower dimensional (N = 1, 2) homoclinic solutions. This complements the work in [1] which focused on this issue for higher dimensional (N = 3) homoclinic solutions where the waves are considerably steeper. For $N \geq 1$ we find that permanent downshifting occurrs whenever nonlinear damping is the dominant effect and, significantly, rogue waves do not develop after the time of permanent downshift. We examine how wind affects downshifting, rogue wave formation, and the lifetime of rogue waves for $N \geq 1$. We find damping by the wind weakens the effect of nonlinear damping and inhibits downshifting while driving by the wind enhances downshifting. The effect of rogue waves on the time of permanent downshift and the effect of proximity to instabilities on rogue waves are also studied.

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SESSION 8
Canceled

SESSION 9
Fully nonlinear Boussinesq models: Theory and practice

Solitary waves for Boussinesq type systems

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Abstract. he aim of this talk is to exhibit specific properties of Boussinesq type models. After recalling the usual asymptotic method leading to BT models, we will present a new asymptotic version and present a local Cauchy theory. We then provide an effective method to compute solitary waves for Boussinesq type equations. We will conclude by discussing shoaling properties of such models. This is a joint work with M. Colin.

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Boussinesq-type modeling of tsunami-like bores generated from storm waves

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Abstract. Many coastal communities are at constant risk from typhoons and storm surges. In recent decades, the increase in computational resources has enabled researchers to develop high fidelity storm forecasting systems. Consequently, progressions in coastal resilience have been made further protecting life and property. Despite research advances, unexpected phenomena still exist which are unaccounted for in coastal hazard assessment, evacuation planning, and structural building codes.

In November 2013, Typhoon Haiyan made landfall in the Philippines where it generated a tsunami-like bore that destroyed the entire village of Hernani in Eastern Samar. The destructive waves astonished both villagers and disaster managers, as the coast near Hernani is sheltered by a broad fringing reef, which was expected to serve as a reliable wave defense.

However, under the extreme storm conditions of Typhoon Haiyan individual waves overtopped the reef and the release of bound infra-gravity waves during wave breaking at the reef edge favored strong surf beat amplification over the reef flat and consequently an exacerbation of the flood hazard. Superposition of incoming wave groups with the oscillating water body caused tsunami-like surges of much longer period than regular swell waves that were responsible for the destruction of the village and the failure of its coastal defense structures. Though this phenomenon has been studied before [1], it had not been known to be a potential cause of the type of destruction seen in Hernani.

We have computed the wave conditions during the peak of the typhoon with the Boussinesq-type model BOSZ. The model accounts for the challenging conditions with very large offshore waves and energetic breaking near the reef edge. The robust formulation reproduces the dangerous resonance of the surf beat with bore formation over the reef flat. We will discuss the mechanisms that are responsible for the outlined counterintuitive wave phenomenon and compare the applicability of Boussinesq-type to RANS-type models for these scenarios.

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An unstructured finite volume numerical scheme for extended Boussinesq-type equations for irregular wave propagation

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Abstract. The interplay between low and high frequency waves is groundwork for the near-shore hydrodynamics for which Boussinesq-type (BT) equations are widely applied during the past few decades to model the waves's propagation and transformations. In this work, the TUCWave code is validated with respect to the propagation, transformation, breaking and run-up of irregular waves. The main aim is to investigate the ability of the model and the breaking wave parametrizations used in the code to reproduce the nonlinear properties of the waves in the surf zone. The TUCWave code numerically solves the 2D BT equations of Nwogu (1993) on unstructured meshes, using a novel high-order well-balanced finite volume (FV) numerical scheme following the median dual vertex-centered approach [1]. The BT equations are recast in the form of a system of conservation laws and the conservative FV scheme developed is of the Godunov-type. The approximate Riemann solver of Roe for the advective fluxes is utilized along with a well-balanced topography source term upwinding and accurate numerical treatment of moving wet/dry fronts. The dispersion terms are discretized using a consistent, to the FV framework, discretization and the friction stresses are also included. High-order spatial accuracy is achieved through a MUSCL-type reconstruction technique and temporal through a strong stability preserving Runge-Kutta time stepping. Wave breaking mechanism have also been developed and incorporated into the model [2]. TUCWave code is applied to benchmark test cases and real case scenarios where the shoaling and breaking of irregular waves is investigated.

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Particle paths and conservation laws in the Serre system

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Abstract. The Serre, Green-Naghdi, system is used to model highly nonlinear weakly dispersive waves propagating at the surface of a shallow layer of a perfect fluid.

In this work, we present a numerical study of particle paths due to the passage of a solitary wave and a dnoidal wave at the surface where the Serre equations are considered as the governing equations [1].

The Serre system has three associated conservation laws which describe the conservation of mass, momentum, and energy due to the surface wave motion. We present a systematic derivation of these conservation laws and we will observe that the balance equations are of the same asymptotic order as the evolution equations over a flat bottom. A study of momentum balance, energy balance and the breaking limits are presented for the undular bores in the context of Serre system. In addition, the system features a fourth conservation law which will be shown how this fourth conservation law can be interpreted in terms of a concrete kinematic quantity connected to the evolution of the tangent velocity at the free surface. The equation for the tangent velocity is first derived for the full Euler equations in both two and three dimensional flows, and in both cases, it gives rise to an approximate balance law in the Serre system which turns out to be identical to the fourth conservation law for this system [2].

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SESSION 10
Mathematical modelling and physical dynamics of solitary waves: From continuum mechanics to field theory

A fast and stable explicit operator splitting method for phase-field models

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Abstract. Numerical simulations of the phase-field models require long time computations, and therefore it is necessary to apply stable large time-stepping methods. In this paper, we propose a fast explicit operator splitting method for both one- and two-dimensional nonlinear diffusion equations for thin film epitaxy with slope selection and Cahn-Hillard equation. The equations are split into the nonlinear and linear parts. The nonlinear part is solved using the method of lines approach together with an efficient large stability domain ODE solver. The linear part is solved by a pseudo-spectral method, which is based on the exact solution and thus has no stability restriction on the size of time step. We demonstrate the performance of the proposed method on a number of one- and two-dimensional numerical examples, where different stages of coarsening such as the initial preparation and the alternating rapid structural transition and slow motion can be clearly observed.

A simulation and dynamical model study of waves in 1D granular tapping flows

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Abstract. We have devised a simulation - dynamical systems approach that sheds light on the propagation of waves in granular flows. The focus is on the dynamics of a vertically tapped column of granular particles. A key ingredient in this approach is our DEM code, with which we have investigated the column's motion and density [2, 3]. Insights from our DEM code simulations led to a relatively simple infinite-dimensional dynamical systems model for the flows called the BSR equation, comprising an equation of the form

$$u_t + uu_y = e + \int_{I(y)} \rho(\eta, t)\Theta(\eta, u, t)d\eta, \qquad (1)$$

coupled with the continuity equation. It was proved [1] that (1) is integrable if the interaction force Θ is perfectly elastic, so it has soliton solutions. For inelastic collisions there are analogs of this and other results that diminish as they evolve, such as decaying solitary waves. Simulations and the results of (1) and other models are found to compare quite well.

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Kinks and their statistical mechanics in higher-order scalar field theories

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Abstract. We study nonlinear Klein–Gordon equations of the form:

$$\phi_{tt} - \phi_{xx} = -V'(\phi), \qquad (2)$$

where V is an even-degree polynomial in ϕ. In 1979, Lohe proposed such scalar field theories with V of degree higher than four in meson physics, and they can also be used to describe successive phase transitions in ferroelastic and ferroelectric crystals.

We obtain [1] exact kink solutions of the ϕ^8, ϕ^{10} and ϕ^{12} field theories with degenerate minima. We find that the higher-order field theories have kink solutions with algebraically-decaying tails and also asymmetric cases with mixed exponential-algebraic tail decay.

The ϕ^{10} field theory is a quasi-exactly solvable (QES) model (akin to ϕ^6). In this case, we obtain [1] analytically the classical free energy as well as the probability distribution function (PDF) in the thermodynamic limit at three different temperatures T. The PDFs are compared to numerical Langevin simulations performed by rewriting (2) as

$$\phi_{tt} - \phi_{xx} = -V'(\phi) - \eta\phi_t + \xi(x, t), \qquad (3)$$

where η and ξ are related by the fluctuation–dissipation theorem.

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On N-soliton interactions of Gross-Pitaevski equation in two space-time dimensions

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Abstract. We consider the N-soliton interactions of the two-component Gross-Pitaevsky eq. [1] in two space-time dimensions:

$$i\vec{u}_t + \frac{1}{2}\vec{u}_{xx} + (\vec{u}^{\dagger},\vec{u})\vec{u} = V(x)\vec{u}(x,t) + c_1\sigma_1\vec{u}(x,t), \tag{4}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

in the presence of inter-channel interaction $c_1 \neq 0$. This equation describes quasi-one-dimensional Bose-Einstein condensates In the adiabatic approximation the propagation of the soliton trains of eq. (22) is described by a perturbed complex Toda chain (PCTC) [2, 3]. More specifically we demonstrate that PCTC correctly models the effects of several types of potentials: a) harmonic $V(x) = v_2x^2 + v_1x + v_0$, b) periodic $V(x) = A\cos(\Omega x + \Omega_0)$ and c) shallow potential wells $V(x) = c_0(\tanh(x - x_{\rm f}) - \tanh(x - x_{\rm in}))$, $c_0 \ll 1$ and $x_{\rm in} < x_{\rm f}$. We will demonstrate that the perturbed CTC adequately models the soliton train dynamics for a wide region of the initial soliton parameters as well as for $c_1 \neq 0$.

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Deformation of solids as wave dynamics

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Abstract. Deformation of solids is discussed as wave dynamics. Conventionally, deformation is viewed as a series of evolving regimes, and discussed based on the constitutive relation of each regime. Specific theories are used for each regime; continuum mechanics for the elastic regime, various plastic theories for the plastic regime, and fracture mechanics for the fracture regime. In reality, elastic and plastic deformations coexist at any stage of deformation, and analysis of transitions from one stage to another is important for engineering. The regime specific approach is unrealistic to deal with these situations.

A recently field theory [1] formulates all stages of deformation comprehensively as wave dynamics. The field equations yield longitudinal/decaying transverse wave solutions for elastic/plastic deformation, and solitary wave solutions for the transitional stage from the elastic to plastic regime. This situation is contrastive to the conventional theories where plastic deformation is not a wave phenomenon. The wave nature of deformation in the plastic regime can be understood through consideration that deformation represents materials' response to the external load and that the work provided by the external load is transferred through the material as a function of space and time. It is known that the formation of shear bands is accompanied by acoustic emission, in some occasion neutron emission as well [2], and that visible light is emitted when rocks are deformed or fractured under impact compression [3]. The interaction between deformation wave and those acoustic and optical waves is not clear at this time, but it is certainly an interesting subject for future investigation.

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Beyond Euler & Navier–Stokes: New theories of compressible flow

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Abstract. With a focus on their specialization to acoustic phenomena, I will survey several generalizations of the Euler and Navier–Stokes equations that have been put forth since the mid-1990s. These include, but are not limited to, a compressible version of the lossless α-model, the original version of which was put forth by Holm *et al.*; Green and Naghdi's (1995) formulation of thermo-acoustics that includes the phenomenon of second-sound; and Straughan's "Cattaneo–Christov" model, which describes the propagation of thermal and mechanical waves in gases. Along the way, the positive and negative features of these models will be discussed, applications to other fields noted, and a number of new results presented. (Work supported by ONR funding.)

Boussinesq paradigm and negative group velocity in a material with double microstructure

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Abstract. In the present paper Mindlin model and approach by Engelbrecht and Pastrone [2] is used for modelling 1D wave propagation in microstructured solids. The particular models considered are the ones with two distinct micro-structures contained within the same matrix [1]. It has been noted that in the investigated models under some parameter combinations there can exist situation where for certain wavelength range the group velocity (which is usually interpreted as the speed of energy propagation in the system) is negative [3]. Following Lagrange formalism where one constructs Lagrangian $L = K - W$, where K is kinetic energy and W is the potential and then uses Euler-Lagrange equations for writing the system of governing equations it is possible to connect coefficients in the model equations to the coefficients in the potential. The focus of the study is on how the potential parameters connect to the existence of the negative group velocity (NGV) zones and if it is possible to detect the effect of the NGV on the solutions in the numerical simulations under the parameter combinations where it exists. Governing equations are solved numerically under localized initial conditions and periodic boundary conditions. For numerical integration Fourier transform based pseudospectral method is used. The effect of the NGV on the solutions is demonstrated.

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Mechanical waves in biomembranes

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Abstract. The goal is to investigate the influence of nonstandard nonlinear terms in a Boussinesq-type model equation. The context of the model equations is a propagation of mechanical disturbance that has been observed to accompany the propagation of electrical nerve pulse (see, for example, [5, 6]). It must be noted that there is a number of models for describing the nerve pulse propagation. These models, however, do not describe all the observed phenomena related to the nerve pulse propagation like the above mentioned mechanical disturbance that has been observed to accompany the propagation of electrical nerve pulse in [5, 6]. Recently Heimburg and Jackson have proposed a model for describing the mechanical waves in cylindrical biomembranes [4, 3]. This model has been modified by Engelbrecht et al in [2]. The model is of the Boussinesq-type [1] but the nonlinearity is of the $f(u) \cdot u_{xx}$-type instead of the $f(u_x) \cdot u_{xx}$-type. The model equations are integrated numerically using Fourier transform based pseudospectral method and different solution types are highlighted.

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A steepest descents method for the study of water waves generated by a moving body

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Abstract. In 1982, M.P. Tulin [1] proposed a simplification of the governing equations for potential flow that could reduce the system into a simpler differential equation, yet preserve important nonlinearities of the problem. However, Tulin discovered certain oddities about the reduced model, such as the fact that it seemed applicable for low speeds, *"but not too low speeds!"*. In this paper, we demonstrate how the reduction process can be extended to yield the correct first-order model, and moreover, how the method of steepest descents allows us to understand the origin of free-surface waves. In particular, we show how changing the geometry of the moving body will relate to changing integration contours on an associated Riemann surface (example shown below). This provides an intuitive methodology related to techniques in exponential asymptotics or asymptotics beyond-all-orders.

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On solitonic structures and discrete spectral analysis

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Abstract. It is well known that solitons and different solitonic structures can form in systems which are characterised by nonlinear as well as dispersive properties. In the present paper formation, propagation and interaction of various solitonic structures is studied making use of numerical experiments. Main attention is paid to modified Korteweg–de Vries equation

$$u_t + u^3 u_x + d u_{3x} = 0,$$

over wide range of dispersion parameter d. Our goal is to demonstrate that (i) (besides the single solitons) groups of equal amplitude solitons form in the considered case; (ii) the number of such a groups depends on the value of the disperison parameter d; (iii) besides visible solitons (which are detecteble in waveprofiles) hidden solitons form and take part in the soliton interaction process. Numerical solutions of other solitonic equations will be used in the presentation in order to emphasise the essence of the considered model. For numerical integration the pseudospectral method is used and for analysing of results discrete spectral analysis is applied [1].

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Modeling the spatially heterogeneous dynamics of HIV *in-vivo*

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Abstract. The Human Immunodeficiency Virus type-1 (HIV) is one of the most intensely studied viral pathogens in human history. Despite this vast effort, many aspects of HIV infection dynamics and disease pathogenesis within a host are still not understood. Here, we propose a new model of viral propagation *in-vivo* that generalizes the standard lumped population model by allowing susceptible and infected T-cells, as well as HIV virions, to move diffusively throughout a host region. In particular, we will elucidate the contributions of spatial fluctuations, correlations, and preferential infection to viral propagation using mathematical results concerning the long-time dynamics of the system. A few well-posedness results for this model, comprised of a nonlinear system of three parabolic PDEs, will also be briefly discussed.

The new semi-implicit Runge-Kutta methods and their applications in shallow water equations with friction terms

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Abstract. In this paper, we develop a family of second-order semi-implicit time integration methods for systems of ordinary differential equations (ODEs) with stiff damping term. The important feature of the new methods resides in the fact that they are capable of exactly preserving the steady states as well as maintaining the sign of the computed solution under the time step restriction determined by the nonstiff part of the system only. The new semi-implicit methods are based on the modification of explicit strong stability preserving Runge-Kutta (SSP-RK) methods and are proven to have a formal second order of accuracy. We illustrate the performance of the proposed SSP-RK based semi-implicit methods on a system of ODEs arising from the semi-discretization of the shallow water equations with friction terms. The presence of the friction term in the models increases the level of complexity in numerical simulations as the underlying semi-discrete system becomes stiff when the water depth is small. We test the designed ODE solver combined with semi-discrete second-order central-upwind schemes on a number of one- and two-dimensional examples that demonstrate robustness and high resolution of the proposed numerical approach. In the last numerical example, we achieve a remarkable agreement between the numerical results and experimental data, which corresponds to the laboratory experiments reported in [L. CEA, M. GARRIDO, AND J. PUERTAS, J. Hydrol, 382 (2010), pp. 88–102] and is designed to mimic the rain water drainage in urban areas containing houses.

Determination of traveling wave front asymptotic

behavior using dominant balance

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Abstract. Many important phenomena in the natural and engineering sciences may be modeled by nonlinear partial differential equations. However, a critical issue is that not only are general solutions not available, but it is not clear that the concept of a general solution has any intrinsic meaning. Consequently, special solutions play an important role in the analysis of the associated physical systems. One class of such solutions are those corresponding to traveling waves. In this talk, we discuss the use of the method of dominant balance (DB) to determine the asymptotic behavior of traveling waves. We consider several explicit examples to illustrate the applicability of DB.

Capturing diffraction using wave confinement

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Abstract. In this paper, we introduce a new method, Wave Confinement (WC), to capture diffraction, which plays a vital role in acoustic or EM wave propagation in realistic environments. The basic enabling concept of this method is to compute propagating Computational Waves (CWs) that follow solutions of the wave equation using WC [1]. This is a discrete Eulerian method in which the wave equation is modified by the addition of a term that results in nonlinear solitary waves without changing the basic conservation properties of the equation. This can then be used to determine the time of arrival and propagation direction. The essential solitary wave property of the CWs (in addition to these integral properties) is that they do not have the dispersive and diffusive truncation error inherent in conventional Eulerian methods: A confined solitary wave may undergo distortions due to local truncation effects, but, due to the shape preservation property, it always returns to its asymptotic shape with no error accumulation in spite of the fact that the term is nonlinear - as a result the pulse never decays. The pulse can remain concentrated as few as 3-4 grid cells regardless of the number of cells traversed in spite of numerical dissipation because the wave form is related to the solution of a discrete eigenvalue equation. Because of this property the grid can be made as coarse as necessary, consistent with overall accuracy (for example, to resolve atmospheric variations).The above mentioned CW can also be used as a carrier to propagate the details of the short physical wave, which will allow us to reconstruct the acoustic/EM signal at any far field point. The idea of carrying the details of short wave features instead of directly propagating them on a coarser grid can only be possible using WC.

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SESSION 11
Nonlinear waves

Solutions of Boussinesq systems for water waves

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Abstract. In this talk, I will summerize joint works over the years on the solutions, which include solitary wave solutions, cnoidal wave solutions, standing wave solutions and two-dimensional wave patterns, and their strong and weak interactions and stabilities for a class of Boussinesq systems.

Solitary waves for some systems of internal wave theory

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Abstract. We consider the Benjamin-Ono and the ILW systems of [1] that model two-way propagation of long internal waves of small amplitude along the interface of two fluid layers under the effects of gravity. After reviewing some theoretical properties of the models at hand, we present numerical evidence of the existence of solitary waves by using several numerical techniques. Some properties of the waves, suggested by the numerical experiments, are discussed including the speed-amplitude relation and their asymptotic decay rate. We also present some numerical studies concerning the dynamics of the waves which involve experiments about their interactions, stability properties along with comparisons with their unidirectional counterparts.

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Regularized long wave equation with white noise dispersion

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Abstract. In this talk we discuss some issues related to generalized Benjamin-Bona-Mahony equations with white noise dispersion. The equations read in the Stratanovich formulation

$$du - du_{xx} + u_x \circ dW + u^p u_x dt = 0, \tag{5}$$

where p is an integer and W denotes a standard Brownian motion.

We address the initial value problem, and the long time behavior of solutions. We present theoretical and numerical results about the decay rate of solutions towards equilibrium.

Dynamics of short and long capillary-gravity waves

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Abstract. We examine the interaction between long gravity waves and short capillary waves on the surface of shallow water in the case when the group velocity of the capillary waves coincides with the phase velocity of the long waves. This gives the coupled nonlinear Schrödinger-Korteweg-de Vries (NLS-KDV) system of equations. We proceed to find all periodic traveling-wave solutions via the method of Conte and Musette and we numerically analyze the spectral stability of these solutions.

Some remarks on the NLS-KdV system

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Abstract. In 1975, Kawahara *et.al.* introduced the linear Schrödinger-KdV system

$$\begin{cases} i\left(\dfrac{\partial u}{\partial t_2} + k\dfrac{\partial u}{\partial x_2}\right) + p\dfrac{\partial^2 u}{\partial x_1^2} = quv, \\ \dfrac{\partial v}{\partial t_3} + \dfrac{\partial v}{\partial x_3} + \dfrac{3}{2}v\dfrac{\partial v}{\partial x_1} + r\dfrac{\partial^3 v}{\partial x_1^3} = -s\dfrac{\partial |u|^2}{\partial x_1} \end{cases} \tag{6}$$

where k, p, q, r and s are real constants as a model for the interaction between long gravity waves and capillary waves on the surface of shallow water. Since then, numerous papers had appeared studying the NLS-KdV system

$$\begin{cases} iu_t + u_{xx} + a|u|^2 u = -buv, \\ v_t + cvv_x + v_{xxx} = -\dfrac{b}{2}(|u|^2)_x \end{cases} \tag{7}$$

where a, b, c are real constants. While these systems appear very interesting as they involve two of the most studied equations, there are several troubles which have gone unnoticed thus far. The main problems at hand can be summed up as
A) Only the system of *linear* Schrödinger-KdV (6) has ever been derived in which the two equations appear at *different* time scales; and
B) Even though numerous papers have studied system (7), unfortunately they all appeared to have quoted one another regarding its derivation and applications. Tracing through hundreds of those references, the exact derivation of system (7) was nowhere to be found. Notice that the first equation in (6) is *linear* whilst that in (7) is *nonlinear*. Thus, it appears that all the works heretofore studying (7) are just dealing with the mathematical aspects of a hypothetical system that has *never* been derived. (Of course, that would have been acceptable still, provided the authors honestly stated that. Except, in the context of the Euler water wave problem–which is the exact regime where the systems (6) and (7) are reportedly derived–it appears impossible to deduce such systems.)

SpecTraVVave: a program package for computing traveling waves of nonlinear dispersive equations

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Abstract. SpecTraVVave is a Python-powered program package capable of solving a broad class nonlinear dispersive equations. The numerical scheme used in the solver is based on the cosine collocation method, [2]-[3]. The user provides a nonlinear flux function, a linear dispersion operator and chooses available specifications for a solution such as boundary condition and wavelength. Then a traveling-wave solution is computed and tested with an evolution integrator. Dynamic simulation results are provided for a number of nonlocal wave equations such as the Whitham equation, the fractional KdV equation, [4]-[5], and generalized Benjamin-Ono equation, [1].
The SpecTraVVave package is available online at the github portal.

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Existence of two-hump solutions for some model equations related to water waves

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Abstract. The talk discusses the recent development on the existence of multi-hump solutions with oscillations at infinity for a class of model equations arising from water wave problems, such as the singularly perturbed 5th-order Korteweg-de Vries equations with a small parameter $\epsilon > 0$. In these equations, the linear operators have a pair of real eigenvalues and a pair of purely imaginary eigenvalues. In general, it can be shown that for such cases, the nonlinear equations have solitary-wave (or single-hump) solutions approaching to periodic solutions at infinity, called generalized solitary-wave solutions. The talk will give ideas on how to prove the existence of two-hump solutions with oscillations at infinity for the nonlinear equations by patching two appropriate generalized solitary-wave solutions together. The amplitude of the oscillations at infinity is algebraically small with respect to ϵ as $\epsilon \to 0$. (This is a joint work with J. W. Choi, D. S. Lee, S. H. Oh, S. I. Whang, and S. Deng).

Dispersive and diffusive-dispersive shock waves for nonconvex conservation laws

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Abstract. The structure of solutions to Riemann problems has been extensively studied in the hyperbolic conservation law community in the context of diffusive and diffusive-dispersive regularization. Another approach, adopted by the nonlinear dispersive wave community, is the purely dispersive regularization of conservation laws. The aim of this talk is to provide a bridge between these two approaches and communities by comparing solutions of the modified Korteweg–de Vries (mKdV) equation and the mKdV–Burgers (mKdVB) equation, yielding regularizations of a hyperbolic equation with nonconvex flux. undercompressive) shock waves. Despite the singular nature of the zero-diffusion limit and different analytical approaches employed in the descriptions of dispersive and diffusive-dispersive regularization, the resulting comparison reveals a number of striking parallels. In particular the mKdV kink solution is identified as an undercompressive DSW. Other prominent features, such as shock-rarefactions, also find their purely dispersive counterparts involving special contact DSWs, which exhibit features analogous to contact discontinuities. The language of hyperbolic conservation laws (e.g., characteristics, admissibility criteria) as well as that of dispersive shock theory (e.g., shock polarity, orientation) provide fruitful means to compare Riemann problem solutions. It is shown that the zero diffusion mapping of solutions is multivalued and not one-to-one. The conditions under which the mKdV and mKdVB Riemann problem classifications persist for more general diffusive-dispersive Eulerian systems that exhibit linear degeneracy are also elucidated.

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On vanishing nonlinear dissipative-dispersive perturbations of conservation laws

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Abstract. In presence of nonlinear diffusion and dispersion [1], we study the well-posedness of the conservation equation

$$u_t + f(u)_x = \delta g(u_{xx})_x + \varepsilon h(u_x)_x.$$

Then, as the right-hand perturbations vanish, we prove convergence of the previous solutions to the entropy weak solution of the hyperbolic conservation law

$$u_t + f(u)_x = 0.$$

When $\delta = 0$ we reduce to the generalized Burgers equation and the approximate solutions $u^{\varepsilon,0}$ converge to the solution of the inviscid Burgers equation. This is the *vanishing viscosity method* [2]. On the other hand, when $\varepsilon = 0$ and considering the flux function $f(u) = u^2$ and the linear dispersion δu_{xxx}, we obtain the Korteweg-de Vries equation for which the approximate solutions $u^{0,\delta}$ do not converge [3]. So, as parameters ε and δ vanish, we are concerned with singular limits and a dominant dissipation regime is needed to ensure convergence [4].

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A Study of well posedness for systems of coupled non-linear dispersive wave equations

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Abstract. To model two-way propagation of waves in physical systems where nonlinear and dispersive effects are equally important, coupled systems of partial differential equations arise. The focus of this study is a particular coupled system of two evolution equations of generalized BBM-type. The coupling in these model systems is through the nonlinearities, which are a pair of homogenous quadratic polynomials. They have the same general mathematical structure as do more complicated models of surface and internal wave propagation.

The present study is concerned with initial-value problems wherein the wave profile and the velocity are specified at a starting time. This is a natural generalization of the initial-value problem for the BBM or regularized long-wave (RLW) equation itself, that was originally proposed as an alternative to the classical Korteweg-de Vries (KdV) equation. Equations like KdV and BBM model unidirectional propagation of small amplitude, long wavelength waves. The initial condition represents a snapshot of an initial disturbance. The coupled systems allow for two-way propagation of waves, and so have a wider range of applicability. They are, however, mathematically more intricate.

These coupled BBM-type systems are shown to be locally well-posed in the L_2-based Sobolev spaces H^s for any $s \geq 0$. The further spatial and temporal regularity of solutions is also investigated. It transpires that there is no smoothing in the spacial variable, but there is smoothing in the temporal variable.

Conditions are derived that imply the local well-posedness theory to extend globally. That is, under exact conditions on the coefficients of the quadratic nonlinearities, solutions are shown to exist and remain bounded for all time provided the initial data is at least in H^1. Using a Fourier splitting technique, global solutions are also inferred even when the initial data is only in L_2.

(Joint work with J. Bona and H. Chen)

Hopf fibrations for turbulent pipe flows

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Abstract. I propose a generalization of Hopf fibrations to quotient the streamwise translation symmetry of turbulent pipe flows viewed as dynamical systems. In particular, I exploit the geometric structure of the associated high dimensional state space, which is that of a principal fiber bundle. The relation between the comoving frame velocity U_d associated with the dynamical phase of an orbit in the bundle and the Taylors hypothesis is investigated. As an application, Laser-Induced-Fluorescence techniques are exploited to capture planar fluorescent dye concentration fields tracing a turbulent pipe flow at the bulk Reynolds number Re = 3200. The symmetry reduction analysis of the experimental data reveals that the speed u of dye concentration bursts is associated with the dynamical and geometric phases of the corresponding orbits in the fiber bundle. In particular, in the symmetry-reduced frame I unveil a pattern-changing dynamics of the passive scalar structures, which explains the observed speed $u = U_d + U_g$ of intense bursting events in terms of the geometric phase velocity $U_g = 0.43U_d$ associated with the orbits in the bundle [1,2].

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SESSION 12
Mathematical progress on nonlinear phenomena in parity-time-symmetric systems

Nonlinear eigenvalue problems and cPT symmetry

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Abstract. We discuss new kinds of nonlinear eigenvalue problems, which are associated with instabilities, separatrix behavior, and hyperasymptotics [1]. First, we consider the toy differential equation $y'(x) = \cos[\pi x y(x)]$, which arises in several physical contexts [2, 3, 4]. We show that the initial condition $y(0)$ falls into discrete classes: $a_{n-1} < y(0) < a_n$ ($n = 1, 2, 3, ...$). If $y(0)$ is in the nth class, $y(x)$ exhibits n oscillations. The boundaries a_n of these classes are strongly analogous to quantum-mechanical eigenvalues and calculating the large-n behavior of a_n is analogous to a semiclassical (WKB) approximation in quantum mechanics. For large n, $a_n \sim A\sqrt{n}$, where $A = 2^{5/6}$. The constant A is numerically close to the lower bound on the power-series constant P,

which plays a fundamental role in the theory of complex variables and which is associated with the asymptotic behavior of zeros of partial sums of Taylor series [5].

The first two Painleve transcendents P_1 and P_2 have a remarkable eigenvalue behavior. As $n \to \infty$, the nth eigenvalue for P_1 grows like $Bn^{3/5}$ and the nth eigenvalue for P_2 grows like $Cn^{2/3}$. We calculate the constants B and C analytically by reducing the Painlevé transcendents to linear eigenvalue problems in cPT-symmetric quantum mechanics [6].

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Families of nonlinear modes in complex potentials

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Abstract. After brief discussion of nonlinear modes in dissipative and parity-time symmetric parabolic and double-well potentials, we address the existence of continuous families of nonlinear modes in a general class of complex asymmetric potentials of the form $w^2(x) - iw_x(x)$, where $w(x)$ is a real function [1], in a medium with Kerr nonlinearity. We show that the families can bifurcate from the linear spectrum (an example of such modes in an asymmetric single-well potential was found numerically [2]). We introduce an asymmetric double-hump complex potential which supports continuous families of stable nonlinear modes [3].

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CPT symmetry in optical systems

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Abstract. We introduce a model of a dual-core optical waveguide with opposite signs of the group-velocity-dispersion (GVD) in the two cores (similar to recent work [1]), and a phase-velocity mismatch between them. The coupler is embedded into an active host medium, which provides for the linear coupling of a gain-loss type [2] between the two cores. The model is based on the system of coupled equations for field variable u and v in the two cores:

$$\begin{aligned} iu_z + (1/2)u_{tt} - qu + \sigma|u|^2u &= i\gamma v, \qquad (8)\\ iv_z - (1/2)v_{tt} + qv + \sigma|v|^2v &= i\gamma u, \qquad (9)\end{aligned}$$

where z is the propagation distance, t is the temporal variable, q is the mismatch, the GVD coefficients are scaled to be ± 1, σ is the nonlinearity coefficient, and γ is the coupling constant. The linear version of this system offers an optical realization of the *charge-parity-time* (CPT) symmetry in the class of models including linear gain and loss, akin to the broad class of PT-symmetric systems, while the inclusion of the cubic nonlinearity breaks the symmetry. By means of direct simulations, and using an analytical approximation based on the averaging method, it is demonstrated that the linear system generates expanding Gaussian states, while the nonlinear one gives rise to broad oscillating solitons, as well as a general family of stable stationary gap solitons. The oscillating solitons are actually located near the edge of the spectral bandgap; some gap solitons are found in an exact analytical form.

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Nonlinear Schrödinger dimer with gain and loss: integrability and PT-symmetry restoration

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Abstract. A PT-symmetric nonlinear Schrödinger dimer is a two-site discrete nonlinear Schrödinger equation with one site losing and the other one gaining energy at the same rate. A physically important example (occasionally referred to as the *standard* dimer) is given by

$$i\dot{u} + v + |u|^2u = i\gamma u, \quad i\dot{v} + u + |v|^2v = -i\gamma v.$$

(See [1-4] for references.) Another model with a wide range of applications is [5]

$$\begin{aligned} i\dot{u} + v + (|u|^2 + 2|v|^2)u + v^2u^* &= i\gamma u, \\ i\dot{v} + u + (|v|^2 + 2|u|^2)v + u^2v^* &= -i\gamma v. \end{aligned}$$

We construct two four-parameter families of cubic PT-symmetric dimers as gain-loss extensions of their conservative, Hamiltonian, counterparts. Our main result is that, barring a single exceptional case, all these damped-driven discrete Schrödinger equations define completely integrable Hamiltonian systems. Furthermore, we identify dimers that exhibit the nonlinearity-induced PT-symmetry restoration. When a dimer of this type is in its symmetry-broken phase, the exponential growth of small initial conditions is saturated by the nonlinear coupling which diverts increasingly large amounts of energy from the gaining to the losing site. As a result, the exponential growth is arrested and all trajectories remain trapped in a finite part of the phase space regardless of the value of the gain-loss coefficient.

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Integrable nonlocal nonlinear Schrödinger equation with PT symmetry

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Abstract. An integrable nonlocal nonlinear Schrödinger equation is introduced. It possesses a Lax pair and an infinite number of conservation laws and is PT symmetric. The inverse scattering transform and scattering data with suitable symmetries are discussed. A method to find pure soliton solutions is given. An explicit breathing one soliton solution is found. Key properties are discussed and contrasted with the classical nonlinear Schrödinger equation. This is a joint work with Mark J. Ablowitz [1].

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Mathematical progress on nonlinear phenomena in parity-time-symmetric systems

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Abstract. In this session we will present recent results on nonlinear parity-time-symmetric models, especially the nonlinear Schrödinger equation and PT-symmetric dimer systems. As for the NLS equation with a complex PT-symmetric potential, we consider using a hydrodynamic formulation and connecting the phase gradient to the field amplitude, which allows for a reduction of the model to a Duffing or a generalized Duffing equation satisfied by the amplitude of the stationary solution. After solving the generalized Duffing equation with cubic or cubic-quintic nonlinearity, we can obtain exact soliton solutions existing in the presence of suitable PT-symmetric potentials, and study their stability and dynamics. In particular, we presented cases where bright solitons in a single-component, PT -symmetric NLS equation are subject to an oscillatory instability when its parameter passes the threshold for PT-symmetry breaking. On the other hand, for defocusing nonlinearities, by continuously varying the PT-symmetric potential and initial guess, a saddle-center bifurcation reminiscent of the PT-phase transition was found between the ground state (plane wave in the Hamiltonian limit) and the first excited state (a dark soliton in the Hamiltonian limit).

This session will also include the case of a general PT-symmetric dimer in the context of two both linearly and nonlinearly coupled cubic oscillators, where the rotating wave approximation is used to convert it into a discrete nonlinear Schrödinger type dimer.

Besides, we'll consider a pair of waveguides which are evanescently coupled but may also be actively coupled and may possess onsite gain and loss, as well as (possibly non-uniform) nonlinearity. The examples include a number of studied PT-symmetric cases, as well as numerous cases that have not been previously considered. With these examples, we'll illustrate how Stokes variables can facilitate the exploration of PT-symmetric and more generalized dimer systems, both analytically and numerically.

Gain-driven discrete breathers in PT-symmetric nonlinear metamaterials

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Abstract. In this talk, we present our recent result ralted with a one-dimensional parity-time (PT)-symmetric magnetic metamaterial consisting of split-ring dimers having both gain and loss. Employing a Melnikov analysis we study the presence of transverse homoclinic orbits and homoclinic bifurcations which allow the emergence of hyperchaos. In the contrary to this result we prove that 'freezing of dimensionality' rules out the occurrence of hyperchaos. Our analytical results are found to be in good agreement with direct numerical computations.

Symmetry breaking of solitons in PT-symmetric potentials

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Abstract. It has been known that in dissipative systems with parity-time (PT) symmetry, continuous families of solitons are admitted, and such solitons are always PT-symmetric. In this talk, we report a surprising phenomenon that in certain special PT-symmetric systems, symmetry breaking of solitons can occur, i.e., continuous families of non-PT-symmetric solitons can bifurcate out from the base branch of PT-symmetric solitons when the base branch's power reaches a certain threshold. At the bifurcation point, the base branch of PT-symmetric solitons changes stability, and the bifurcated branch of non-PT-symmetric solitons can be stable. This symmetry breaking is demonstrated in the nonlinear Schrödinger equations with certain special forms of PT-symmetric potentials in both one and two dimensions. This talk is based on results from Refs. [1] and [2].

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PT symmetry in optics

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Abstract. Interest in complex non-Hermitian Hamiltonian problems in quantum mechanics was triggered after the seminal work of Bender and Boettcher [1]. In fact, they showed that a wide class of non-Hermitian Hamiltonians can exhibit entirely real spectra as long as they simultaneously respect parity (P) and time (T) symmetries. Of course, quantum mechanics is by nature a Hermitian theory and thus any evidence of PT symmetry in such settings is fundamentally out of reach. On the other hand, non-Hermiticity can be easily introduced in optical systems where gain or loss is involved. Based on this property, it has been recently suggested that notions from PT symmetry can be directly introduced in the optical domain [2,3]. In optics, the refractive index and gain/loss profiles play the role of the real and imaginary parts of the aforementioned complex potential respectively. In general an optical potential is PT-symmetric if its complex refractive index distribution ($n(\mathbf{r}) = n_R(\mathbf{r}) + in_I(\mathbf{r})$) satisfies the following condition: $n_R(-\mathbf{r}) = +n_R(\mathbf{r})$ and $n_I(-\mathbf{r}) = -n_I(\mathbf{r})$. As it has been indicated in several studies, PT-symmetric optical structures can exhibit peculiar properties that are otherwise unattainable in traditional Hermitian structures. Among them, is an abrupt phase transition from real to complex eigenvalues, band merging effect and unidirectional invisibility. Here we review recent developments in the field of PT-symmetric optics and photonics.

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SESSION 13
Water waves

On the Whitham equation

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Abstract. The Whitham equation was proposed by Whitham [6] as an alternative to the well known Korteweg-de Vries equation as an approximate water-wave model. In this context, the equation is given by

$$\eta_t + \frac{3}{2}\frac{c_0}{h_0}\eta\eta_x + K_{h_0} * \eta_x = 0,$$

where the convolution kernel K_{h_0} is given in terms of the Fourier transform by $FK_{h_0}(\xi) = \sqrt{g \tanh(h_0\xi)/\xi}$. Here g is the

gravitational acceleration, h_0 is the undisturbed depth of the fluid, and $c_0 = \sqrt{gh_0}$ is the corresponding long-wave speed.

In this talk, we will present recent results on the Whitham equation concerning existence and stability of traveling wave solutions, numerical approximation of solutions, and the validity of the equation as a water-wave model.

This is joint work with John Carter, Denys Dutykh, Mats Ehrnström, Kerry Kodama, Daulet Moldabayev, Nathan Sanford and Olivier Verdier.

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Traveling water waves with point vortices

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Abstract. We construct small-amplitude solitary traveling gravity-capillary water waves with a finite number of point vortices along the vertical axis, on finite depth. This is done using a local bifurcation argument. The properties of the resulting waves are also examined: We find that the properties depend significantly on the position of the point vortices in the water column.

Oscillations in hyperasymptotic series

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Abstract. The method of hyperasymptotic series was invented by Berry and Howls [1] to approximate more accurately solutions of Schrdinger-type ordinary differential equations (ODEs). We present here a variation on their method to find the hyperasymptotic series for the Airy function, *Ai(z)*, which solves an ODE of this type. Berry and Howls applied their method to exactly this problem in [1]; we analyze the same problem as they did in order to make clear how the two methods differ. Our most surprising result is that the hyperasymptotic series for *Ai(z)* for *z* ¿ 0 exhibits small oscillations, even though *Ai(z)* is not oscillatory for *z* ¿ 0. We show that these oscillations are a natural consequence of the formulation of a hyperasymptotic series.

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Toward a general solution of the three-wave resonant interaction equations

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Abstract. The resonant interaction of three wavetrains is the simplest form of nonlinear interaction for dispersive waves of small amplitude. Such interactions arise frequently in applications ranging from nonlinear optics to internal waves in the ocean through the study of the weakly nonlinear limit of a dispersive system. The slowly varying amplitudes of the three waves satisfy a set of integrable nonlinear partial differential equations known as the three-wave equations. If we consider the special case of spatially uniform solutions, then we obtain the three-wave ODEs. The ODEs have been studied extensively, and their general solution is known in terms of elliptic functions. Conversely, the universally occurring PDEs have been solved in only a limited number of configurations. For example, Zakharov and Manakov (1973, 1976) and Kaup (1976) used inverse scattering to solve the three-wave equations in one spatial dimension on the real line. Similarly, solutions in two or three spatial dimensions on the whole space were worked out by Zakharov (1976), Kaup (1980), and others. The known methods of analytic solution fail in the case of periodic boundary conditions, although numerical simulations of the problem typically impose these conditions.

To find the general solution of an nth order system of ordinary differential equations, it is sufficient to find a function that satisfies the ODEs and has n constants of integration. The general solution of a PDE, however, is not well defined and is usually difficult, if not impossible, to attain. In fact, only a small number of PDEs have known general solutions. We seek a general solution of the three-wave equations, which has the advantage of being compatible with a wide variety of boundary conditions and any number of spatial dimensions. We show that the general solution of the three-wave equations can be constructed using the known general solution of the three-wave ODEs. In particular, we try to construct the general solution of the three-wave equations using a Painlevé-type analysis. For now, we consider a convergent Laurent series solution (in time), which contains two real free constants and three real-valued functions (in space) that are arbitrary except for some differentiability constraints. In order to develop a full general

solution of the problem, the two free constants must also be allowed to have spatial dependence, and one more function must be introduced. That is, a full general solution of the problem would involve six of these real-valued functions.

Dissipation of narrow-banded, linear, deep-water waves

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Abstract. Studies ([1], [2], [3], [6], [4]) of narrow-banded, weakly nonlinear, deep-water waves show that dissipation is important in their propagation and evolution. Here we model the dissipation due to a contaminated air-water interface by considering two fluids separated by a thin fluid layer. This approach is a generalization of several models available in the literature that take into account various dissipative physical mechanisms. We use the new model to conduct a parameter study to determine which mechanisms are important for different frequency regimes. We further compare the new model and available models with laboratory and field data to find theoretical models that (i) agree with measured dissipation rates in laboratory and field experiments, and (ii) have the mathematical properties required to be useful in weakly nonlinear models of the evolution of waves with narrow-banded spectra, as they propagate over long distances on deep water. Much of this work is reported in [5] and [7].

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A scalar time-dependent equation for water waves

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Abstract. The equations that govern the motion of the free interface of an inviscid irrotational incompressible fluid are given by Euler's equations. However this set of equations has multiple reformulations. These equations have a Hamiltonian character as emphasized in the work of Zakharov [1] with canonical structure [2]. More recently, [1] provided a non-local formulation. In this talk we present a reformulation that combines ideas from the above works to obtain a single scalar equation for the free surface. We then derive various reduced asymptotic models obtained from this single equation and compare them to well known models for water waves.

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Frequency downshift in a viscous fluid

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Abstract. Frequency downshift, *i.e.* a shift in the spectral peak to a lower frequency, in a train of gravity waves was first reported by Lake *et al.* [2]. Many physical phenomena, including wave breaking and wind, have been proposed as mechanisms for frequency downshift. However, the precise cause of frequency downshifting remains an open question as the only generally agreed upon point is that it is related to the Benjamin-Feir instability.

In 2008, Dias, Dyachenko and Zakharov [1] added a viscous correction to the Euler equations in order to derive the dissipative NLS equation (DNLS). The dissipation in DNLS "stabilizes" the Benjamin-Feir instability. In this talk, we present a

generalization of the DNLS equation which predicts many of the effects associated with frequency downshift and attributes them to viscosity.

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Comparison of stability of solutions to Hamiltonian water wave models

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Abstract. The goal of this work is to compare and contrast the stability results for solutions to different models for water waves. It is known that high frequency instabilities exist for the nonlinear solutions to Euler's equations describing water waves [1, 2], however not all models exhibit these instabilities. We will use a generalization of the theory used to predict existence of high frequency instabilities in periodic Hamiltonian systems first proposed by MacKay [3] as well as Mackay and Saffman [4] to see which water wave models meet the necessary conditions for these instabilities to arise.

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SESSION 14
Coupled systems with hyperbolic components

The free-boundary Euler equations: existence and convergence results

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Abstract. We study the free boundary Euler equations in three spatial dimensions with surface tension. Under natural assumptions, we prove that solutions of the free boundary fluid motion converge to solutions of the Euler equations in a fixed domain when the coefficient of surface tension κ tends to infinity. The well-posedness of the free-boundary equations under the relevant hypotheses for the study of the singular limit $\kappa \to \infty$ is also established. Our existence theorem overlaps with known results in the literature, but parts of it are new.

Shape differentiability and hidden boundary regularity for the wave equations with Neumann boundary conditions

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Abstract. Shape differentiability is a fundamental question in shape optimization and control problems. While the shape derivative analysis has been considered and resolved for many classical linear problems, the hyperbolic situation is more delicate, due to the lack of good boundary regularity for the wave solution, which is a key ingredient in the differentiability analysis. We provide a full analysis of shape differentiability for the solution to the second order hyperbolic equation with Neumann boundary conditions and also discuss a hidden boundary regularity result, which we obtained through a new pseudo-extractor technique.

"Hidden" trace regularity of a tangentially degenerate wave equation and supersonic panel flutter

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Abstract. In the study of panel flutter (the principal phenomenon of interest in *aeroelasticity*) a canonical model [1] involves the coupling of: (i) a convected (or perturbed) wave equation of the form

$$(\partial_t + U\partial_x)^2\phi = \Delta_{x,y,z}\phi \tag{10}$$

on the half-space, and (ii) a clamped von Karman plate on the boundary. (U corresponds to the unperturbed flow velocity in the x-direction.) The strong coupling occurs in a Neumann condition for the wave equation and in the dynamic pressure term for the plate. In studying well-posedness and stability of this model in the *supersonic regime* $(U > 1)$—where flutter is a principal concern—we note that the (10) becomes *degenerate* in the sense that traditional hyperbolic "energies" are no longer positive. By considering the state variable $D_t\phi \equiv (\partial_t + U\partial_x)\phi$ (rather than ϕ_t) we are able to show well-posedness of the supersonic model *contingent upon* the trace term $\int_{\partial \mathbf{R}^3_+} tr[D_t\phi]u_x d\Omega$ being "energy bounded" in the sense of finite-energy solutions [3].

Thus, owing to the importance of the trace regularity associated to (10) [2], we investigate a more general *tangentially degenerate* wave equation on the half-space (or on a bounded domain). We show, using microlocal methods, that the equation admits hidden trace regularity of the form: $\mathrm{tr}[\phi_t + U\phi_x] \in L^2(0,T;H^{-1/2}(\partial\Omega))$, where $\Omega \subset \mathbf{R}^3_+$ is a compact, flat portion of the Neumann boundary of the flow domain.

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Optimal control in a free boundary fluid-elasticity interaction

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Abstract. We consider an optimal control for the problem of minimizing flow turbulence in the case of a nonlinear fluid-structure interaction model. If the initial configuration is regular, then a class of sufficiently smooth control inputs contains an element that minimizes, within the control class, the vorticity of the fluid flow around a moving and deforming elastic solid. We establish this existence and derive the first order optimality conditions on the optimal control.

Global attractors for Full von Karman systems with thermal effects

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Abstract. We consider long time behavior of a dynamics prescribed by full von Karman system which consists of nonlinear plate equation strongly coupled to a dynamic system of elasticity. This model is used in representing oscillations of a shell on a compact manifold. The nonlinearity displayed by the system is supercritical with respect to finite energy space -say H. However, it can be shown by using compensated compactness methods that the PDE system is a dynamical system on H and the solutions exhibit undamped oscillations. It has been also known that mechanical dissipation added to the system allows to eliminate the oscillations. The main goal of this talk is to show that mechanical dissipation is not necessary and one can forge the desired long time behavior by taking an advantage of thermal effects. This result is obtained by showing that the solutions satisfy quasi-stability estimate which is the key in proving attractiveness properties of the dynamics. The latter owns its validity due to "hidden" compactness and propagation of partial analyticity through the interface between plate and the wave.

This is a joint work with Rodrigo Monteiro from San Paolo University, Brasil.

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Global uniqueness and stability of an inverse problem for the Schrödinger equation on a Riemannian manifold via one boundary measurement

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Abstract. We consider a mixed problem for the Schrodinger equation on a finite dimensional Riemannian manifold with magnetic and electric potential coefficients and non-homogeneous Dirichlet boundary term. The goal is the nonlinear problem of the recovery of the electric potential coefficient by means of only one boundary measurement on an explicitly identified portion of the boundary. We obtain global uniqueness of the recovery and Lipschitz stability of the recovery exclusively on an arbitrarily short time-interval in terms of the data of the problem. Two key ingredients are: (i) a Carleman estimate for the Schrodinger equation on a Riemannian manifold [2]; (ii) optimal interior and boundary regularity of the direct mixed problem [1].

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Fluid-composite structure interaction

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Abstract. Composite materials appear in virtually all areas of engineering and in nature. Examples include engineering structures such as boats and aircrafts, or, in biological applications, blood vessels of major human arteries. No mathematical results exist so far that analyze solutions to fluid-structure interaction problems with composite structures.

In this talk we make a first step in this direction. We present an *existence result* for a weak solution to a fluid-structure interaction (FSI) problem between an incompressible, viscous fluid flowing in a cylinder with elastic walls composed of two layers: a thin layer modeled by the linearly elastic membrane shell, and the thick layer modeled by the equations of linear elasticity. This set up was motivated by blood flow in human arteries whose walls are composed of several different layers, each with different mechanical characteristics and thickness. The coupling between the three different models (the fluid, the thin structure, and the thick structure) is accomplished at a deformed fluid-structure and structure-structure interface, via two sets of coupling conditions: continuity of velocity (no-slip) and balance of contact forces. The resulting mathematical model is a time-dependent, *nonlinear moving-boundary problem* of hyperbolic-parabolic type. The existence proof, see [1], is based on an operator splitting method. This method was used by the authors in the design on a *partitioned numerical scheme* for the solution of the underlying multi-physics FSI problem [2]. Our theoretical and numerical results reveal a *new physical regularizing mechanism* for this class of problems: the inertia of the fluid-structure interface with mass regularizes the evolution of the entire solution.

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SESSION 15
Geometric, algebraic and analytic approaches to integrable systems

Lax pars of discrete Painlevé equations arising from the integer lattice

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Abstract. Construction of the Lax pairs of the ordinary difference equations called discrete Painlevé equations from the those of the partial difference equations called ABS equations via the periodic reduction are well investigated (for example, see [1]).

In this talk we will show a method to obtain the Lax pairs of discrete Painlevé equations by using the integer lattice associated with ABS equations in detail by taking an example of q-Painlevé equations of $A_5^{(1)}$-surface type in Sakai's classification [2]. This work supported by the Australian Research Council grant ♯DP130100967.

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Exceptional orthogonal polynomials and generalized Jacobi matrix

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Abstract. Classical orthogonal polynomials (COPs), which are defined as a polynomial eigenfunction of the Sturm-Liouville problem:

$$\Big(A(x)\partial_x^2 + B(x)\partial_x + C(x)\Big)\, p_n(x) = \lambda_n p_n(x), \qquad (11)$$

have been fundamental objects in mathematical physics, approximation theory and so on. Recently, an extension of COPs was introduced as a polynomial eigenfunction of the Sturm-Liouville problem whose several degrees are missed but still

complete with respect to the corresponding space [1, 2]. They are called exceptional orthogonal polynomials (XOPs) and are applied to construct new superintegrable models.

As XOPs are no longer ordinary COPs, the properties of XOPs are different from those of COPs. For instance, COPs hold three term recurrence relations which are related to Jacobi matrices although XOPs do not. In this talk, we will show that XOPs hold recurrence relations with more than three terms regarded as generalized Jacobi matrices. This talk is based on jointwork with S. Tsujimoto at Kyoto University [3].

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Painlevé equations and weight systems

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Abstract. A weight (a tuple of integers) is one of the invariants of the Painlevé equation determined by the Newton diagram of the equation. The Painlevé equations are systematically studied by means of the weighted projective spaces associated with the weights.

Furthermore, it will be shown that the weights for the Painlevé equations are closely related to Saito's theory of weight systems. For each regular weight in Saito's theory, the corresponding Painlevé equation and its Hamiltonian function can be uniquely recovered. In this talk, several 4-dim Painlevé equations will be constructed from suitable Saito's weights.

Here, I give a (part of) list of weights of the 2-dim and 4-dim Painlevé equations. Notations will be explained in my talk.

	$p, q; H$	κ
(P_I)	2, 3 ; 6	6
(P_{II})	1, 2 ; 4	4
(P_{IV})	1, 1 ; 3	3

	$p_1, q_1, p_2, q_2; H$	κ
$(P_I)_2$	2, 5, 3, 4 ; 8	2,5,8
$(P_{II})_2$	1, 4, 2, 3 ; 6	2,3,6
$(P_I) \times (P_I)$	2, 3, 2, 3 ; 6	2,3,6
no name(?)	1, 3, 2, 2 ; 5	1,3,5
coupled (P_{II})	1, 2, 1, 2 ; 4	1,2,4
coupled (P_{IV})	1, 1, 1, 1 ; 3	1,1,3

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Irregular conformal field theory and Painlevé tau functions

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Abstract. The tau functions defined by the Hamiltonians H_{J} of the Painlevé equations $\mathrm{P_J}$ $(\mathrm{J} = \mathrm{I}, \dots, \mathrm{VI})$ as

$$H = \frac{d}{dt} \log \tau(t)$$

play a central role in the study of the Painlevé functions, such as the construction of Bäcklund transformations.

Recently, O. Gamayun, N. Iorgov and O. Lisovyy found the sixth Painlevé tau function has the Fourier expansion in terms of the four point conformal block of Virasoro conformal field theory. Hence, the sixth Painlevé tau function has an explicit series expansion because the conformal blocks admit explicit representation by the AGT correspondence. Later, by taking scaling limits, explicit series expansions of the fifth and third Painlevé tau functions were obtained in [2], which are expansions at regular singular points.

On the other hand, series expansions of the Painlevé tau functions in terms of irregular conformal blocks at irregular singular points have been not obtained. This is because of the following two reasons: (i) We could not take confluent limits or scaling limits of known series expansions of the tau functions at regular singular points becoming irregular singular points after taking limits. (ii) We lack a precise definition of irregular conformal blocks.

In my talk, I propose a definition of irregular conformal blocks with at most two irregular singular points. Then, we give conjectural formulas of the fourth and fifth Painlevé tau functions in terms of irregular conformal blocks at an irregular singular point.

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Symmetry and combinatorics of discrete integrable systems

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Abstract. We explore the relation between the geometric representation of affine Weyl groups and different classes of discrete integrable systems. In particular, the two classes of systems of interest are the discrete Painlevé equations [1] and discrete analogues of the integrable PDEs known as quad-equations [2]. By using the tools from representation theory of affine Weyl groups, we give a unified description of the different discrete integrable systems based on the geometric and combinatorial aspects of the root systems. The fundamental object in this construction is the Voronoi cell of the root lattice. The Painlevé variables and the variables of the quad-equation are represented as different sub-structures of the Voronoi cell, and their dynamics are described by the actions of the root systems.

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Delay Painlevé equations

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Abstract. Delay Painlevé equations are nonlinear differential-functional equations governing a function $f(z)$ of one variable, which also contain iterates of f, such as $f(z-1)$. These are integrable in the sense that they have Lax pairs, that is, they are compatibility conditions for associated linear equations. In this talk, we focus on the so-called first delay Painlevé equation which is obtained as a reduction of the Toda lattice [1, 2] and describe new properties of such equations.

This research was supported by an Australian Laureate Fellowship #FL120100094 from the Australian Research Council.

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New algebro-geometric approach to the Schlesinger equations and Poncelet polygons

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Abstract. In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI $(1/8, -1/8, 1/8, 3/8)$ equation starting with any Poncelet polygon inscribed in a conic and circumscribed about another conic. We will show that Hitchins construction can be generalized and embedded in the Okamoto transformation between the Picard solution and the general solution of the Painlevé VI $(1/8, -1/8, 1/8, 3/8)$ equation. Moreover, we will show that this Okamoto transformation can be presented in an invariant, geometric way, in terms of an Abelian differential of the third kind on the associated elliptic curve. The last observation allows us to obtain solutions to the corresponding 2×2 Schlesinger system with four poles in terms of this differential as well. The solution of the Schlesinger system admits a natural generalization to the case with 2g + 2 poles and corresponding Riemann surfaces of genus g. These higher genera solutions, specialized to the case of rational parameters, are related to higher-dimensional Poncelet polygons associated to g confocal quadrics in g + 1 dimensional space, closing the loop with the initial Hitchins remarkable observation.

Bergman tau-function and Witten classes

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Abstract. he Bergman tau-function can be viewed as a generalization of Dedekind’s eta-function to higher genus. In physical terms, it is a partition function of free bosons on a Riemann surface with flat singular metric. Mathematically, it arises as isomonodromic tau-function of special Riemann-Hilbert problems; it also appears in the problem of holomorphic factorization of determinant of Laplace operator. Analyzing analytical properties of the tau-function one can derive several (both new and known before) results on geometry of moduli spaces (including, for example, Mumford’s relation between determinantal line bundles on moduli spaces). Applying similar machinery to moduli spaces $M_{g,n}$ of punctured Riemann surfaces, one can give an analytical derivation of the formula for Hodge class on $M_{g,n}$ in terms of Witten’s cycles, which are special combinatorially defined submanifolds of $M_{g,n}$. The talk is based on joint works with M.Bertola, A.Kokotov and P.Zograf.

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SESSION 16
Advances using the unified transform method

Gibbs phenomenon for dispersive PDEs on the line

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Abstract. The classical Gibbs phenomenon is an artifact of non-uniform convergence. More precisely, it arises from the approximation of a discontinuous function with the analytic partial sum of the Fourier series. It is known from the work of DiFranco and McLaughlin [1] that a similar phenomenon occurs when a box initial condition is taken for the free Schrödinger equation in the short-time limit. Our work is focused extending the linear theory of their work in two ways. First, we establish sufficient conditions for the classical smoothness of the solutions of linear dispersive equations for positive times. Second, we derive an oscillatory and computable short-time asymptotic expansion of the solution. The Wilbraham–Gibbs constant is identified as a limiting overshoot value. Boundary-value problems can also be treated via the Unified Transform Method.

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New transform methods for Laplace's equation in circular domains

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Abstract. This paper presents a new transform approach to solving boundary value problems for Laplace's equation in circular domains [1], including multiply connected cases [2]. Circular domains are those with boundaries made up of circular arc and/or straight line segments. Our results are an extension of the formulation of Fokas & Kapaev [3] which is restricted to convex polygonal domains (just straight line boundaries). The generalized formulation is developed by first giving an elementary rederivation of the results of [3] for polygons and then extending it in a natural way to circular domains. Illustrative examples of the versatility of the new method will be showcased, including a presentation of quasi-analytical solutions to some classic boundary value problems previously only solvable by purely numerical means or approximate techniques.

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The time-dependent Schrödinger equation with piecewise constant potential

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Abstract. In quantum mechanics, the linear Schrödinger equation with piecewise constant potential receives a lot of attention, as it is one of the few solvable models which shares a lot of qualitative features with many physically important potentials. Textbook examples include the particle in a box, tunneling, etc. In all of these examples, attention is restricted to the time-independent Schrödinger equation. To our knowledge the solution of the time-dependent problem is open. Combining the Unified Transform Method due to Fokas with recent insights about interface problems, we present fully explicit solutions of the time-dependent problem.

Birkhoff regularity and well-posedness of linear initial-boundary value problems

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Abstract. A classification of two-point ordinary differential operators into regular, simply irregular and degenerate irregular has evolved since Birkhoff's pioneering work a century ago. Regular operators have many of the desirable properties of self-adjoint operators, including basis-liked properties of their eigenfunctions. It is known that degenerate irregular operators lack such properties. Nevertheless, we show that the Unified Transform Method can be used to solve initial-boundary value problems associated with certain degenerate irregular spatial operators, and that the solution representation thus obtained is an expansion in certain spectral functionals of the operator.

SESSION 17
Waves, dynamics of singularities, and turbulence in hydrodynamics, physical, and biological systems

Two color filament beam interaction in air

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Abstract. Filamentation in air is a fast growing area of research both in the experimental and theoretical fronts. Most of the research has been conducted on quasimonochraomatic filament dynamics. In the search for improving filament stability over longer propagation distances, efforts have shifted towards using non-gaussian beams such as Bessel, Airy and ring vortex beams. In particular, the recent experimental work in [1], considers a new type of filament propagation using a ”dressed” state. This idea, lead us to theretically investigate the co-propagation of two color (Ultraviolet and Infrared) beams with corresponding temporal widths in the nanosecond (UV) and femtosecond(IR) regimes. In this paper, we first present results in a simplified time-independent model demonstrating the existence of co-propagating UV and IR stationary beams [2]. We then extend our investigation to a more realistic model where temporal effects are included [3].

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New perspectives on wave turbulence

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Abstract. Weak turbulence theory is a kinetic theory for weakly nonlinear dispersive wave turbulence. Recently, there are many new theoretical progresses. Here, we discuss a theoretical framework that allows us to treat the non-perturbative nature of wave turbulence in strongly nonlinear regimes by emphasizing the importance of renormalized dispersion relationship in controlling wave resonances in turbulence regime. We demonstrate that nonlinear wave interactions renormalize the dynamics, thus modifies classical pictures of wave turbulence. Finally, we present an extension of the weak turbulence kinetic theory to systems with strong nonlinearities.

Finding the Stokes wave: From low steepness to the highest wave

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Abstract. A fully nonlinear periodic wave travelling over a free surface of incompressible ideal fluid of infinite depth is known as the Stokes wave.

In this work [1] we solve the Euler equations with free surface in the framework of conformal variables via Newton Conjugate Gradient method and find Stokes waves in regimes dominated by nonlinearity. By investigating Stokes waves with increasing steepness we observe peculiar oscillations occur as we approach Stokes limiting wave. Finally by analysing Pade approximation of Stokes waves we infer that analytic structure associated with those waves has branch cut nature.

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Necklace solitary waves on bounded domains

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Abstract. We present a new type of solitary waves of the two-dimensional cubic nonlinear Schrödinger equation on bounded domains. These multi-peak “necklace” solitary waves consist of several identical localized profiles (“pearls”), such that adjacent “pearls” have opposite signs. We observe numerically that necklace solitary waves on bounded domains are stable at low powers, but become unstable at powers well below the critical power for collapse. In contrast, the corresponding ground-state (“single-pearl”) solitary waves are always stable. The necklace instability is excited by perturbations that break the antisymmetry between adjacent pearls, and thus lead to power transfer between pearls. In particular, necklace instability is unrelated to collapse. In order to compute numerically the profile of necklace solitary waves on bounded domains, we introduce a non-spectral variant of Petviashvili’s renormalization method.

The derivation of 1D focusing NLS from 3D quantum many-body evolution

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Abstract. We consider the focusing 3D quantum many-body dynamic which models a dilute bose gas strongly confined in two spatial directions. We assume that the microscopic pair interaction is attractive and given by $a^{3\beta-1}V(a^{\beta}\cdot)$ where $\int V \leq 0$ and a matches the Gross-Pitaevskii scaling condition. We derive rigorously the 1D focusing cubic NLS as the mean-field limit of this 3D focusing quantum many-body dynamic and obtain the exact 3D to 1D coupling constant.

Circular instability of a standing wave: numerical simulation and wave tank experiment

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Abstract. We performed full-scale numerical simulation of instability of weakly nonlinear standing waves on the surface of deep fluid in the framework of the primordial dynamical equations. This instability, which was discovered in [1], is present for both capillary and gravity standing waves. Both cases were studied separately in the numerical simulation. The instability offers a new approach for generation of nearly isotropic spectrum using parametric excitation in the laboratory wave tank experiments.

Ina laboratory wave tank experiment we used parametric excitation in order to generate a standing wave which through the new instability mechanism mentioned above generate nearly isotropic spectrum. Direct measurements of spacial Fourier spectrum confirm existence of the instability in a real life conditions for gravity waves.

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Evolution equations in Mathematical Biology: Cellular and network processes

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Abstract. Due to the impressive recent progress in both acquiring new experimental data and in the computational power needed to analyze and model these data, research efforts in Mathematical Biology are becoming crucial for our understanding of the biological processes that take place at the cellular and network level. Recognizing the need for communicating advances in mathematical approaches to biological systems, our workshop brings together researchers working on various aspects of the biological systems such as chemotaxis, cellular processes, dynamics of biological fluids, cancer, infectious diseases, connectomics, neuron and brain population dynamics, to name a few. The exchange of ideas, results, and mathematical approaches to biological problems will foster a creative environment for better understanding of the structures studied. Furthermore, this exchange will likely also result in applying some of the insights from one area to other systems and in generating new exciting questions as a way to establish new research directions.

Long-time numerical integration of the generalized nonlinear Schrödinger equation with time steps exceeding the instability threshold

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Abstract. Some applications, e.g., modeling of wave turbulence, rogue events, propagation in random potential, and pulse oscillations in potential wells, require simulation of many realizations of nonlinear wave evolution over long times. In such applications, accuracy of an individual simulation does not need to be too high (say, 0.1–0.01% would typically suffice); rather, the focus is on obtaining accurate *statistical* information of the evolution. For such simulations, with relatively low individual accuracy, it is natural to use relatively large time steps. This may be especially relevant for 2D and 3D simulations, where one long-time simulation may take tens of hours. However, using larger time steps may cause numerical instability (NI). Therefore, finding ways to suppress such numerical instability while retaining the ability to use larger time steps appears to be of interest.

We considered two methods of suppressing NI for the generalized nonlinear Schrödinger equation (gNLS):

$$iu_t + u_{xx} + (|u|^2 + V(x))u = 0.$$

These methods differ by the type of problem that they can be applied to: with vanishing and non-vanishing boundary conditions. The same ideas will apply in 2D and 3D.

When zero boundary conditions can be used (example: pulse oscillations in a potential well), we have theoretically explained and numerically demonstrated that a smooth absorber at the boundaries suppresses NI of the split-step Fourier method (SSM) and allows one to use time steps that exceed the stability threshold by a factor of about three. The fact that an absorber at the boundary suppressed NI of the SSM for the 1D NLS was pointed out in [1]; however, that paper did not present either an explanation of that phenomenon or its study in the presence of a potential in the gNLS. We will do so here; in particular, we will show that the absorber is effective for bounded potentials, but not for the "unbounded" quadratic potential $V \propto x^2$.

In simulations of wave turbulence and rogue events, absorbing boundaries cannot be used. We have theoretically explained an numerically demonstrated that using an exponential time-differencing leapfrog scheme (ETD-LF),

$$e^{ik^2\Delta t}F[u_{n+1}] - e^{-ik^2\Delta t}F[u_{n-1}] = 2i\gamma\Delta t\,\mathrm{sinc}(k^2\Delta t)\,F[(|u_n|^2 + V(x))u_n] \quad (12)$$

instead of the SSM, significantly reduces the growth rate of numerically unstable modes. (In (12), $F[u]$ is the Fourier transform of u and Δt is the time step.) Using (12), we have demonstrated a speedup of two to three times compared to the case where the SSM operates just below its instability threshold; an even greater speedup is achieved over the ETD Runge–Kutta method. The ETD-LF scheme, however, is prone to certain nonlinear numerical instabilities [2], which do not occur for the SSM. We will discuss when those instabilities can be avoided or ameliorated.

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Effective dispersion relation of the nonlinear Schrödinger Equation

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Abstract. The linear part of the Nonlinear Schrödinger Equation (NLS) ($iq_t = q_{xx}$) has dispersion relation $\omega = k^2$. We don't necessarily expect solutions to the NLS to behave nicely or have any kind of effective dispersion relation, since we expect nonlinear waves to be strongly coupled and not sinusoidal in time. However, I have seen that solutions to the NLS are actually weakly coupled and are often nearly sinusoidal in time with a dominant frequency. In fact, when I look at long-time average of either a solution with many solitons or with many unstable modes, the power spectral density does indicate a quadratic dispersion relation that has been shifted by a constant proportional to the amplitude of the initial condition: $\omega = k^2 - 2A$ where $A = \frac{\|\hat{q}(k,0)\|^2}{2\pi}$. I will show a number of plots confirming this.

Branch cut singularity of Stokes wave on deep water

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Abstract. Complex analytical structure of Stokes wave for two-dimensional potential flow of the ideal incompressible fluid with free surface and infinite depth is analyzed. Stokes wave is the fully nonlinear periodic gravity wave propagating with the constant velocity [1,2]. We consider Stokes wave in the conformal variables which maps the domain occupied by fluid into the lower complex half-plane. Then Stokes wave can be described through the position and the type of complex singularities in the upper complex half-plane. Similar idea was exploited for other hydrodynamic systems in different approximations, see e.g. [3-6]. We studied fully nonlinear problem and identified that this singularity is the square-root branch point. That branch cut defines the second sheet of the Riemann surface if we cross the branch cut. Second singularity is also the square-root and located in that second (nonphysical) sheet of the Riemann surface in the lower half-plane. Crossing corresponding branch cut in second sheet one arrives to the third sheet of Riemann surface with another singularity etc. As the nonlinearity increases, all singularities approach the real line forming the classical Stokes solution (limiting Stokes wave) with the branch point of power 2/3. We reformulated Stokes wave equation through the integral over jump at the branch cut which provides the efficient way for finding of the explicit form of Stokes wave [7]. Simulations with the quadruple (32 digits) and variable precisions (up to $\sim$ 200 digits) are performed to find Stokes wave with high accuracy and study the Stokes wave approaching its limiting form with $2\pi/3$ radians angle on the crest.

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Low-damping transition times in a ferromagnetic model system

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Abstract. I study an idealized model of the stochastic Landau-Lifshitz-Gilbert equation to determine waiting times between noise-induced magnetization reversals. The zero-damped spatially-extended model is a wave equation with nonlinear forcing and stochastic initial conditions. I expose the non-trivial dependence on the spatial domain size: large domains have exponentially-scaling transition times due to entropic effects, whereas transitions in small domains depend on initial conditions; the system is effectively described by a single degree of freedom hamiltonian system. These observations then guide selecting the correct technique for determining the asymptotic scaling of transition times in the low-damping regime of the model: large deviation theory for large domains, and stochastic averaging on the effective dynamics for small domains.

Waveaction spectrum for fully nonlinear MMT model

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Abstract. The Majda-McLaughlin-Tabak (MMT) model describes a one-dimensional system exhibiting dispersive wave turbulence. In this model, the derivative of each Fourier mode includes linear and nonlinear terms. We consider a modified version of the model where the linear term is absent. We predict, based on the trivial resonances in the nonlinear term, the presence of an effective dispersion relation. This effective dispersion relation can then be used, along with an effective kinetic equation, to predict the waveaction spectra of the system. These spectra are found to agree with the results of time-dynamics simulations obtained using fourth-order Runge Kutta iteration. Measurements of the power spectral density for such simulations are consistent with the predicted effective dispersion relation. Accurate predictions for the spectra are also made using arguments based on scaling symmetry and the principle of flux balance. For the case that the driving and damping are removed from the system, there are special cases of the initial conditions for which accurate predictions for the spectra and other statistical measures may be obtained analytically by ensemble averaging.

Dynamics and singularity formation under the mean curvature flow

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Abstract. The mean curvature flow appears naturally in the motion of interfaces in material science, physics and biology. It also arises in geometry and has found its applications in topological classification of surfaces. In this talk I will discuss recent results on formation of singularities under this flow. In particular, I will describe the 'spectral' and 'dynamical' picture of singularity formation and the notion of effective dynamics. Some of the results are obtained jointly with Zhou Gang and Dan Knopf.

Cascades in nonlocal turbulence

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Abstract. We consider developed turbulence in 2D Gross-Pitaevsky model, which describes wide classes of phenomena from atomic and optical physics to condensed matter, fluids and plasma. Well-known difficulty of the problem is that the hypothetical local spectra of both inverse and direct cascades in weak-turbulence approximation carry fluxes which either are zero or have a wrong sign; such spectra cannot be realized. We derive analytically the exact flux constancy laws (analogs of Kolmogorov's 4/5-law for incompressible fluid turbulence), expressed via the fourth-order moment and valid for any nonlinearity. We confirmed the flux laws in direct numerical simulations. We show that a constant flux is realized by non-local wave interaction both in direct and inverse cascades. Wave spectra (second-order moments) are close to slightly (logarithmically) distorted thermal equilibrium in both cascades.

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Stability of modelocked lasers with slow saturable absorbers

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Abstract. Sinclair *et al.* [1] have developed a highly robust and transportable fiber laser comb source that operates at 1.55 μm. It is built with telecom-grade polarization maintaining (PM) components, including highly nonlinear PM fiber (HNLF), a highly-doped erbium-doped fiber (EDF), and a semiconductor saturable absorber mirror (SESAM). The HNLF provides nonlinearity and dispersion; the EDF provides slow saturable gain, and the SESAM provides saturable loss. Using all PM components enables the laser to be highly robust and transportable. Using telecom-grade components enables the laser to be a relatively inexpensive comb source ($< \$50,000$ in parts). The SESAM is a slow saturable absorber, in which the response time is much greater than the pulse duration, leading to a gain window that follows the pulse as it propagates. As a consequence, background noise can grow unstably in the wake of the pulse. This instability leads to the generation of a new pulse that steals the energy from the original pulse and ultimately replaces it. This behavior repeats periodically or quasi-periodically. The appearance of this wake instability sets a lower limit on the magnitude of the chromatic dispersion and an upper limit on the pulse power [2].

We have constructed both an averaged model of this system — in which the effect of all components is averaged over a single round trip — and a lumped model — in which the effect of each laser component on a pulse is treated separately. We use both models to study the stability of the laser system and to optimize its parameters. We study the stability using both evolutionary techniques and analysis of the linearized spectrum [3]. We discuss tradeoffs among simplicity, accuracy, and computational speed with both models.

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Cnoidal wave as a limit of N-solitonic solutions and aperiodic one-gap potentials

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Abstract. We derive a system of singular integral equations describing one-gap reflectionless potential in the stationary one-dimensional Schrodinger equation. We show that all these potentials are limits of N-solitonic solutions if $N \to \infty$. In a general case one-gap potentials are described by two positively defined continuous "dressing functions" satisfying the Holder condition. We found the dressing functions for a very special case when potentials are periodic and are expressed in terms of elliptic functions. For these cases, the potential describes knoidal waves for Hamiltonian evolutionary equations belonging to the KdV hierarchy.

Subtracting complex singularity from the Stokes wave

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Abstract. Two-dimensional potential flow of the ideal incompressible fluid with free surface and infinite depth has class of solutions called Stokes waves which is the fully nonlinear gravity waves propagating with the constant velocity. The increase of the scaled wave height H/λ, where H is the wave height and $H\lambda$ is the wavelength, from $H/\lambda = 0$ to the critical value H_{max}/λ marks the transition from almost linear wave to a strongly nonlinear limiting Stokes wave. Fully nonlinear Euler equations describing the flow can be reformulated in terms of conformal map of the fluid domain into the complex lower half-plane, with fluid free surface mapped into the real line. This description is more convenient for analysis and numerical simulations. Analyzing the spectrum of the solution in conformal variables one can see that the distance v_c from the closest singularity in the upper half-plane to the real line goes to zero as we approach the limiting Stokes wave [1]. Here we subtract

this singularity from the solution in order to have faster decay in the spectrum so we can consider the limit $v_c \to 0$.

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SESSION 18

Advances in integrable systems and nonlinear wave theory

On the zeros of large degree Vorob'ev-Yablonski polynomials

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Abstract. It is well known (cf. [4]) that all rational solutions of the second Painlevé equation and its associated hierarchy can be constructed with the help of Vorob'ev-Yablonski polynomials and generalizations thereof. The zero distribution of the aforementioned polynomials has been analyzed numerically by Clarkson and Mansfield [4] and the authors observed a highly regular and symmetric pattern: for the Vorob'ev polynomials itself the roots form approximately equilateral triangles whereas they take the shape of higher order polygons for the generalizations.

Very recently Buckingham and Miller [2, 3] completely analyzed the zero distribution of large degree Vorob'ev-Yablonski polynomials using a Riemann-Hilbert/nonlinear steepest descent approach to the Jimbo-Miwa Lax representation of PII equation. In our work [1] we rephrase the same problem in the context of orthogonal polynomials on a contour in the complex plane. The polynomials are then analyzed asymptotically and the zeroes localized through the vanishing of a theta divisor on an appropriate hyperelliptic curve.

Our approach starts from a new Hankel determinant representation for the square of the Vorob'ev-Yablonski polynomial. This identity is derived using the representation of Vorob'ev polynomials as Schur functions.

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Higher-rank Schlesinger transformations and difference Painlevé equations

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Abstract. The relationship between isomonodromic deformations and differential Painlevé equations is quite well-known: Schlesinger equations describing isomonodromic deformations of a two-dimensional Fuchsian system with four poles on the Riemann sphere reduce to the most general Painlevé VI equation, and other differential Painlevé equations can be obtained from this by coalescence. In fact, the isomonodromic approach is one of the most powerful ways to study properties of Painlevé equations and their solutions, the Painlevé transcendents. Similar isomonodromic representation exists for difference Painlevé equations as well. In this case, instead of continuous deformations of the poles of our Fuchsian system we consider Schlesinger transformations, which are a special kind of gauge transformations that change the characteristic indices of the system by integral shifts, and so they are also isomonodromic. In [DST13] we wrote explicit evolution equations for a discrete dynamical system given by rank-one elementary Schlesinger transformations and considered their reductions to difference Schlesinger equations. However, classes of difference Painlevé equations that have the largest symmetry groups, $E_7^{(1)}$ and $E_8^{(1)}$ in the Sakai's classification scheme [Sak01], correspond to Schlesinger transformations of Fuchsian systems with degenerate eigenvalues. In this talk we explain how to extend our evolution equations to higher-rank elementary Schlesinger transformations and, show, as an example, how to obtain difference Painlevé equations with the symmetry group $E_7^{(1)}$ from Schlesinger transformations.

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Long time asymptotics for Gross-Pitaevskii and asymptotic stability of N-soliton solutions

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Abstract. We consider the Cauchy problem for the Gross–Pitaevskii (GP) equation. Using the $\bar{\partial}$ generalization of the nonlinear steepest descent method of Deift and Zhou we derive the leading order approximation to the solution of GP in the solitonic region of space–time $|x| < 2t$ for large times and provide bounds for the error which decays as $t \to \infty$ for a general class of initial data whose difference from the non vanishing background possess's a fixed number of finite moments and derivatives. Using properties of the scattering map of NLS we derive as a corollary an asymptotic stability result for initial data which are sufficiently close to the N-dark soliton solutions of (GP).

A close-up look at the semiclassical sine-Gordon equation

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Abstract. We will focus on semiclassical sine-Gordon equation with pure impulse initial data. It is known for sine-Gordon equation to have two types of soliton solutions, the breather and the kinks. In the pure impulse case, the type of waves generated depends on the magnitude of initial data. If initial data small, then only breathers are generated at first. However, when time evolves, the system starts to exhibit a phase transition. The phase transition is universal in the sense that it only depends on initial data, not parameter ϵ. In this talk, I want to give some analytical and numerical results about what is happening near these phase transitions.

Hurwitz numbers and random matrix theory

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Abstract. The Harish-Chandra-Itzykson-Zuber (HCIZ) integral, $I_{A,B,\tau} := \int_{U(n)} e^{\tau tr(AUBU^*)} dU$ where dU is the Haar measure on $U(n)$, can be computed explicitly as an expression involving only the eigenvalues of A and B. This integral plays an important role in matrix models: in particular, it appears in the two-matrix model for Hermitian matrices with probability distribution $\tilde{Z}_n^{-1} e^{-ntr(V(X)+W(Y)-2\tau XY)} dXdY$. Ercolani and McLaughlin constructed a complete family of biorthogonal polynomials associated to this two-matrix model. In this talk, I will discuss how a recent alternative formula for the HCIZ integral discovered by Goulden, Guay-Paquet, and Novak, involving the monotone Hurwitz numbers, relates to the family of biorthogonal polynomials associated to the two-matrix model and to the Riemann-Hilbert problem for these polynomials.

Global solutions for the zero-energy Novikov-Veselov equation

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Abstract. We construct global solutions to the Novikov-Veselov equation at zero energy for initial data q_0 having sufficient decay and so that associated Schrödinger operator $-\partial\bar{\partial} + q$ is nonnegative. We use the inverse scattering method to construct the solutions. These results considerably extend previous results of Lassas-Mueller-Siltanen-Stahel [1] and Perry [2]. Our analysis draws on previous work of the first author and on ideas of S. P. Novikov and P. G. Grinevich.

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Soliton and breather turbulence in ocean surface waves

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Abstract. The study of *integrable wave turbulence* in nonlinear physical systems such as the Korteweg-deVries (KdV) equation has recently been stimulated by the discovery of *soliton turbulence* in ocean surface waves [1]. More recently we have also found nonlinear Schrödinger (NLS) equation *breather turbulence* in ocean waves [2]. An important aspect of these experimental studies has been the use of finite gap theory (FGT) for both the KdV and NLS equations to analyze the data: The procedure projects measured time series onto the nonlinear modes of KdV and NLS. Remarkably, in this way, two types of coherent structures, densely packed solitons and breather trains, are seen in the same oceanic data sets. Fully understanding the physics of this coupled, nonlinear dynamics continues to be a challenge.

The goal of this talk will be to discuss how FGT can be used to construct *nonlinear statistical theories* for both KdV and NLS. To this end analytic expressions for the *exact correlation function* and *power spectrum* are computed in terms of the the Riemann spectrum which contains solitons for KdV and breathers for NLS. The key to the approach is to recognize that the Its-Matveev formula $(u(x,t) = 2\partial_{xx} \ln \theta(x,t))$ for KdV can be reduced to a multidimensional Fourier series.

In this way there is a direct connection between the Riemann spectrum of FGT and the coefficients of the multidimensional Fourier series. This provides a route for computing the exact correlation function for solutions of the KdV equation. A similar approach is also applied to the NLS equation. Thus we have the tools for computing exact statistical properties of integrable systems without having to evoke closure arguments or approximations. I apply the statistical formulations to a number of numerical examples of densely interacting solitons and breather trains and provide insight into the nonlinear dynamics of ocean wave data. This requires the numerical computation of Riemann theta functions of genus $g \sim 1000$.

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Explorations of the solution space of the fourth Painlevé equation

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Abstract. Over the last several decades, solutions to the six Painlevé equations (P_I-P_{VI}) have been found to play an increasingly central role in numerous areas of mathematical physics. However, due to vast dense pole fields in the complex plane, numerical evaluation of the solutions to P_I-P_{VI} remained challenging until the recent introduction of a fast 'pole field solver' [Fornberg and Weideman, J. Comp. Phys. 230 (2011), 5957-5973].

P_{IV}, in particular, has two free parameters in its coefficients and two free initial or boundary conditions, amounting to a four-dimensional solution space. Apart from existing analytic and asymptotic knowledge about the equation, there are vast, unexplored regions of the space of initial conditions and parameters to which the 'pole field solver' can be efficiently applied.

Solutions that are free of poles over extensive regions of the complex plane are of considerable interest, dating back as far as the tronquée and tritronquée solutions of P_I discussed by Boutroux [Boutroux, Ann. École Norm., 30 (1913), 255-375]. This talk presents parameter choices and initial conditions that exhibit these pole-free sectors in the complex plane, highlighting the existence of the families of solutions discussed in [1] and [2] that are free from (or have only a finite number of) poles on the real axis.

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Formation of rogue waves in the small dispersion limit of the focusing NLS

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Abstract. Focusing NLS is considered to be one of the most commonly used models describing the formation of the rogue waves. It is widely accepted that the NLS rogue waves are described by various breather solutions, such as, for example, the Peregrine breather. In a recent paper ([2]) it was argued that the small dispersion limit of the focusing NLS can be also useful in describing the formation and propagation of the rogue waves. Namely, it was demonstrated that a slowly modulated plane wave can burst into (growing in size) chains of Peregrine breathers after passing through a point of gradient catastrophe. The hight of each breather is 3 times the the hight of background oscillations and location of the breathers is determined by the location of the poles of the tritronquee solution to Painleve 1.

In this talk we discuss collision of dispersive shock waves (DSW) as another possible mechanism of formation of rogue waves. In the context of the focusing NLS, DSWs may appear in the case of discontionuous initial potential. Evolution (in the small dispersion limit) of such data, namely, of the box potential, was studied in [1], where the regions of modulated plane waves (genus 0) and of DSW (genus 1) were considered. We use Whitham theory and numerical simulations to describe further evolution of the box potential and discuss the obtained results.

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Direct scattering and small dispersion for the Benjamin-Ono equation with rational initial data

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Abstract. The Benjamin-Ono (BO) equation describes the weakly nonlinear evolution of one-dimensional interface waves in a dispersive medium [1, 2, 9]. It is an integrable system with a known inverse scattering transform and can be viewed as a prototypical problem for the study of multi-dimensional integrable systems and Riemann-Hilbert problems with a nonlocal jump condition [3, 5]. In this talk, we propose a construction for the scattering data of the BO equation with a rational initial condition, under mild restrictions. The construction procedure consists in building the Jost function solutions explicitly to recover the reflection coefficient, eigenvalues, and phase constants. For this class of initial conditions, all of these steps are explicit and the recovery of the scattering data can be done by using the analyticity properties of the Jost functions. We finish by showing that this procedure validates well-known formal asymptotic results obtained in the zero-dispersion limit [6, 7, 8]. This work can be seen as a significant extension of that of Kodama, Ablowitz, and Satsuma [4] who discovered that for the Lorentzian initial condition $u_0(x) = 2\nu/(x^2+1)$, with special values of ν, the eigenvalues of scattering transform are given by the roots of certain Laguerre polynomials.

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Effective integration of ultra-elliptic solutions of the cubic nonlinear Schrödinger equation

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Abstract. An effective integration method for quasi-periodic ultra-elliptic solutions of the cubic nonlinear Schrödinger equation is studied. The real quasi-periodic two-phase solutions are constructed using elementary methods and expressed in terms of Kleinian ultra-elliptic functions explicitly parametrized by the branch points of a genus-two Riemann surface. The method generalizes the effective integration of one-phase solutions introduced by Kamchatnov [1] and later applied to one-phase solutions of the Manakov system [2].

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On the use of fast multipole methods for the numerical solution of Riemann-Hilbert problems

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Abstract. We describe a method for the numerical solution of matrix-valued Riemann-Hilbert problems. The Riemann-Hilbert problem is reformulated as a singular integral equation which is then solved by an iterative multigrid solver with matrix-vector products accelerated by the use of a fast multipole method. We will then solve a nonlinear PDE by relating it to a classic Riemann-Hilbert problem and applying our method.

SESSION 19
Modeling, geometry, integrability, and analysis of nonlinear (dispersive) waves

Wave-breaking phenomena for the nonlocal Whitham-type equations

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Abstract. In this talk, the formation of singularities for the nonlocal Whitham-type equations is studied. It is shown that if the lowest slope of flows can be controlled by its highest value with the bounded Whitham-type integral kernel initially, then the finite-time blow-up will occur in the form of wave-breaking. This refined wave-breaking result is established by analyzing the monotonicity and continuity properties of a new system of the Riccati-type differential inequalities involving the extremal slopes of flows. Our theory is illustrated via the Whitham equation, Camassa-Holm equation, Degasperis-Procesi equation and their μ-versions as well as hyperelastic rod equation.

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Wave phenomena in population biology and application to cancer

Abstract. By spanning multiple scales from genes, to cell and organs, biological systems create the phenotypes of malignancy. The authors will present mathematical models, theory and numerical simulations of cancer systems pertaining to their dynamical behavior as well as to their biological and clinical implications. The models will span the molecular, cellular, and organ scales; the malignancies include brain cancer, melanomas, and GIST.

Liouville correspondence between the modified KdV hierarchy and its dual integrable hierarchy

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Abstract. We study an explicit correspondence between the integrable modified KdV hierarchy and its dual integrable modified Camassa-Holm hierarchy. A Liouville transformation between the isospectral problems of the two hierarchies also relates their respective recursion operators, and serves to establish the Liouville correspondence between their flows and Hamiltonian conservation laws. In addition, a novel transformation mapping the modified Camassa-Holm equation to the Camassa-Holm equation is found. Furthermore, it is shown that the Hamiltonian conservation laws in the negative direction of the modified Camassa-Holm hierarchy are both local in the field variables and homogeneous under rescaling.

Dispersive quantization of linear and nonlinear waves

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Abstract. The evolution, through spatially periodic linear dispersion, of rough initial data leads to surprising quantized structures at rational times, and fractal, non-differentiable profiles at irrational times. Similar phenomena have been observed in optics and quantum mechanics, and lead to intriguing connections with exponential sums arising in number theory. Ramifications of these observations for numerics and nonlinear wave models will be discussed.

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Continuous dependence on the density for steady stratified water waves

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Abstract. There are two distinct regimes commonly used to model traveling waves in stratified water: continuous stratification, where the density is smooth throughout the fluid, and layer-wise continuous stratification, where the fluid consists of multiple immiscible strata. The former is the more physically accurate description, but the latter is frequently more amenable to analysis and computation. By the conservation of mass, the density is constant along the streamlines of the

flow; the stratification can therefore be specified by prescribing the value of the density on each streamline. We call this the streamline density function. Intuitively speaking, one expects that it is possible to use layer-wise smooth waves (for which the streamline density function is piecewise smooth) to approximate smoothly stratified waves (for which the streamline density function is of course smooth).

In this talk, we will discuss some recent work in this direction. Our main result states that, for every smoothly stratified periodic traveling wave in a certain small-amplitude regime, there is an L^∞ neighborhood of its streamline density function such that, for any piecewise smooth streamline density function in that neighborhood, there is a corresponding traveling wave solution. Moreover, the mapping from streamline density function to wave is Lipschitz continuous in a certain function space framework. As this neighborhood includes piecewise smooth densities with arbitrarily many jump discontinues, this theorem provides a rigorous justification for the ubiquitous practice of approximating a smoothly stratified wave by a layered one.

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Local discontinuous Galerkin methods for the generalized Korteweg-de Vries equations

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Abstract. The Korteweg-de Vries (KdV) equation is a nonlinear mathematical model for the unidirectional propagation of waves in a variety of nonlinear, dispersive media. Recently it has attracted increasing attention as test-bed for the competition between nonlinear and dispersive effects leading to a host of analytical issues such global existence and finite time blowup, etc.

In this presentation, we construct, analyze, and numerically validate a class of discontinuous Galerkin schemes for the generalized KdV equation. We will provide a posteriori error estimate through the concept of dispersive reconstruction, i.e. a piecewise polynomial function which satisfies the GKdV equation in the strong sense but with a computable forcing term enabling the use of a priori error estimation techniques to obtain computable upper bounds for the error. Both semi-discrete and fully discrete approximations are studied.

Approximation of polyatomic FPU lattices by KdV equations

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Abstract. Famously, the Korteweg-de Vries equation serves as a model for the propagation of long waves in Fermi-Pasta-Ulam (FPU) lattices. If one allows the material coefficients in the FPU lattice to vary periodically, the "classical" derivation and justification of the KdV equation go awry. By borrowing ideas from homogenization theory, we can derive and justify an appropriate KdV limit for this problem. This work is joint with Shari Moskow, Jeremy Gaison and Qimin Zhang.

SESSION 20
Numerical simulations for solving nonlinear waves equations

A discontinuous Galerkin least-squares finite element - finite difference simulation of the Nagumo equation

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Abstract. The Nagumo equation is one of the simplest nonlinear reaction-diffusion equations, which has been widely used in studies in biological electricity. This equation has traveling wave solutions. If the equation has strong reaction term, oscillations may appear in the numerical solutions due to the singular perturbation in the reaction-dominated regime. Finding stable numerical solutions to reaction-dominated Nagumo equations is very challenging due to moving wave fronts and high wave speeds.

In the present study, a discontinuous Galerkin least-squares finite element-finite difference algorithm is developed to solve the Nagumo equation. Numerical simulation results are given to demonstrate the performance of the algorithm in relatively long time domains. Convergence rates with respect to spatial and temporal discretization are obtained experimentally.

Runup of nonlinear long waves in U-shaped bays of finite length: analytical theory and numerical

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Abstract. he long nonlinear wave runup on sloping U-shaped bays of finite length is studied analytically in the framework of the 1D nonlinear shallow-water theory with averaging on the cross-section. By assuming that the wave flow is uniform in cross-section the nonlinear shallow-water equations are reduced to a linear semi-axis variable-coefficient 1D wave equation by use of the generalized Carrier-Greenspan [1] transformation developed recently for channels of arbitrary cross-section [2]. An analytic solution is then found via separation of variables for the linear wave equation. To compute the details for the long wave runup, the Carrier-Greenspan transformation is numerically implemented and a numerical method is developed to find the needed eigenvalues/eigenfunction decomposition for the analytic solution. The runup of a long wave in a real world narrow wave tank is then discussed.

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A method of 3-D seabed terrain generation based on data fusion theory

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Abstract. Natural neighboring point interpolation algorithm shows high efficiency and good simulation effect in 3-d seabed terrain generation. But considering the defects of the electronic chart itself, we proposed a new method.

In this paper we consider the following problem. First, interpolate water depth data of scattered points, which extracted from the same area of many electronic charts, with natural neighboring points interpolation algorithm. Second, determine the weighted coefficient by the information fusion theory of weighted fusion and fuzzy logic. Third, determine the random increment by the fractal characteristics of the seabed terrain. Finally, generated a new 3-d seabed terrain map, and compared with the map which directly generated by interpolation algorithm. Results showed that the method we proposed can reduce the crease obviously, and make terrain more lively and natural.

Creation and manipulation of temporal solitons in optical microresonators by pump phase modulation

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Abstract. The generation of temporal solitons from a whispering-gallery-mode (WGM) microresonator can lead to chip-scale, low-cost, and stable ultra-short pulse sources and replace the state-of-the-art table-top pulsed lasers. Recently, femtosecond solitons were demonstrated by the careful sweeping of the pump frequency in the vicinity of a microresonator resonant mode [1]. This technique, however, faces challenges for integration on a chip because it requires an on-chip tunable laser source.

In this paper, we propose an alternative method for microresonator-based soliton formation by input phase modulation. Starting from a set of coupled nonlinear equations for the temporal evolution of the comb's modal fields, we show that the spatiotemporal evolution of frequency combs seeded by pump phase modulation is governed by a damped and driven nonlinear Schrödinger equation. Through theoretical analysis and split-step Fourier numerical simulations, we show that solitons generated by this technique are more stable against perturbations. We further show that this method makes it possible to manipulate the time delay between the short pulses at the output of a microresonator. Combined with the available optoelectronic modulators, our proposed method provides a viable approach for ultra-short pulse formation on a chip.

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SESSION 21
Functional analysis and PDEs

Finite time blow-up and global solutions for a semilinear parabolic equation with linear dynamical boundary conditions

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Abstract. For a class of semilinear parabolic equations with linear dynamical boundary conditions in a bounded domain

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u, & x \in \Omega,\ t > 0, \\ u = 0, & x \in \Gamma_0,\ t > 0, \\ u_\nu + u_t + u = 0, & x \in \Gamma_1,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{13}$$

where Ω is a bounded domain in $\mathbb{R}^n$, ν is the ourter normal on Γ_1, $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\lambda_{n-1}(\Gamma_0) > 0$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, $1 < p < \infty$ if $n = 1,2$; $1 < p < \frac{n+2}{n-2}$ if $n \geq 3$, we obtain both global solutions and finite time blow-up solutions when the initial data varies in the phase space $H^1(\Omega)$. Our main tools are the comparison principle, the potential well method and the concavity method. In particular, we discuss the behavior of the solutions with the initial data at critical and high energy level.

This work was supported by International Exchange Plan of Harbin Engineering University.

Global and blowup solutions for general Lotka-Volterra systems

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Abstract. The classical Lotka-Volterra cooperating, reaction-diffusion system of two species is given by

$$u_t = \nabla \cdot (D_1 \nabla u) + u(a_1 - b_1 u + c_1 v)$$

$$v_t = \nabla \cdot (D_2 \nabla v) + v(a_2 - b_2 v + c_2 u)$$

in a bounded domain Ω, together with suitable boundary and initial conditions. The two species interaction model Lotka-Volterra arises from ecology, where the presence of each species influences the growth of the other species, but also in a positive way through mutualisms.[1]

This paper employs functional methods to obtain lower and upper bounds for the solution of the parabolic system $u_t = \alpha(v)\nabla \cdot (u^p \nabla u) + f(u,v)$ and $v_t = \beta(u)\nabla \cdot (v^q \nabla v) + g(u,v)$ with homogeneous Dirichler boundary conditions . We will find relations among diffusion terms, nonlinear terms and the size of the domain. We show that the blowup property only depends on the first eigenvalue of $-\Delta$ with the Dirichlet boundary condition.

This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

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Global well-posedness of semilinear hyperbolic equations

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Abstract. It is well known that semilinear hyperbolic equation is one of the most fundamental and important nonlinear evolution equation. The global existence of solution for this equation without positive energy was resolved by Sattinger by introducing potential well in 1968. After that the potential well method become one of most significant method while studying the global well-posedness of nonlinear evolution equation without positive energy.

In this paper we study the initial-boundary value problem of semilinear hyperbolic equations

$$u_{tt} - \Delta u = f(u),\ x \in \Omega,\ t > 0 \tag{14}$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega \tag{15}$$

$$u(x,t) = 0,\ x \in \partial\Omega,\ t \geq 0. \tag{16}$$

Under some completely new assumptions on $f(u)$, we discuss the invariant sets W and V of solutions for the semilinear hyperbolic equation when the initial value satisfy $u_0(x) \in H_0^1(\Omega)$, $u_1(x) \in L^2(\Omega)$ and $E(0) < d$, then we prove the global existence and nonexistence of solutions and obtain some sharp conditions for global existence and nonexistence of solution by using potential well method and convexity method.

This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

The application of Kalman filter in dynamic obstacle perception technology for AUV

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Abstract. The classical kalman filtering, which is based on least mean-square error rule, is widely used in engineering. However, when the noise statistics are inaccurate the classical kalman filtering will lose the optimal estimation quality. It can reduce the filtering accuracy or lead the filter divergence. The adaptive filtering is able to restrain the divergence effectively.

This paper makes a research on the classical kalman filtering theory systematically, and analyzes the causes of its divergence. Firstly, we bring in the Sage-Husa adaptive filtering to work the divergence out, and though the fading memory exponent method we emphasize the new measure's role in adaptive filtering to get the improved Sage-Husa time-varying noise statistics estimators. And then we make simulations about the performance of the improved filter.Finally, to verify the performance of the Sage-Husa adaptive filtering algorithm in the dynamic obstacle sensing system of AUV, we make simulations by combining the Sage-Husa algorithm with CA model and Singer model respectively, and the prediction results are compared in order to obtain a obstacle motion model which is the most appropriate.

The results show that Sage-Husa adaptive Kalman filter algorithm with Singer model meets the requirements of prediction accuracy and real-time AUV system at the same time. Therefore, we can apply this algorithm into the technology of AUV dynamic obstacle sensing to provide theoretical support for the subsequent obstacle avoidance and path planning.

This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

LSH-based active strategy for visual categories learning

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Abstract. This paper addresses the problem of constructing a training set from a large-scale image database at minimal labeling costs. This problem usually occurs in the situations of obtaining sufficient labeled images to build a classifier for recognizing thousands of visual categories. In this paper, an active strategy is used in which the unlabeled images are selected from the whole database and added into the training set after being labeled. A new selection strategy based on local-sensitive hash (LSH) is proposed in order to return the most informatively unlabeled images without scanning the whole database in every round. The training set is derived by iteratively selecting the images which are both close to the classification boundary and similar to the existing positive images. Experimental results on MIRFLICKR database are reported, and the results prove the efficiency and the effectiveness of the proposed method.

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A method of image binarization for bubble detection

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Abstract. Image binarization methods [1] are commonly used in the tasks of bubble image detection in order to explore information contained in given images. Global threshold and local threshold [2, 3] are two main methods, but they are both time-consuming and complex-computing which make them difficult in processing a sequence of bubble images. In order to deal with a sequence of bubble images, this paper proposed two novel binarization methods, respectively using in global and local threshold. In the proposed methods, the threshold is computed by considering the commonly feature in all images, and then the binary images are obtained faster. Experimental results have showed the effectiveness and efficiency of the proposed methods.

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Kelvin-Helmholtz instability & acoustic waves in twin-jet flow

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Abstract. ecause of the need for large thrust, many high performance jets are propelled by two jet engines housed close to each other. The objectives of this work are to study the characteristics of acoustic waves and Kelvin-Helmholtz instability waves in twin jet flow. The dispersion relations are derived for four familys solutions
Using the Trace Theorem proved by Davidenko, we are able to compute the numerical results to interpret the characteristics of Kelvin-Helmholtz instability waves and upstream propagated acoustic waves.

This work was supported by International Exchange Plan of Harbin Engineering University.

The Cauchy problem for generalized Benney-Luke equation

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Abstract. We investigate the Cauchy problem for n-dimensional general Benney-Luke equation, where $1 \leq n < 10$. The local solution is considered in energy space $\dot{H}^1 \cap \dot{H}^3$ by the contraction mapping principle. The Hölder inequality, Sobolev inequalities and the estimates of the kernel function of Bessel potential are the main tools. We prove the existence and uniqueness of the global solution in energy space according to the energy equality which being proved using regularization.

SESSION 22
Spectral methods in stability of traveling waves

Stability of transition front solutions in multidimensional Cahn-Hilliard systems

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Abstract. We consider spectral and nonlinear stability for planar transition front solutions arising in multidimensional Cahn-Hilliard systems. A critical feature of such systems in one space dimension is that the linear operator that arises upon linearization of the system about a transition front has essential spectrum extending up to the imaginary axis, a feature that is known to complicate the step from spectral to nonlinear stability. The analysis of planar waves in multiple space dimensions is further complicated by the fact that the leading eigenvalue for this linearized operator (after Fourier transform in the transverse variable) moves into the stable half plane at cubic rate in the Fourier variable. We discuss the spectrum of this linearized operator, and identify spectral conditions (checked in certain cases) that imply nonlinear stability.

Numerical stability analysis for thin film flow

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Abstract. We discuss various aspects of numerical stability analysis of periodic roll wave solutions arising in equations of inclined thin film flow, with an eye toward the development of guaranteed error bounds. In particular, we rigorously verify stability of a family of periodic wave solutions arising in a generalized Kuramoto-Sivashinsky equation in the Korteweg-de Vries limit, that is, in the limit viscosity goes to zero.

Orbital stability of waves traveling along vortex filaments

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Abstract. By the term vortex filament, we mean a mass of whirling fluid or air (e.g. a whirlpool or whirlwind) concentrated along a slender tube. The most spectacular and well-known example of a vortex filament is a tornado. A waterspout and dust devil are other examples. In more technical applications, vortex filaments are seen and used in contexts such as superfluids and superconductivity. One system of equations used to describe the dynamics of vortex filaments is the Vortex Filament Equation (VFE). The VFE is a system giving the time evolution of the curve around which the vorticity is concentrated.
In this talk, we develop a framework for studying the orbital stability solutions of the VFE, based on the correspondence between the VFE and the NLS provided by the Hasimoto map. This framework is applied to VFE solutions that take the form of soliton solutions. If time permits, we will also tackle the case of closed vortices, which take the form of torus knots.

An Evans function for the Riccati equation on the Grassmannian

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Abstract. Evans functions have been used to investigate the point spectrum in travelling wave problems since their inception in the 1970's. Computing them numerically usually requires tracking the evolution of a subspace under a linear, nonautonomous ordinary differential equation (ODE).

Subspaces of a fixed dimension of an ambient space (say $\mathbb{C}^k$ inside $\mathbb{C}^n$) form a manifold, the so-called Grassmannian. Given a linear ODE, one can then translate said flow to a nonlinear, non-autonomous ODE on the Grassmannian. This is a generalisation of the Riccati equation in two dimensions. Tracking a subspace is then reduced to tracking a single solution on the Grassmannian under flow of this associated ODE.

In this talk, I will given an example of such a setup and how it can be used to numerically establish the stability or instability of travelling waves in a system of partial differential equations (PDEs) in one spatial and one temporal dimension. This will require writing an Evans function in terms of the Plücker coordinates on the Grassmannian. In translating an Evans function to these coordinates, what results is not Evans function per se (in terms of a traditional, axiomatic definition), however it does share some of the traditional properties of the classical Evans function, and moreover it is readily extendable into the essential spectrum.

If there is time I will discuss how further assumptions on the system (i.e. if it is Hamiltonian, a gradient flow, etc) can be used to gain more information on the spectrum of the linearised PDE (and how the Evans function can be related to the Maslov index in these cases), as well as how this new, Riccati-Evans function can be used to resolve (in one case anyway) the essential singularity at the leading edge of the absolute spectrum of the classical Evans function.

Instability index theory and applications

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Abstract. We construct an Evans-like function that identifies real instabilities for certain class of eigenvalue problems on the real line R^1. We then apply this theory to prove that various special solutions such as traveling waves and explicit peakon solutions are unstable.

Linear instability and invariant manifolds for Hamiltonian PDEs

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Abstract. We will discuss a general framework to study instability of coherent states (traveling waves, standing waves, steady states etc.) for Hamiltonian PDEs with an energy functional bounded from below. First, an instability index theorem is obtained for the eigenvalues of the linearized problem. Then, we prove the exponential trichotomy estimate for the linearized equation. Applications to construct invariant manifolds of the nonlinear equations will be briefly discussed. Some examples include Gross-Pitaevskii equation for superfluids, generalized BBM, KDV and Boussinesq equations, and 2D Euler equation for ideal fluids.

The Morse and Maslov indices for Schrödinger operators

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Abstract. We study the spectrum of Schrödinger operators with matrix valued potentials utilizing tools from infinite dimensional symplectic geometry. Using the spaces of abstract boundary values, we derive relations between the Morse and Maslov indices for a family of operators on a Hilbert space obtained by perturbing a given self-adjoint operator by a smooth family of bounded self-adjoint operators. The abstract results are applied to the Schrödinger operators with θ-periodic, Dirichlet and Neumann boundary conditions. In particular, we derive an analogue of the Morse-Smale Index Theorem for the multidimensional Schrödinger operators with periodic potentials. For quasi-convex domains in $\mathbb{R}^n$ we recast the results connecting the Morse and Maslov indices using the Dirichlet and Neumann traces on the boundary of the domain.

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Avoided Hamiltonian-Hopf bifurcations

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Abstract. arameter dependent nonlinear waves lose stability through Hamiltonian-Hopf bifurcations. We discuss our new findings on how these bifurcations are avoided in particular nongeneric cases.

The Hamiltonian-Hopf bifurcations are associated with collisions of purely imaginary eigenvalues of the linearization of the system that may happen only if the colliding eigenvalues have opposite Krein signatures. However, opposite Krein signatures do not provide a sufficient condition for such a bifurcation, although it is true "generically". It was demonstrated earlier that symmetries of the system may prevent the bifurcation but there are other possible scenarios that are very nongeneric. The extension of the Krein signature to the graphical Krein signature allows to study particular cases when the bifurcation is avoided due to a preservation of the intersection of eigenvalue curves of an extended spectral problem. We show that (unexpectedly) these intersections can be preserved even for the cases when the determinant of the associated eigenvalue pencil is irreducible. Furthermore, we present a simple example of such a system and show how it is connected with a resonance in the spectrum of the extended eigenvalue problem, i.e. passing of eigenvalues of opposite signature on the imaginary axis cannot be predicted from the original problem. This is a joint work with Peter Miller (U Michigan).

Phase locking and stability in the Kuramoto model

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Abstract. We consider the stability of phase-locked states of the Kuramoto system of coupled oscillators. We give a geometric stability criterion for counting the number of unstable directions to a given phase-locked configuration, and use this to show that in the thermodynamic limit there is a phase transition: as the coupling strength is increased the probability of phase-locking goes from zero to one in a finite interval.

Multidimensional instability criteria via the Maslov index

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Abstract. This talk will highlight new applications of the Maslov index, a symplectic invariant, to the spectral analysis of Schrödinger operators, i.e. $L = -\Delta + V$. We focus on the case of a bounded domain $\Omega \subset \mathbb{R}^n$ decomposed into sublevel sets of a Morse function,

$$\Omega_t = \{x \in \Omega : \psi(x) < t\},$$

and use the Maslov index to describe how the spectrum of L on Ω_t varies with t. This is advantageous when ψ has some geometric significance to the problem at hand—for instance, it could depend on the potential $V(x)$, or a steady state about which one has linearized. Of particular interest is when t passes through a critical value of ψ, hence the topology of Ω_t changes. These considerations lead to a general decomposition formula that should prove useful in analyzing the stability of localized structures such as solitons and standing waves.

$O(2)$ Hopf bifurcation of viscous shock waves in a channel

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Abstract. We study $O(2)$ transverse Hopf bifurcation, or "cellular instability", of viscous shock waves in an infinite channel, with periodic boundary conditions, for a class of hyperbolic-parabolic systems including the equations of thermoviscoelasticity. Due to the refection symmetry property of our model, the underlying bifurcation is not of planar Hopf type, but, rather, a four-dimensional $O(2)$ Hopf bifurcation: roughly speaking, a "doubled" Hopf bifurcation coupled by nonlinear terms. Since the linearized operator about the wave has no spectral gap, the standard center manifold theorems do not apply; indeed, existence of a center manifold is unclear. To prove the result, we use the Lyapunov–Schmidt reduction method applied to the time-T evolution map of the underlying perturbation equations, resulting in a 4-dimensional stationary bifurcation problem with $O(2)$ symmetry plus an additional "approximate S^1 symmetry" induced by the underlying rotational linearized flow.

SESSION 23
Solitons and nonlinear dispersive waves

Generalized Willmore surfaces, flow and applications

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Abstract. Our report studies a Generalized Willmore Equation (GWE) (i.e., the Euler-Lagrange equation of a Generalized Willmore Energy). Certain associated boundary value problems were introduced, such as Dirichlet-Navier, and respectively, Dirichlet-Newman. As motivated by our study of beta-barrels in protein biology, our study includes (but is not limited to) the case of Generalized Willmore Surfaces of Revolution. We have been studying 1-parameter families of solutions to certain associated boundary value problems, and their geometric meaning, properties and classifications. Among these solutions, we obtained significant classes of minimal and CMC surfaces, as well as some novel, interesting extensions. A Generalized Willmore flow is under our current investigation.

The techniques we used include: classical variational calculus, differential forms, Cartan theory, Hodge theory, nonlinear PDE analysis. COMSOL Multiphysics was used extensively in solving the boundary value problems we considered. It made the analysis of the GW surfaces relevant, rich and significant.

On nonintegrable semidiscrete Hirota equations

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Abstract. In this talk, we investigate nonintegrable semidiscrete Hirota equations including nonintegrable semidiscrete Hirota$^-$ equation and nonintegrable semidiscrete Hirota$^+$ equation. We focus on the topics on gauge equivalent structures and dynamical behaviours for the two nonintegrable semidiscrete equations. By using the concept of the prescribed discrete curvature, we show that under the discrete gauge transformations, nonintegrable semidiscrete Hirota$^-$ equation and nonintegrable semidiscrete Hirota$^+$ equation are respectively gauge equivalent to the nonintegrable generalized semidiscrete modified Heisenberg ferromagnet equation and the nonintegrable generalized semidiscrete Heisenberg ferromagnet equation. We prove that the two discrete gauge transformations are reversible. We study the dynamical properties for the two nonintegrable semidiscrete Hirota equations. The exact spatial period solutions of the two nonintegrable semidiscrete Hirota equations are obtained through the constructions of period orbits of the stationary discrete Hirota equations. We discuss the topic that the spatial period property of the solution to nonintegrable semidiscrete Hirota equation is whether preserved to that of the corresponding gauge equivalent nonintegrable semidiscrete equations or not under the action of discrete gauge transformation. By using of the gauge equivalent, we obtain the exact solutions to the nonintegrable generalized semidiscrete modified Heisenberg ferromagnet equation and nonintegrable generalized semidiscrete Heisenberg ferromagnet equation. We also give the numerical simulations for the stationary discrete Hirota equations. We find that their dynamics are much richer than the ones of stationary discrete nonlinear Schrödinger equations. This talk is based on the paper jointed with Li-yuan Ma [1].

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Localized waves in nonlinear Schrödinger equations with external potentials and nonlinearities

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Abstract. In this talk, we mainly report on wave structures of nonlinear Schrödinger equation (a.k.a. the Gross-Piteavskii equations in Bose-Einstein condensates) and its extensions with some external potentials including harmonic and double-well potentials. We give their some localized wave solutions and analyze them wave propagations under the effection of controlling parameters

Integrable multi-component nonlinear wave equations from symplectic structures

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Abstract. I will show how the bi-Hamiltonian structure of integrable systems can be used to derive various related integrable wave equations, in particular, a hyperbolic equation and a peakon equation, as well as a Schrodinger equation and peakon-like kink equation if the original integrable system is unitarily invariant.

An analysis of financial data with wavelets and fractal functions

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Abstract. The study of fractal functions starts from Weierstrass's nowhere differentiable function and beyond. Seemingly different types of nowhere differentiable functions are unified under the fractal point of view. This unification led to new mathematical methods and applications in areas that includes: dimension theory, dynamical systems and chaotic dynamics, image analysis, and wavelet theory. Now Wavelet analysis is an exciting new method for solving difficult problems in mathematics, physics, and engineering, with modern applications as diverse as wave propagation, data compression, image processing and financial engineering. In this paper, our goal is to find the deeper connection between fractal functions and wavelets, and find the application in finance to make predictions as to the risk involved for particular stocks or options.

Nonlinear steady two layer interfacial flow about a submerged point vortex

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Abstract. Two dimensional two layer steady interfacial flow about a point vortex is studied in a uniform stream for each layer, the upper layer is finite depth with lid in the upper surface and the lower layer is infinite depth. The point vortex located at lower layer fluid. The fully nonlinear problem is formulated by an integrodifferential equation system to determine the unknown steady interfacial surface exactly. The problem is formulated as an integrodifferential equation system in terms of the complex potential function and the interfacial surface function. This equation system are solved by Newton's method numerically. The numerical results of the downstream wave is provided. Comparison with the infinite depth single layer about a point vortex, it is investigate that the interfacial waves between two layers has an obvious greater amplitude than that in one single layer, nearly triple times. The effects of vortex strength and Froude number are also studied in terms of the amplitude of the downstream waves. Numerical results of the downstream waves are given, the effects between the two layer flows is analyzed in terms of the vortex strength, Froude number and compared with the free surface of single layer flows. It is investigated that the amplitude of the interfacial waves is much greater than it for free surface waves. It is may be useful to analysis the environment condition and wave loads in ocean engineering.

Well-posedness and blowup phenomena for a new integrable system with peakon solutions

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Abstract. In the paper, we first establish the local well-posedness for a new integrable system with peakon solutions. We then present a precise blowup scenario and several blowup results of strong solutions to the system.

Fully nonlinear numerical simulation of large-amplitude obliquely interacting solitary waves

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Abstract. The "Mach Reflection" occurs when two identical solitary waves interact with an angle less than the critical threshold [1]. The unique feature of Mach reflection is, besides the incident wave and the reflected wave, a third wave, called (Mach) stem wave, continuously grows from the intersection, and it is perpendicular to the symmetric plane. It is a strongly nonlinear process that might result in four-fold stem wave in theory [2]. In the experiments, a maximum amplitude of the stem wave has been observed [3]. Although higher-order numerical simulations have been carried out to study Mach reflection [4], fully nonlinear numerical simulations are needed to reveal the physics of the extreme stem wave.

In this paper we use Volume Of Fluid (VOF) method to takle the fully nonlinear free surface boundary condition. The governing equations, imcompressible Euler equations, are discretized by Finite Volume Method (FVM) based on the open source libarary OpenFOAM. The third-order analytical solution of a solitary wave is used to initialise the calculation. Several runs have been carried out to examine the grid-size independence. Both the run-ups at the symmetric plane and stem lengths given by the numerical simulation for the extreme case agree well with the experimental measurements.

Some distinct characteristics have been discovered, for example, the stem wave is not perpendicular to the symmetric plane and the stem length does not grow proportionally as the wave is approaching its maximum height. Other data, such as velocities and pressure of the flow field are obtained, which are significant for the offshore and coastal engineering applications.

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Multi-solitons and rogue waves for the coupled Hirota equation

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Abstract. By using the iterative algorithm of the Darboux transformation, multi-soliton solutions of the coupled Hirota equation are generated from the zero seed solution. With the help of the generalized Darboux transformation, the higher-order rogue waves of the coupled Hirota equation are presented from the plane wave solution. In particular, the dynamics of the above obtained solitons and rogue waves are discussed and illustrated through some figures.

The unified transform method for nonlinear evolution equations

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Abstract. The initial-boundary value problem for nonlinear evolution equations on the half-line with data in Sobolev spaces is analysed via the formulae obtained through the unified transform method of Fokas, and a contraction mapping approach. First, the linear problem with initial and boundary data in Sobolev spaces is solved and the basic space and time estimates of the solution are derived. Then, using these estimates well-posedness of the nonlinear problem with data in the same spaces is established via a contraction mapping argument. The nonlinear Schrödinger equation will be used as an illustrative example. This work places Fokas' unified transform method for evolution equations into the broader Sobolev spaces framework.

Single peak soliton and periodic cusp wave of the generalized Schrödinger-Boussinesq equations

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Abstract. In this talk, we study peakon, cuspon, smooth soliton and periodic cusp wave solutions for the generalized Schrödinger-Boussinesq equations from a dynamical systems theoretical point of view. The generalized Schrödinger-Boussinesq equations are shown to have the parametric representations of peakon, cuspon, smooth soliton, and periodic cusp wave solutions. As a result, the conditions under which peakon, cuspon, smooth soliton, and periodic cusp wave solutions appear are also given.

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Periodic solitary waves of shear flow of barotropic atmosphere from the shallow water equations

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Abstract. Using the multi-scale disturbed analysis method, a new nonlinear evolution equation about the disturbed height field is deduced according to the shallow water equations. After one transplant to the solution of Poisson equation, the new analytic solution with two arbitrary plural functions is obtained. The real solutions obtained by rationalizing several kinds of special plural functions to the solution are studied. Two types of periodic solitary waves formed by the mutual shearing between the periodic waves and solitary waves are discussed. Thereinto, one type of wave has relation with effect, but the other has no. The results indicate that there exit the periodic solitary waves not only with plane wave but also with hyperbolic wave in barotropic atmosphere. Therefore, the nonlinear evolution equation and its solutions can supply meaningful theoretical reference value for studying atmospheric circulation and weather phenomena.

Dust acoustic waves in magnetized dense plasmas with dust-neutral collisions

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Abstract. In order to investigate the linear and nonlinear propagation characteristics of the dust acoustic waves for a uniform dense dusty magnetoplasma, a two-dimensional nonlinear equation is derived by employing the quantum hydrodynamic model to account for dust-neutral collisions. The linear dispersion relation shows that quantum parameter modifies the scale lengths of the system, and that wave motion decays gradually leading the system to stable state eventually. The variations of the dispersion frequency with the dust concentration, collision frequency and magnetic field strength are discussed. For the coherent nonlinear dust acoustic waves, new analytic solutions are obtained, and it is found that shock wave and explosive wave may be easily produced in the background of high dust density, strong magnetic field and weak collision. The relevance of obtained results to dense astrophysical environments is pointed out.

Key words: Dusty magnetoplasma, Quantum hydrodynamic model, Dust acoustic wave
PACS numbers: 52.27.Lw, 52.25.Xz, 52.35.Mw, 52.35.Fp

Approximate similarity solutions of the modified (1+1)-dimensional displacement shallow water wave system

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Abstract. A (1+1)-dimensional displacement shallow water wave system (1DDSWWS) for perfect fluid has been constructed, recently [1]. The term related to the fluid viscosity was added to the 1DDSWWS by means of mechanics analysis in this paper. The new model is called as modified 1DDSWWS (M1DDSWWS). For perfect fluid, the M1DDSWWS will degenerate to 1DDSWWS. Approximate symmetries are researched and calculation shows the M1DDSWWS is invariant under some Galilean transformations, and space-time translations. Three types of approximate similarity solutions and reduction equations of the M1DDSWWS are obtained. A type of exact solution is obtained, which reveals the decay characteristics of water displacement with space and time.

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The extended Estabrook-Wahlquist method

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Abstract. Variable Coefficient Korteweg de Vries (vcKdV), Modified Korteweg de Vries (vcMKdV), and nonlinear Schrodinger (NLS) equations have a long history dating from their derivation in various applications. A technique based on extended Lax Pairs has been devised recently to derive variable-coefficient generalizations of various Lax-integrable NLPDE hierarchies. The resulting Lax- or S-integrable NLPDEs have both time- AND space-dependent coefficients, and are thus more general than almost all cases considered earlier via other methods such as the Painlevé Test, Bell Polynomials, and various similarity methods.

However, this technique, although operationally effective, has the significant disadvantage that, for any integrable system with spatiotemporally varying coefficients, one must 'guess' a generalization of the structure of the known Lax Pair for the corresponding system with constant coefficients. Motivated by the somewhat arbitrary nature of the above procedure, we embark in this paper on an attempt to systematize the derivation of Lax-integrable sytems with variable coefficients. An ideal approach would be a method which does not require knowledge of the Lax pair to an associated constant coefficient system, and also involves little to no guesswork. Hence we attempt to apply the Estabrook-Wahlquist (EW) prolongation technique, a relatively self-consistent procedure requiring little prior infomation. However, this immediately requires that the technique be significantly generalized or broadened in several different ways, including solving matrix partial differential equations instead of algebraic ones as the structure of the Lax Pair is deduced systematically following the standard Lie-algebraic procedure of proceeding downwards from the coefficient of the highest derivative. The same is true while finding the explicit forms for the various 'coefficient' matrices which occur in the procedure, and which must satisfy the various constraint equations which result at various stages of the calculation.

The new and extended EW technique whch results is illustrated by algorithmically deriving generalized Lax-integrable versions of the NLS, generalized fifth-order KdV, MKdV, and derivative nonlinear Schrodinger (DNLS) equations.

Peakon Solutions for (2+1)-dimensional Camassa-Holm Equation

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Abstract. In recent years, the Camassa-Holm equation has attracted much attention to the theory of integrable systems and solitons. In this study, we generalize the CH equation to $(2+1)$-dimensions and derive single peakon and multi-peakon solutions. We also look at other topics such as smooth soliton solutions, cuspons and peakon stability.

SESSION 24
Long time dynamics of nonlinear dispersive waves

Long time dynamics of nonlinear dispersive waves

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Abstract. For the nonlinear Dirac equation in (1+1)D with scalar self-interaction (Gross–Neveu model), with quintic and higher order nonlinearities (and within certain range of the parameters), we prove that solitary wave solutions are asymptotically stable in the "even" subspace of perturbations (to ignore translations and eigenvalues $\pm 2\omega i$). The asymptotic stability is proved for initial data in H^1. The approach is based on the spectral information about the linearization at solitary waves which we justify by numerical simulations. For the proof, we develop the spectral theory for the linearized operators and obtain appropriate estimates in mixed Lebesgue spaces, with and without weights.

Uniqueness of conservative solution for Camassa-Holm and variational wave equations

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Abstract. Relying on the analysis of characteristics, we prove the uniqueness of conservative solutions for both the Camassa-Holm equation modeling water wave and variational wave equation $u_{tt} - c(u)(c(u)u_x)_x = 0$. Given a solution $u(t,x)$, even if the wave speed $c(u)$ is only Hölder continuous in the t-x plane, we show that one can still define characteristics in a unique way. Using a new set of independent variables and dependent variables, we prove that any conservative solution satisfies a semilinear system with smooth coefficients. From the uniqueness of the solution to this semilinear system, one obtains the uniqueness of conservative solutions to the Cauchy problem with general initial data.

Long time dynamics of nonlinear dispersive waves

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Abstract. I will present a recent results about a resonance problem, in the consideration of the asymptotic dynamics of ground states of semilinear Schrödinger equations. Different from previous results, I do not exclude the possibility of the presence of certain resonance. This is achieved by applying a better understanding of normal form transformation. The key argument is to demonstrate that if certain denominators are small or are zeros, resulted by the presence of resonances, then the corresponding nominators are proportionally small. Certain related small divisor problems will be discussed.

The energy critical limit of ground state solitons

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Abstract. We consider the energy-critical limit of the ground state solitary waves of the pure power, focusing, energy sub-critical nonlinear Schroedinger equation in 3D. After a suitable rescaling, driven by the resonance present in the energy critical linearized problem, we identify the limit as the Aubin-Talenti function, and discuss implications for the sub-critical linearized problem, and for the best constant in the Gagliardo-Nirenberg inequality. This is joint work with Tai-Peng Tsai and Ian Zwiers.

Recent progress on global regularity of rotating BEC

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Abstract. As is known, the magnetic nonlinear Schrödinger equation (mNLS) is generated by the Hamiltonian $H_{A,V}$, where A represents the magnetic potential and V the electric potential. The mNLS appears in modeling dilute, trapped boson gases with rotation in ultra-cold temperature, whose mechanism in the semiclassical regime obeys the Newton's law in the transition from quantum to classical mechanics. Such systems may exhibit interesting symmetries as well as stationary wave phenomenon, accompanied by spinor and quantized vortex, a remarkable signature for the superfluidity of Bose-Einstein condensation (BEC).
We study the fundamental solution for the propagator $e^{-itH_{A,V}}$. The earliest related result traces back to Feynman's work on path integrals. Rigorous mathematical derivations were given by Fujiwara and Yajima. We further consider the threshold for the global regularity and blowup for mNLS. We will also briefly review current development in this perspective. Some numerical simulations are provided. Part of the work is collaboration with Luigi Galati.

Long time dynamics of nonlinear dispersive waves

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Abstract. Title: Pointwise decay for the Maxwell system on black holes

Abstract: We consider both the inhomogeneous wave and the inhomogeneous Maxwell system on nonstationary black hole space times. Under the assumption that uniform energy bounds and a weak form of local energy decay hold forward in time, we establish sharp pointwise decay rates for the solutions. In the case of the wave equation, the rate of decay is $t^{-1}(t-r)^{-2}$, and the result is sharp. For the Maxwell system, we recover peeling estimates that are direction dependent near the light cone, as well as a t^{-4} rate of decay on compact regions for all the components of the Maxwell tensor. This is joint work with J. Metcalfe and D. Tataru

When is it possible to have wellposedness of the fully non-linear KdV equation without resorting to weighted spaces?

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Abstract. We study the well-posedness of the initial value problem on the real line for the fully nonlinear evolution equations for which the leading-order terms have three spatial derivatives. This problem generalizes KdV to scenarios, where there is a competition between the dispersive effects which stem from the leading- order term, and anti-diffusion which stems from the lower-order terms with two spatial derivatives. In the preceding works [2] and [3] we have established sharp results for linear problems of this type with non-degenerate dispersion. This analysis of the linear equations suggested that for generic non-linear problems on a real line, weighted spaces were needed for wellposedness to hold, which was confirmed in [1].

This talk reports on a wide class of equations for which weighted Sobolev spaces are not needed. In particular, it include quasilinear equations which also exhibit this competition between dispersion and anti-diffusion: a Rosenau-Hyman compacton equation, the Harry Dym equation, and equations which arise in the numerical analysis of finite difference schemes for dispersive equations. For these quasilinear equations, the well-posedness theorem requires that the initial data be uniformly bounded away from zero.

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On the mass-less limit for the Euler-Poisson model

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Abstract. We consider the two fluid Euler-Poisson equation in plasma physics, namely (after rescaling)

$$\begin{aligned} \partial_t n_e + \operatorname{div}(n_e v_e) &= 0, & (17)\\ \varepsilon^2 n_e\left\{\partial_t v_e + v_e\cdot\nabla v_e\right\} + \nabla p_e(n_e) &= n_e \nabla V, & (18)\\ \partial_t n_i + \operatorname{div}(n_i v_i) &= 0, & (19)\\ n_i\left\{\partial v_i + v_i\cdot\nabla v_i\right\} + \nabla p_i(n_i) &= -\nabla V, & (20)\\ -\Delta V &= n_i - n_e. & (21) \end{aligned}$$

In this equation, (n_e, v_e) (resp. (n_i, v_i)) stand for the density and velocity of the electrons (resp. ions), V is the electric potential and $\varepsilon^2 = m_e/m_i$ is the ratio of the inertia of each species. This ratio can be no larger than 10^{-3}. We investigate the behavior of solutions as $\varepsilon \to 0$ on R^3 and T^3 and we derive the well-known Euler-Poisson equation for the ions. For well prepared initial data, this is relatively easy, but the case of ill-prepared initial data (initial data with energy excess of size $O(\varepsilon^2)$ relative to well-prepared initial data) leads to an interesting analysis.

Stability and instability of standing waves for the nonlinear fractional Schrödinger equation

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Abstract. In this talk, we consider the orbital stability of standing waves for the nonlinear fractional Schrödinger equation by the profile decomposition theory. We prove that when $0 < \gamma < 2s$, the standing waves are orbitally stable; when $\gamma = 2s$, the standing waves are strongly unstable to blow-up.

Global well-posedness for the hyperbolic nonlinear Schrödinger equation on subcritical Sobolev spaces

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Abstract. In this talk, we describe a new method to obtain a priori uniform bounds in time for solutions to the nonlinear Schrödinger equations (NLS) on $\mathbb{R}^2$ having power nonlinearities of arbitrary odd degree, and with large initial data in Sobolev space. The method presented here applies to both the usual NLS equations associated to the Laplacian and with a nonlinearity of defocusing sign, as well as to the more difficult so-called "hyperbolic" NLS which is associated to an indefinite or "hyperbolic" signature. The latter is particularly interesting since its long time behavior is to date unknown.

We show how, by rigorously justifying that these equations govern the modulation of wave packet-like solutions to an artificially constructed equation with an advantageous structure, every subcritical (with respect to scaling) Sobolev norm of the solution is a priori non-increasing in time. Global existence in all Sobolev spaces then follows immediately from standard local well-posedness results for Schrödinger equations.

SESSION 25
Solitons, vortices, domain walls, their dynamics and their progenitors

Quantum gravity and quantum groups

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Abstract. Methods of nonlinear steepest descent or nonlinear stationary phase based on Riemann-Hilbert analysis have proved to be effective in understanding the *propagation of oscillations* in a nonlinear environment. 2D quantum gravity arose in the study of discrete approximations for Polyakovs formulation of Liouville field theory and has been a fertile source for questions and ideas in the analysis of random combinatorial structures and their relation to various problems at the interface between probability theory and mathematical physics. In particular, it bears on the problem of constructing a measure on random surface metrics. This connects directly to the analysis of nonlinear oscillations within the framework of random matrix theory.

Our work applies in this latter context. Specifically we study scaling limits for discrete models of the conformal geometry of surfaces. These limits lead naturally to a class of conservation laws that describe a propagation of oscillations that can in turn be analyzed via the bionomial Hopf algebra and its associated umbral calculus. Non-commutative extensions of this bring one to the setting of quantum groups and a new class of problems relating the characteristic geometry of conservation laws to the classical study of ruled surfaces in algebraic geometry.

On the uniqueness and scattering for Gross Pitaevskii hierarchies and quantum de Finetti

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Abstract. This talk addresses some recent results related to the Cauchy problem for the cubic Gross-Pitaevskii (GP) hierarchy, an infinite system of coupled linear PDEs which emerges in the derivation of the cubic nonlinear Schroedinger equation from interacting Bose gases. In particular, a new proof of unconditional uniqueness of solutions is presented, as well as a proof of scattering in the defocusing 3D case. The techniques involved include an application of the quantum de Finetti theorem, combined with recursive Strichartz estimates and tree graph expansions. This is joint work with C. Hainzl, N. Pavlovic and R. Seiringer.

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Finite energy bound states for the stationary Klein-Gordon and magnetic Ginzburg-Landau

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Abstract. We prove the existence of infinitely many solitary waves for the nonlinear Klein-Gordon or Schrödinger equation

$$\Delta u - u + u^3 = 0, \;\; u \in H^1$$

in $\mathbf{R}^2$, which have finite energy and whose group of symmetry reduces to the identity. More precisely, given any balanced, flexible and closable network in $\mathbf{R}^2$ we find a sequence of bounded energy solutions whose energy approaches to infinity and with large number of positive or negative bumps arranged on the edges of the network. We then extend these constructions to construct multi-vortex, non-radial, finite energy solutions to the magnetic Ginzburg-Landau equation and magnetic Chern-Simons-Higgs equations on all of $\mathbf{R}^2$. The constructions borrow some ideas of geometric gluing in CMC surfaces.

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Flows related to one-dimensional Gagliardo-Nirenberg inequalities

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Abstract. One dimensional Gagliardo-Nirenberg inequalities are equivalent to Sobolev's inequality on the sphere, at least when the dimension is an integer, or to the critical interpolation inequality for the ultraspherical operator in the general case. Viewing these inequalities as Lyapunov functionals associated with nonlinear diffusion equations provides an alternative proof of these inequalities in their sharp form [1].

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Near amplitude crossing of mKdV double solitons and applications to effective dynamics of perturbed mKdV

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Abstract. The mKdV equation $\partial_t u = \partial_x(-u_{xx} - 2u^3)$ admits double-soliton solutions, obtained via inverse scattering theory, with parameters of position and amplitude. We prove a new asymptotic decomposition formula for the double-soliton profile into the sum of two single-solitons that remains valid as the amplitude parameters coincide. Using this formula, we give an explanation of the avoided-crossing phenomena discussed in Holmer, Perelman, & Zworski [1], that emerges when one considers the dynamics of a double-soliton under the influence of a slowly-varying potential, i.e. as an approximate solution to $\partial_t u = \partial_x(-u_{xx} + Vu - 2u^3)$, for $V(x,t) = W(hx, ht)$ and $0 < h \ll 1$. The avoided-crossing is a dynamical scenario in which the scales of the two solitons become exponentially close inducing an abrupt switch in position of the two solitons. The results are supported by numerical computations.

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Monopole wall dynamics

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Abstract. Monopoles are prototypical solitons of gauge theory. Their dynamics can be viewed as a geodesic motion on the full space of monopole solutions, called the monopole moduli space. Such moduli spaces are interesting geometrically, and have numerous applications in quantum gauge theory, in string theory, and in supergravity. Finding the metrics on these spaces, however, is a difficult problem. Since the explicit metric on the moduli space of two monopoles was found by Atiyah and Hitchin in 1985, metrics for higher monopole charge or for periodic monopoles remain elusive.

In this work [1, 2] we study doubly periodic monopoles, which we call monopole walls. For a general monopole wall we find the number of its moduli. For large values of moduli we find that the wall splits into far-separated sub-walls. By studying the interaction between such moving sub-walls we find the asymptotic metric on the moduli space of the original monopole wall. The answer we obtain is formulated in terms of spectral curves, their amoebas, and melting crystal volumes. We also identify which, often drastically different, monopole walls have identical dynamics.

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SESSION 26

Evolution equations in mathematical biology: Cellular and network processes

Exploiting network structure in models of biochemical reaction systems

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Abstract. Systems biology is concerned with understanding the dynamics of complicated biochemical reaction systems. When modeled with mass action kinetics, many classical results allow dynamical and steady state properties of the system to be inferred from the structure of the reaction network alone [1]. This a powerful tool in cutting through the complexity of the systems, which may consist of dozens or hundreds of interacting molecules. The network property of being weakly reversible has played a powerful recurring role; however, many very simple biochemical motifs are not weakly reversible.

In this paper, we summarize recent extensions to the study of mass action systems which significantly expand the scope of networks which can be analyzed by these classical methods [2]. The approach consists of associating the original network to a weakly reversible one which has generalized kinetics, i.e. interaction terms which are not necessarily implied by the network's stoichiometry. We are able to obtain results regarding the algebraic structure of the steady state set typically reserved for weakly reversible networks. We then present a new computational procedure for determining when a biochemical reaction network may be associated to a weakly reversible one in this manner [3].

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Perceptual alternation and neuronal dynamics in auditory streaming

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Abstract. Several acoustic features have been shown to influence perceptual organization of sound, in particular, the frequency of tones and the rate of presentation. Subjects listening to sequences of pure tones that alternate in frequency (presented in patterns of repeating doublets or repeating triplets) report alternations in perception between one auditory object (“stream integration”) and two separate auditory objects (“stream segregation”). In addition, several animal studies using such stimuli have characterized the single-unit and multiunit neural activity and event-related local field potentials in monkeys' primary auditory cortex (A1) while systematically varying the difference ΔF between tones or the presentation rate [1, 2, 3]. They found that when presenting triplets $ABA\_$, the responses to the non-preferred tones B decreased with increasing ΔF. There was also a general decrease in response magnitude after the first triplet in the sequence. Thus it has been suggested that the suppression of the responses to B tones is indicative of stream segregation.

In this talk we introduce a firing rate model for stream segregation and integration in primary auditory cortex and compare its dynamics to data reported in [2]. Using the model, we then investigate the validity of the hypothesis that stream segregation results from suppression of the neural response to the non-preferred tones.

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The effect of radiation-induced dedifferentiation on cancer cell lineage

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Abstract. A substantial amount of evidence indicates that solid tumors exhibit cellular hierarchy in which only a fraction of thus called cancer stem cells (CSCs) possess the ability to regenerate and differentiate. Remarkably, recent discoveries indicate that, besides these inherent CSCs, previously differentiated cells can also revert to a stem-like state. This process, appropriately referred to as "*de*differentiation," has been shown to escalate when cancer cells are exposed to radiation [1]. Clearly, this finding has major implications in tumor treatment protocol since radiation prevails as a routine anticancer therapy and tumor regrowth remains a continual concern.

In this work we modify a cell lineage model [2] to describe a tumor population and to include cellular dedifferentiation. We construct a coupled ODE system which tracks stem, committed progenitor, and differentiated cells and investigate various internal feedbacks. Using experimental data to set the model parameters, we observe an increased dedifferentiation rate for irradiated cells. Furthermore, our findings suggest that this elevated dedifferentiation rate can profoundly impact the long-term population size and composition.

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Bacterial chemotaxis and stochastic particle systems

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Abstract. I will discuss coalescing diffusion processes, associated numerical methods and applications to the Keller-Segel model of bacterial chemotaxis.

Walking, sliding, and detaching: time series analysis for kinesin

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Abstract. Kinesin is a molecular motor that, along with dynein, moves cargo such as organelles and vesicles along microtubules through axons. Studying these transport process is vital, since non-functioning kinesin has been implicated in a number of neurodegenerative diseases, such as Alzheimers disease. Over the last twenty years, these motors have been extensively studied through in vitro experiments of single molecular motors using laser traps and fluorescence techniques. However, an open challenge has been to explain in vivo behavior of these systems when incorporating the data from in vitro experiments into straightforward models.

In this talk, I will discuss recent work with experimental collaborator, Will Hancock (Penn State), to understand more subtle behavior of a single kinesin than has previously been studied, such as sliding and detachment and how such behavior can contribute to our understanding of in vivo transport. Data from these experiments include time series taken from fluorescence experiments for kinesin. In particular, we will use novel applications of switching time series models to explain the shifts between different modes of transport.

Hemodynamic model of malaria infection with detailed immune response

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Abstract. Half the world population is currently at risk of malaria infection, with 200 million clinical cases and 600,000 deaths in 2012. Even though this disease has attracted substantial research resources in the last century, the detailed characterization of the dynamics of malaria is still an open question. Existing mathematical models of malaria infection are rudimentary, and lack the immune data to expand the level of detail to useful predictive levels. The Malaria Host Pathogen Interaction Center (MaHPIC), a research consortium comprised by the University of Georgia, Emory University, Georgia Tech, and the Center for Disease Control is producing information about the disease at unprecedented levels of detail. In this talk I will present recent developments by our MaHPIC group in the mathematical modeling of the blood stage of malaria infection using a coupled system of differential equations comprised of two transport PDEs and a set of ODEs. I will also present the challenges in calibrating this type of model with 'omic technologies (transcriptomics, lipidomics, proteomics, metabolomics, and clinical data). Our preliminary model is able to reproduce the clinical presentation of malaria: severe anemia on first infection, and coexistence of host and parasites in subsequent infections.

Pattern formation of folds in the brain

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Abstract. The folding patterns of the brain vary across individuals in their size, depth, and location. This individual variability presents a challenge when studying and quantifying changes in brain folding patterns during development, aging, and disease. Furthermore, there are a number of biological theories that hypothesize about the mechanisms involved in the formation and location of cortical fold development, but no consensus has been reached.

In this paper we present a spatio-temporal Turing reaction-diffusion system to model folding pattern formation in the brain. A Turing system consists of two reaction-diffusion equations representing chemical morphogens. One biological hypothesis suggests that during neurogenesis intermediate progenitor (IP) cells are formed in the subventricular zone (SVZ) of the lateral ventricle (LV), leading to ridge (gyrus) and valley (sulcus) formation in the brain. We assume that folding pattern formation is driven by chemical activator and inhibitor morphogens that drive IP cell formation. We model the SVZ with a prolate spheroid surface and use exponential and logistic growth to create dynamically growing prolate spheroid domains.

We derive the mathematical equations and conditions needed to construct a growing domain Turing system that generates a genetic chemical prepattern for cortical folding. Parameters representing growth rate and domain scale are varied during numerical simulations. Altering the growth rate parameter R allows the size of the domain to be controlled, and thus the size of the SVZ and LV. Changing the domain scale parameter ω represents a change in the overall level of genetic expression of activator and inhibitor morphogens. Results demonstrate that R and ω control the complexity of the labyrinthine pattern that evolves.

We use our growing domain Turing system to model various diseases of cortical folding. Polymicrogyria is a group of cortical folding diseases where the folds of the cortex are unusually high in number and small in size. Lissencephaly is a group of diseases where cortical folds appear broader in width and fewer in number. We are able to capture various manifestations of polymicrogyria and lissencephaly by altering R and ω. For example, polymicrogyria with lateral ventricle enlargement can be modeled by increasing the value of R, resulting in a complex pattern with an increased number and width of folds. Our results demonstrate that dynamically growing domain Turing models represent an important step in improving our understanding of cortical folding pattern formation in the brain and the influence that domain growth and genetic expression can have on cortical folding.

Stochastic reaction-diffusion methods for modeling cellular processes

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Abstract. Particle-based stochastic reaction diffusion methods have become a popular approach for studying the behavior of cellular processes in which both spatial transport and noise in the chemical reaction process can be important. While the corresponding deterministic, mean-field models given by reaction-diffusion PDEs are well-established, there are a plethora of different stochastic models that have been used to study biological systems, along with a wide variety of proposed numerical solution methods.

In this talk I will motivate our interest in such methods by first summarizing several applications we have studied, focusing on how the complicated ultrastructure within cells, as reconstructed from X-ray CT images, might influence the dynamics of cellular processes. I will then introduce our attempt to rectify the major drawback to one of the most popular particle-based stochastic reaction-diffusion models, the lattice reaction-diffusion master equation (RDME). We propose a modified version of the RDME that converges in the continuum limit that the lattice spacing approaches zero to an appropriate spatially-continuous model.

The essential role of phase delayed inhibition in decoding synchronized oscillations within the brain

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Abstract. he widespread presence of synchronized neuronal oscillations within the brain suggests that a mechanism must exist that is capable of decoding such activity. Two realistic designs for such a decoder include: 1) a read-out neuron with a high spike threshold, or 2) a phase-delayed inhibition network motif. Despite requiring a more elaborate network architecture, phase-delayed inhibition has been observed in multiple systems, suggesting that it may provide inherent advantages over simply imposing a high spike threshold. We use a computational and mathematical approach to investigate the efficacy of the phase-delayed inhibition motif in detecting synchronized oscillations, showing that phase-delayed inhibition is capable of detecting synchrony far more robustly than a high spike threshold detector. Furthermore, we show that in a system with noisy encoders where stimuli are encoded through synchrony, phase-delayed inhibition enables the creation of a decoder that can respond both reliably and specifically to a stimulus, while a high spike threshold does not.

Wave patterns in an excitable neuronal network

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Abstract. This talk describes a study of spiral- and target-like waves traveling in a two-dimensional network of integrate-and-fire neurons with close-neighbor coupling. The individual neurons are driven by Poisson trains of incoming spikes. Each wave nucleates as a result of a fluctuation in the drive. It begins as a target or a spiral, and eventually evolves into a straight "zebra"-like grating. Some of the waves contain defects arising from collisions with other waves. The wavelength and wave speed of the patterns were investigated, as were the temporal power spectra of the oscillations experienced by the individual neurons as waves were passing through them.

Theoretical modeling of spatiotemporal dendritic integration

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Abstract. Neurons process information via spatiotemporal integration of synaptic inputs from dendrites. Many experimental results demonstrate dendritic integration could be highly nonlinear, yet few theoretical analyses have been performed to obtain a precise quantitative characterization analytically. Based on asymptotic analysis of a passive cable-type neuron model, given a pair of time-dependent synaptic inputs, we derive a bilinear spatiotemporal dendritic integration rule. The summed somatic potential can be well approximated by the linear summation of the two postsynaptic potentials elicited separately, plus a third additional bilinear term proportional to their product with a proportionality coefficient. The coefficient encodes the spatial and temporal input information and is nearly independent of input strengths. The rule is valid for a pair of synaptic inputs of all types, including excitation-inhibition, excitation-excitation, and inhibition-inhibition. In addition, the rule is valid during the whole dendritic integration process for a pair of synaptic inputs with arbitrary input time differences and input locations. This rule is then verified through simulation of a realistic pyramidal neuron model and in electrophysiological experiments of rat hippocampal CA1 neurons. The rule is further generalized to describe the spatiotemporal dendritic integration of multiple excitatory and inhibitory synaptic inputs. The integration of multiple inputs can be decomposed into the sum of all possible pairwise integration, where each paired integration obeys the bilinear rule. This decomposition leads to a graph representation of dendritic integration, which can be viewed as functionally sparse.

Asymptotic and numerical methods for metastable events in stochastic gene networks

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Abstract. Stochasticity in gene regulation circuits is known to cause rare extreme shifts in the expression of a gene, which can have a profound effect on the behavior of a cell. This leads to the question of how cells coordinate and regulate different sources of biochemical fluctuations, or noise, to function within a genetic circuit. Rare, noise-induced dynamical shifts in a stochastic process are known as metastable events. For example, Brownian motion in a double well potential, where the fluctuations are weak compared to the force of the potential, displays bistable switching. In general, metastable events occur when fluctuations are weak compared to deterministic forces, and the stochastic process is said to be under weak noise conditions. The challenge for stochastic modeling is to predict and explain the possible metastable behaviors and offer a testable hypothesis by quantifying the timescale on which those events are likely to occur.

A computational model of the influence of depolarization block on initiation of seizure-like activity

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Abstract. The dynamics of networks of excitatory and inhibitory neurons exhibit pathological activity when excessive excitation is not balanced by inhibition. Impairment of inhibitory neurons has been observed to precede the initiation of epileptiform activity in biological neural networks. However, the exact cause of the failure of inhibition is not well understood. We hypothesized that inhibitory neurons become vulnerable to depolarization block when extracellular potassium concentration is increased and investigated different types of pathological dynamics arising from the network. Using a Wilson-Cowan type model, we found that the transition from a physiological to a pathological state can occur via a saddle-node bifurcation or a homoclinic bifurcation. For networks exhibiting the saddle-node bifurcation, transient perturbations switched the network to seizure state, in which inhibitory neurons entered into depolarization block and excitatory neurons fired at their maximum rate. A tonic to clonic phase transition was observed in the network model with a homoclinic bifurcation when the recovery of the extracellular potassium concentration to normal levels shifted the network states. The predictions of the mean field model were tested in network simulations. Our network model with the proposed mechanism for failure of inhibition reproduces network activity observed in brain slices at the onset of seizure-like events and provides an explanation for the tonic to clonic transition as an emergent phenomenon of the network.

Effects of synaptic connectivity inhomogeneities on traveling waves of activity in integrate and fire neural networks

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Abstract. Large-scale networks containing integrate-and-fire models of the neurons have been successfully used activity propagation in disinhibited cortical slices. For these models, a common assumption is that while the strength of the synaptic connections between two neurons changes as a function of distance, this interaction does not depend on other local parameters. In this work we examine how inhomogeneities affect the dynamics of the activity propagation, both for small inhomogeneities that modulate constant speed traveling vaves as well as for large inhomogeneities that induce propagation failure. While an explicit formula for speed modulations for activity propagation is difficult to determine, we established a series of progressively more accurate approximations that we use to compare with results from numerical simulations.

New parameter-free results on the bistability of chemical reaction networks

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Abstract. Much attention has been paid recently to bistability and switch-like behavior that is resident in important biochemical reaction networks. It is often the case that large classes of complex networks cannot exhibit bistable behavior, no matter what their reaction rates might be. In those frequent cases, the absence of bistable behavior is a consequence of the network structure alone, and is independent of the values of the various parameters involved in the model.
In this talk I will discuss some new developments relating the topology of a reaction network with its capacity for bistable behavior. These results fall broadly into the field of chemical reaction network theory.

Dimension reduction for stochastic conductance based neural models

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Abstract. Random gating of ion channels is a significant source of noise in neural systems. Finding accurate and effective low-dimensional representations of neural dynamics that incorporate stochastic effects is an important step towards understanding the role of channel noise in network level behavior. The "stochastic shielding approximation" introduced by Schmandt and Galán [1] is a dimension reduction technique for generating approximate sample paths from a finite state Markov process in which only a subset of states are observable. We conducted a rigorous analysis of this stochastic shielding heuristic, deriving a new quantitative measure of the contribution of individual edges in the graph to the accuracy of the approximation.
In this talk, I will discuss our analysis and our extension of this method for a broad class of random graph models and for the Hodgkin-Huxley ion channel model. I will show how these results shed new light on the contributions of different ion channel transitions to the variability of neural systems [2]. I will also briefly discuss how well stochastic shielding works if we introduce a small parameter via separation of time scales.

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Basal ganglia dynamics during the transition to Parkinson's disease

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Abstract. Muscle rigidity associated with Parkinson's disease (PD) is thought to be correlated with the loss of dopamine and the emergence of beta oscillations in the basal ganglia. As dopamine-producing neurons die off, synaptic connection

strengths change leading to a favoring of the Indirect Pathway. Due the the sheer number of neurons in the basal ganglia, a model of individual neurons would be a heavy computational load and too complicated to analyze. Instead a firing rate model using the populations of neurons allows us to simplify the hundreds of thousands neurons in each population to just one equation per population based on the average incoming firing rates of neighboring nuclei. By first modeling a healthy basal ganglia and then a PD basal ganglia (based on available experimental data) we can show the transition between the two stages and which connections are the biggest culprits, as this is still under debate in the scientific community.

As this is a nonlinear problem, getting analytic results poses a real problem. To counter this, we simplified the nonlinear system of ODEs to a set of piecewise-linear differential equations by splitting the space up into 27 different regions. With this new set of equations, we are able to solve the system exactly in each individual region, while still keeping the structure of our nonlinear forcing term. Though the equations in each region became linear we are still able to show the existence of limit cycles due to this piecewise structure and the system's dynamics through the region boundaries. Since this behavior is seen in the PD basal ganglia, it is crucial we are able to observe this behavior.

Determination of optimal targeting performances in a stochastic neural growth model with low branching probabilities

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Abstract. We examine the target performances of a simplified growth model under a finite resource constraint. Starting from an initial branch, each active neurite branches independent of other branches with set probability (p), while total tree length is limited to maximal value (L). Since the probability of success is proportional to the number of active branches, we seek to determine the distance (D) at which a family of trees, generated with branching probability p, has a maximal number of branches. We determine how distance D depends on variables p and L when the branching probabilities are low. We show that our model is in agreement with experimental data from tissue cultures.

SESSION 27
Mechanisms for computations in neuronal networks

Integrate-and-fire model of insect olfaction

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Abstract. When a locust detects an odor, the stimulus triggers a series of synchronous oscillations of the neurons in the antenna lobe. These oscillations are followed by slow dynamical modulation of the firing rates which continue after the stimulus has been turned off. I model this behavior by using an Integrate-and-Fire neuronal network with excitatory and inhibitory neurons. The inhibitory response of both types of neurons contains a fast and slow component. The fast component, together with the excitation, creates the initial oscillations while the slow component suppresses them and aids in the creation of the slow patters that follow. During the initial oscillations the stimulus can be identified by determining which excitatory neurons participate consistently in every cycle of the oscillations. I have also derived a firing-rate model that produces the same spiking behavior as the network simulations and allows for more detailed bifurcation analysis and additional insight into the plausible mechanism by which insects detect and identify odors.

The role of gap junctions between excitatory neurons in synchronizing cortical dynamics

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Abstract. Brain networks are known to give rise to global oscillations that are linked to synchronized neuronal activity, which has been shown to contribute to cognitive processes such as perception, motor performance, learning and memory [1]. How these oscillations arise, however, is not yet completely understood. Researchers believe that electric coupling through sites called gap junctions may facilitate their emergence, and determine some of their properties. Gap junctions between inhibitory neurons in the mammalian cerebral cortex have been well studied, but experimentalists have only recently discovered the existence of gap junctions between excitatory, pyramidal neurons.

In this paper we closely follow data from experimental papers to construct a detailed, comprehensive model with both synaptic and electric coupling for both excitatory and inhibitory neurons using a modified version of the Hodgkin Huxley equations. Through this model, we examine the resulting dynamical regimes from the inclusion of both electric and synaptic connections, with a specific interest in the emergence and properties of synchrony.

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Efficient reconstruction of structural connectivity from neuronal dynamics

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Abstract. Recovering structural connectivity in large neuronal networks is an unresolved yet fundamental problem in characterizing neuronal computation. Taking into account the prominence of sparsity in neuronal connectivity, we develop a framework for efficiently reconstructing neuronal connections in a sparsely-connected, feed-forward network model with nonlinear integrate-and-fire (I&F) dynamics. Driving the network with a small ensemble of random stimuli, we derive a set of underdetermined linear systems relating the network inputs to the corresponding firing rates of the downstream neurons via the feed-forward connection matrix. In reconstructing the network connections, we utilize compressive sensing theory, which allows for the recovery of sparse solutions to such underdetermined linear systems. Using the reconstructed connection matrix, we also accurately recover network inputs distinct from the training set of stimuli. We conclude by analyzing the timescale over which neuronal firing rates should be measured for successful recovery and then similarly consider the reconstruction of feed-forward connectivity in networks with both recurrent and feed-forward connections. We expect this work to be useful in understanding the structure-function relationship for neuronal networks, giving insight into a possible mechanism for unconscious inference of natural stimuli.

Stochastic synchronization of neural activity waves

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Abstract. We show waves in distinct layers of a neuronal network can become phase-locked by common spatiotemporal noise, analyzing stationary bumps, traveling waves, and breathers. A weak noise expansion is used to derive an effective equation for the position of the wave in each layer, yielding a stochastic differential equation with multiplicative noise. Stability of the synchronous state is characterized by a Lyapunov exponent, which we can compute analytically from the reduced system. Our results extend previous work on limit-cycle oscillators [1], showing common noise can synchronize waves in a broad class of models.

We analyze a pair of uncoupled neural field models driven by common noise:

$$\mathrm{d}u_j(x,t) = \left[-u_j(x,t) + \int_{-\pi}^{\pi} w(x-y) f(u(y,t))\mathrm{d}y\right]\mathrm{d}t + \varepsilon \mathrm{d}W(x,t), \qquad j = 1,2,$$

where $u_j(x,t)$ is population activity of population, $w(x-y)$ describes synaptic connectivity, and $f(u)$ is a firing rate nonlinearity. Small amplitude ($\varepsilon \ll 1$) spatiotemporal noise $\mathrm{d}W(x,t)$ is white in time and has spatial correlations $C(x-y)$. We begin by studying noise-induced synchronization of bumps, $u_j \approx U_j(x+\Delta_j(t))$. Noise perturbations cause each bump to wander diffusively about the spatial domain. Once both bumps' position Δ_1 and Δ_2 meet, they are phase-locked indefinitely. Using perturbation theory, we can derive effective Langevin equations for the bumps' positions

$$\mathrm{d}\Delta_j = \varepsilon \mathrm{d}W(\Delta_j, t),$$

driven by multiplicative noise

$$W(\Delta, t) = \sum_{k=1}^{\infty} a_k \cos(k\Delta) X_k + b_k \sin(k\Delta) Y_k.$$

The phase difference $\phi = \Delta_1 - \Delta_2$ will have geometric mean $\phi_0 \mathrm{e}^{\lambda t}$, and we can approximate the Lyapunov exponent $\lambda = -\varepsilon^2\ sum_{k=1}^{\infty}\left[a_k^2 + b_k^2\right]$ [2]. We will discuss similar results for the case of traveling waves and breathers.

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Low-dimensional dynamics embedded in echo-state networks

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Abstract. Biological neural circuits display both spontaneous asynchronous activity, and complex, yet ordered activity while actively responding to input. Recently, researchers have demonstrated this capability in large, recurrently connected neural networks, or "echo-state" networks [1]. In this talk, we address the question of how network connectivity structure may affect the performance of echo-state networks.

We choose a family of networks in which the neurobiological constraint of Dale's Law — that most neurons are either excitatory or inhibitory — is satisfied. We first study the transition to chaos in this setting, using principal orthogonal decomposition techniques to provide a lower-dimensional description of network activity. We find that key characteristics of this transition differ in constrained networks, versus unconstrained networks with similar variability. This is a consequence of the fact that the constrained system may be described as a perturbation from a system with non-trivial symmetries, implying the presence of both fixed points and periodic orbits that organize solutions, even for large perturbations.

We next investigate the impact of this phenomenon on the ability of constrained networks to reproduce a learning task recently investigated in unconstrained networks [2]. Surprisingly, the delayed transition to chaos has little effect on statistical measures of training effectiveness. We hypothesize that the addition of the feedback loop quickly moves effective network connectivity away from symmetry and into the chaotic regime at the onset of training.

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What does efficient coding theory tell us about the benefits of pathway splitting in sensory coding?

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Abstract. In many sensory systems the neural signal splits into multiple parallel pathways, suggesting an evolutionary fitness benefit of a very general nature. The purpose of this profuse and early pathway splitting remains unknown. One hypothesis for pathway diversification is efficient coding: that sensory neurons have evolved to an optimal configuration for maximizing information transfer given the structure of natural stimuli. Using efficient coding, I explore the benefits of cell diversification into ON and OFF types, that respond to stimulus increments and decrements, respectively.

I will discuss the conditions under which sensory coding by a mixture of ON and OFF cells is more efficient than coding by a population of only one cell type (ON or OFF) using two measures of efficiency: the mutual information between stimulus and response, and the accuracy of a linear decoder. Surprisingly, our results show that the information transmitted by a neuronal population is independent of the way the population splits into ON and OFF cells. However, an equal mixture of ON and OFF subpopulations is the most efficient in the sense that it uses fewest spikes. In addition to computing information, if we ask for downstream neurons to linearly decode the stimulus, the ON-OFF system can achieve a better stimulus decoding. I relate the statistics of natural stimuli to the optimal ON-OFF ratio in a population of sensory neurons, and derive predictions for the optimal response properties of the population. Our work suggests that the evolution of ON-OFF diversification in sensory systems may be driven by the benefits of lowering average metabolic cost, especially in a world in which the relevant stimuli are sparse.

Coding natural stimuli through correlated neural activity

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Abstract. Efficient coding and recognition of natural stimuli that often display complex spatiotemporal characteristics is essential for an organism's survival. Here we focused on understanding how correlated neural activity can provide information about natural stimuli that is not captured by single neuron activity. Using a combination of theoretical, computational, and experimental approaches, we show that correlated neural activity can provide information about behaviorally relevant second order stimulus attributes. In particular, we demonstrate a novel form of stochastic resonance as such coding is optimal for a non-zero value of neuronal variability. Moreover, time permitting, we will also show how correlated activity can provide a feature invariant representation of natural communication stimuli.

Pulse-coupled mixed-mode oscillators: rhythms and clusters in the olfactory system

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Abstract. In the presence of odors the mammalian olfactory system displays fast and slow rhythms of coherent neuronal activity. Experiments have shown that the faster gamma rhythm can be generated in the olfactory bulb alone while the slower beta rhythm requires the interconnectivity of the olfactory bulb with piriform cortex. The connectivity of three relevant populations of neurons within these two brain areas comprises two recurrent pathways, one within the olfactory bulb and one that passes through both the bulb and piriform cortex. The principle cells of the olfactory bulb, which excite the other populations via feedforward connections and receive inhibition in feedback, feature mixed-mode oscillations. We investigate the role of these mixed-mode oscillations and the recurrent pathways in the generation of these two rhythms.

To address the influence of mixed-mode oscillations on rhythmic activity, we introduce a minimal model in which the interactions of the three populations is reduced to an effective inhibitory pulse-coupling within a single population of the principle cells. The two different pathways are captured by employing two types of pulses that differ, in particular, in their propagation delays. The principle cells are modeled by single-compartment Hodgkin-Huxley-type equations and exhibit spikes and non-spike (sub-threshold) oscillations along a single stable limit cycle. Odor input is modeled as noisy current injected into each cell.

In the limit of weak coupling, the system is captured by a generalized stochastic Kuramoto model. The complexity of the mixed-mode oscillations is reflected here by the complex interaction between the oscillators, which arises from the phase resetting curve of the mixed-mode oscillators. For the case of all-to-all coupling and a large population, we analyze the system via a non-linear Fokker-Planck equation. Weakly nonlinear analysis around the incoherent state describes its destabilization by various cluster states, which we identify with different rhythms. We investigate transitions between different rhythms as the shapes and delays of the pulses are varied and compare these theoretical results with numerical simulations of the single-population, spiking-neuron model.

Sensory adaptation tunes visual cortex to criticality

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Abstract. Cortical circuits adapt when presented with changing sensory input. Such adaptation is a homeostatic process known to prevent excessive firing rates and correlations. Adaptation also optimizes aspects of sensory function by tuning gain, stimulus selectivity, and sensory information transmission. What regime of cortical network dynamics supports the moderate firing rates, the intermediate correlations, and the anticipated functional advantages brought about by adaptation? What is the network-level modus operandi that emerges during adaptation?

A possible answer to these questions is offered by recent computational models in which activity-dependent adaptation dynamically tunes the network to a critical regime at the boundary between two extreme types of network dynamics: one with low firing rates and weak correlations, the other with high firing rates and strong correlations. The critical regime strikes a balance between these extremes, with moderate firing rates and correlations. Moreover, the critical regime can provide functional advantages including optimal dynamic range and information transmission. Thus, optimized function together with moderate firing rates and correlations are associated with both adaptation and the critical regime. This commonality condenses the two questions raised above to one: Does adaptation to dynamic sensory input tune cortex network dynamics towards the critical regime?

Here, we addressed this question in turtle visual cortex during naturalistic stimulation of the retina and in a companion computational model. We obtained long-duration 96-electrode recordings of population activity from visual cortex, allowing us to measure multi-scale spatiotemporal patterns of visually-evoked activity and quantify their statistics. We examined statistics of neuronal avalanches, which are bouts of elevated population activity with correlations in space and time. We carried out a parallel study in a model network, implementing adaptation in the form of synaptic depression.

In both the experiments and the model, we found multifaceted evidence for the critical regime after a transient period of adaptation. In contrast, during the transient, avalanches were predominantly large scale, inconsistent with the critical regime. Our findings suggest that adaptation plays a crucial role in tuning cortical circuits towards the critical regime during vision.

SESSION 28

Analytical and computational techniques for differential and difference equations

Inverse scattering transform for nonlinear Schrödinger equations with non-zero boundary conditions

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Abstract. We will discuss some recent results on the Inverse Scattering Transform (IST) as a means to solve the initial value problem for the focusing and defocusing nonlinear Schrödinger equations:

$$iq_t = q_{xx} + 2\sigma|q|^2 q\,, \qquad \sigma = \pm 1\,,$$

with nonzero boundary values $q_\pm(t) \equiv |q_\pm|e^{i\theta_\pm(t)}$ as $x \to \pm\infty$. In both dispersion regimes, the direct problem is shown to be well-posed for potentials $q(x,t)$ such that $q(x,t) - q_\pm(t) \in L^{1,1}(R^\pm)$ with respect to x for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data can be rigorously established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations, and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_\pm = \sqrt{k^2 + \sigma|q_\pm|^2}$, k being the usual complex scattering parameter in the IST.

Besides setting up the stage for a rigorous study of the long-time asymtptotics of fairly general NLS solutions, the IST formulation enabled us to further investigate the spectrum of the associated scattering operators. Specifically, for the defocusing NLS equation with symmetric boundary values $|q_+| = |q_-|$, by considering a specific kind of piecewise constant initial condition we were able to clarify two issues, concerning: (i) the (non)existence of an area theorem relating the presence/absence of discrete eigenvalues to an appropriate measure of the initial condition; and (ii) the existence of a contribution to the asymptotic phase difference of the potential from the continuous spectrum.

This talk is based on joint work with Gino Biondini and Emily Fagerstrom (SUNY Buffalo), Francesco Demontis and Cornelis van der Mee (Università di Cagliari), and Federica Vitale (Università del Salento).

Darboux transformations of type I (generalization of Laplace-Darboux transformations method to operators of higher orders)

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Abstract. Darboux transformations by Gaston Darboux of classical differential geometry have been adopted for analytical solution of partial differential equations, linear and nonlinear by the Soliton theory. There are well-known short compact formulas for Laplace-Darboux transformations for Schrödinger operators of first and second orders, and the corresponding solution algorithms are widely used.

Here I shall present the first significant results in my project of constructing some analogues of Darboux transformations method for operators of order three:

1) own algebraic formalization for the notion of Darboux transformations which led to a more consistent theory.

2) a special class of Darboux transformations with remarkable properties: a) transformations are invertible; b) formulas for the inverse are compact, and explicit.

3) examples showing that some new types of partial differential equations can be solved by the corresponding algorithm.

This is an ongoing project.

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Application of the Euler and homotopy operators to integrability testing

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Abstract. It was shown in [2] how the homotopy operator can be used to invert the total divergence, thereby reducing an integration in multiple dimensions to a standard one-dimensional integration where the integration variable parameterizes a homotopic path.

In [2], the homotopy formula is expressed in terms of (coordinate independent) differential forms and involves the higher-order Euler operators. In this talk, a calculus-based formula for the continuous homotopy operator will be presented in its simplest form. In essence, the formula does integration by parts on jet spaces but without explicit use of higher-order Euler operators. This makes the formula computationally very efficient when implemented in computer algebra systems such as *Mathematica* or *Maple*.

The discrete homotopy operator (see [1] for references) allows one to invert the forward difference operator and carry out summation by parts on discrete jet spaces. A simplified formula for the discrete homotopy operator will be shown. Despite its simplicity, the discrete homotopy is not as efficient as telescopic summation to carry out summation by parts.

Euler operators (also known as variational derivatives) are universal tools to testing exactness. Likewise, homotopy operators allow one to invert divergence and difference operators no matter what the application is. One area of application is the symbolic computation of conservation laws of nonlinear PDEs (in particular, in multiple space dimensions) and nonlinear differential-difference equations. Using the short pulse equation, the Zakharov-Kuznetsov equation, and the Toda lattice as examples, it will be shown how the homotopy operators are used to compute conserved fluxes.

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Application of the simplified Hirota (homogenization) method to a (3+1)-dimensional evolution equation for deriving multiple soliton solutions

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Abstract. In this talk, the simplified Hirota (homogenization) method [1] will be applied to a (3+1)-dimensional evolution equation for deriving multiple soliton solutions, and the steps of the method will be illustrated using *Mathematica*, a computer algebra system.

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A numerical study of eventual periodicity of the Korteweg-de Vries type equation using sinc collocation method

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Abstract. We demonstrate numerically the eventual time periodicity of solutions to the Korteweg-de Vries equation with periodic forcing [1, 2, 3] at the boundary using the sinc-collocation method. This method approximates the space dimension of the solution with a cardinal expansion of sinc functions, thus allowing the avoidance of a costly finite difference grid for a third order boundary value problem. The first order time derivative is approximated with a θ-weighted finite difference method. The sinc-collocation method was found to be more robust and more efficient than other numerical schemes when applied to this problem.

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SESSION 29
Nonlinear Schrödinger models and applications

On-site and off-site solitary waves of the discrete nonlinear Schrödinger equation in multiple dimensions

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Abstract. We construct several families of symmetric localized standing waves to the one-, two-, and three-dimensional discrete nonlinear Schrödinger equation (DNLS) with nearest neighbor coupling using bifurcation methods about the continuum nonlinear Schrödinger limit. We also show that their energy differences are exponentially small (beyond all polynomial orders) in a natural parameter which is related to the frequency and lattice spacing.

We will also discuss the stability properties of these families of states. Finally, we turn to lattices with more general interactions than nearest neighbor, in particular short- and long-range interactions. If the non-locality is strong, then the governing continuum limit is a nonlocal and nonlinear Schrödinger equation with a fractional Laplace operator, whose fractional-order is set by the degree of non-locality. See [1] and [2] for related results.

These families and their energy differences are related to the Peierls-Nabarro (PN) barrier [3], the energy required for a localized state to travel by one lattice site, and play a significant role in DNLS dynamics. In particular, traveling pulses radiate energy away to infinity until becoming pinned at a lattice site as a stable standing wave. Applications include optical waveguide arrays, tight binding limits in Bose-Einstein condensates, numerical analysis, and the dynamics of crystals and biological molecules.

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Vortex Scattering Across Interfaces

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Abstract. e will discuss the behavior of a vortex dipoles in a nonlinear Schrodinger equation with a variable background. For a particular asymptotic regime, a vortex will interact with both the background potential and the other vortex. The resulting dynamical equation is attained by an examination of the conservation laws for the initial Schrodinger equation. This is joint work with Matthias Kurzke (Nottingham) and Jeremy Marzuola (UNC).

Dark-bright solitons in coupled nonlinear Schrödinger equations with unequal dispersion coefficients

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Abstract. In this talk, we will present a two-component NLS system with equal, repulsive cubic interactions and different dispersion coefficients in the two components. We will consider states that support a dark solitary wave in the one component, and explore the possibility of the formation of bright solitonic bound states in the other component. Initially, based on the linear limit for the bright component, we identify bifurcation points of such states and explore their continuation in the nonlinear regime afterward. Then, we will identify regimes of potential stability (in the realm of linear stability analysis) for the single-peak ground state (the dark-bright soliton) as well as excited states with one or more zero crossings in the bright component. Finally, for unstable such states, we will demonstrate results on direct numerical simulations and discuss the dynamics of the instability.

Vortex nucleation in a dissipative variant of the nonlinear Schrödinger equation under rotation

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Abstract. We motivate and explore the dynamics of a dissipative variant of the nonlinear Schrodinger equation under the impact of external rotation. As in the well stablished Hamiltonian case, the rotation gives rise to the formation of vortices. We show, however, that the most unstable mode leading to this instability scales with an appropriate power of the chemical potential mu of the system, increasing proportionally to $\mu^(2/3)$. The precise form of the relevant formula, obtained through our asymptotic analysis, provides the most unstable mode as a function of the atomic density and the trap strength. We show how these unstable modes typically nucleate a large number of vortices in the periphery of the atomic cloud. However, through a pattern selection mechanism, prompted by symmetry-breaking, only few isolated vortices are pulled in sequentially from the periphery towards the bulk of the cloud resulting in highly symmetric stable vortex configurations with far fewer vortices than the original unstable mode. These results may be of relevance to the experimentally tractable realm of finite temperature atomic condensates. This is a joint work with Ricardo Carrettero and Panoyotis Kevrekidis.

Finite temperature effects on vortex motion in ring Bose–Einstein condensates

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Abstract. In a recent experment [1] a ring Bose–Einstein condensate (BEC) with zero circulation (with winding number $m = 0$) and stirred by a weak link jumped to an $m = 1$ state when stirred faster than a certain critical speed, Ω_{+c}. Conversely an $m = 1$ condensate dropped to $m = 0$ when stirred below a critical speed, Ω_c, which was lower than Ω_{+c}. In a second experiment performed on the same system [2], two weak links stirred the condensate by being swept towards each other through an angle of 90°. The difference between the densities of the squeezed and unsqueezed regions was measured at the end of the sweep. The hysteresis loop areas, $\Omega_{+c} - \Omega_{-c}$ measured in the first experiment, disagreed significantly with the predictions of the zero–temperature Gross–Pitaevskii (GPE) equation while the results of the second experiment agreed quite well. The mechanism needed to understand the Gross–Pitaevskii behavior of the system in both of these experiments depends on the behavior of vortex/antivortex pairs that appear in the weak–link region during the stirring. We report the results of simulating these experiments with the Zaremba–Nikuni–Griffin (ZNG) theory. The ZNG theory models a Bose–Einstein condensate in the presence of a weakly perturbed,

thermal equilibrium non–condensate cloud. We compare the results of these simulations with the experimental data and describe how the dynamics of vortex/antivortex pairs formed in the weak–link region during the stirring is modified by the presence of a thermal cloud. Systems such as a ring BEC stirred with a weak–link laser beam have the potential to serve as rotation sensors in ultra–precise navigation applications. Such devices can only be made practical if the behavior of the underlying ultracold atom system is understood quantitatively at the microscopic level.

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Nonconservative variational approximations for NLS: Application to symmetry-breaking bifurcation

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Abstract. We review three different nonconservative variational approaches to the nonlinear Schrödinger (NLS) equation. In comparison, all three variational techniques produce equivalent equations of motion. To showcase the relevance of these nonconservative approaches we present some basic examples incorporating density dependent and linear gain.

These methods are applicable to understanding the basic properties of nonlinear Schrödinger equation with linear and density dependent loss and gain. Therefore, nonconservative variational approximations are ideally suited to describe an exciton-polariton condensate modeled via the nonlinear Schrödinger equation with linear loss and density dependent gain, describing the decay of polaritons and pumping of new excitons from an external reservoir, respectively.

Specifically, we are exploring the application of the nonconservative approaches to symmetry-breaking bifurcations observed in coherently driven passive optical Kerr resonators. The equations of motion through the nonconservative variational approach offer a theoretical approximation for the symmetry-breaking (pitchfork) instability found experimentally above a certain pump power threshold.

Engineering dispersion relations with a spin-orbit coupled Bose-Einstein condensate

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Abstract. Spin-orbit coupling, i.e. the coupling between linear or angular motion and an internal degree of freedom of a particle, plays a prominent role in many physical systems. In gaseous Bose-Einstein condensates spin-orbit coupling can be induced by dressing the atoms with suitably chosen laser fields. Such Raman dressing schemes have turned out to be a powerful tool to engineer interesting dispersion relations, including double-well structures in momentum space. In this paper we will present recent and ongoing experiments conducted in our lab at Washington State University that investigate the dynamics of spin-orbit coupled condensates. Using Bragg spectroscopy we directly probe the excitation spectrum and demonstrate mode-softening of roton-like modes [1]. We also test the dynamical instability of spin-orbit coupled condensates that are placed into a moving one-dimensional optical lattice [2]. The points of maximum instability can be related to features in the effective band structure describing such a system whose dispersion is affected by both the lattice and the spin-orbit coupling at the same time. Observed differences for opposite directions of motion of the moving lattice can be interpreted as a breaking of Galilean invariance, which is a peculiar feature of spin-orbit coupled systems.

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SESSION 30

Wave phenomena in population biology and application to cancer

Effects of anti-angiogenesis on glioblastoma growth and migration: Model to clinical predictions

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Abstract. Glioblastoma multiforme (GBM) is a malignant brain tumor that continues to cause significant neurological morbidity and short survival times. Brain invasion by GBM is associated with poor prognosis. In 1971, Folkman proposed that the growth of tumors depends on angiogenesis (new blood vessel formation). This hypothesis catalyzed the development of anti-angiogenic therapy (AA) leading to the development and clinical use of bevacizumab in patients diagnosed with GBM. To decode the reaction of GBM to AA therapy, we previously constructed a mathematical model of GBM growth and hypoxia-driven brain invasion; simulations supported an exception to the Folkman hypothesis [1]. Here, we present a concise pde model at the scale of clinical MRI, which includes a small number of equations and embodies two different mechanisms of brain invasion. Simulations and model predictions uncover a new pattern of recurrence in 11/70 bevacizumab-treated patients, which is associated with statistically- significant short survival times. The findings also support the conclusion that GBM grows in the absence of angiogenesis by a cycle of proliferation and brain invasion that expands necrosis. Furthermore, necrosis is positively correlated with brain invasion in 26 newly-diagnosed GBM. The unintuitive results explain the unusual clinical effects of bevacizumab, recently reported in two phase III clinical trials, and suggest new hypotheses on hypoxia-driven migration.

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Spatial modeling of tumor drug resistance: The case of GIST liver metastases

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Abstract. This work is devoted to modeling gastrointestinal stromal tumor (GIST) metastases to the liver, their growth and resistance to therapies. More precisely, resistance to two standard treatments based on tyrosine kinase inhibitors (imatinib and sunitinib) is observed clinically. Using observations from medical images, we build a spatial model consisting in a set of nonlinear hyperbolic partial differential equations, which describes the front propagation of the metastatsis. After calibration of its parameters with clinical data, this model reproduces qualitatively and quantitatively the spatial tumor evolution of one specific patient. Important features of the growth such as the appearance of spatial heterogeneities and the therapeutical failures may be explained by our model. We then investigate numerically the possibility of optimizing the treatment in order to increase the progression free survival time and the minimum tumor size reachable by varying the dose of the first treatment. We find that according to our model, the progression free survival time reaches a plateau with respect to this dose. We also demonstrate numerically that the spatial structure of the tumor may provide much more insights on the cancer cell activities than the standard RECIST criteria, which only consists in the measurement of the tumor diameter.

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A constructive existence proof for a coupled system of equations to model tumor growth

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Abstract. Gastro-intestinal stromal tumor can create lesions to the liver in the metastatic phase. To control the growth of the tumor, clinicians use targeted therapies, such as cytotoxic and anti-angiogenic drugs, and follow the effects of these treatments using CT-scans. In general, these therapies are efficient, however, a relapse can occur in some cases at some time. We provide a spatial model of coupled PDE's based on scans observations which can reproduce not only the spatial tumor evolution, but also the heterogeneity in the tumor composition and the effects of the treatment. The model is similar to the one presented in [1] and can be written as follows

$$\begin{gathered}
\partial_t P_1 + \nabla \cdot (\mathbf{v} P_1) = (\gamma_1(M) - \gamma_2(M)) P_1 - \delta M P_1, \\
\partial_t P_2 + \nabla \cdot (\mathbf{v} P_2) = (\gamma_1(M) - \gamma_2(M)) P_2, \\
\partial_t N + \nabla \cdot (\mathbf{v} N) = \delta M P_1 + \gamma_2(M)(P_1 + P_2) - \frac{1}{\tau} N, \\
\partial_t S + \nabla \cdot (\mathbf{v} S) = 0, \\
\nabla \cdot \mathbf{v} = \gamma_1(M)(P_1 + P_2) - \frac{1}{\tau} N, \text{ with } \mathbf{v} = -k \nabla \Pi, \\
\partial_t \xi = \alpha \int_\Omega \frac{\gamma_2(M)}{\max \gamma_2} (P_1 + P_2) dx - \lambda \xi,
\end{gathered}$$

$$\begin{aligned}
&\partial_t M - \Delta M + \nabla \cdot (M \xi \nabla (P_1 + P_2)) \\
&\qquad\qquad = -\eta M (P_1 + P_2) + C_0 S (1 - M),
\end{aligned}$$

with boundary conditions on the bounded domain Ω and initial conditions. We will present a constructive proof of well-posedness for this model. The proof is based on the fact that the tumor remains compactly supported in the domain until

it reaches the boundary, if the initial tumor is compactly supported. We will end with an asymptotic result for the solution of our model when $\tau \to 0$ and the application of this result.

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Invadopodia modeling

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Abstract. *Invadopodia* is the initial and crucial point in the metastatic process of most of the invasive cancer diseases. At this early stage, protrusions form at the cytoplasmic membrane of a cancer cell. These highly proteolytic structures are able to degrade the neighboring biological tissue, which makes it possible for the cancer cell to detach from the primary tumor and migrate through the tissue to join a blood vessel. Then, in a latter stage, the metastatic cells will spread in the body and initiate secondary tumors, called metastases, that are the main responsible of death.

We propose a mathematical model at the cell level, based on a nonlinear PDE system. In contrast to the work of Saitou *et al.* [1], where a system of reaction-diffusion equations is provided without considering the cell membrane, we derive a free boundary problem by introducing a level-set function whose level zero represents the cell membrane, that delineates the cell medium from the external environment. Chemical species diffuse on both time-dependant areas. The level-set velocity is given by the gradient of an internal biological signal (according to the principle of chemotaxis), which accounts for protrusion formation.

A second-order numerical method is proposed to solve the free boundary problem on a cartesian grid. The core of the numerical method is the use of superconvergence properties [2] to get a consistent and convergent global method. The superconvergence properties are needed for the first and second derivatives of the solutions, which requires very specific definitions for the operators and very accurate discretization schemes. Our numerical experiments provide satisfying scheme behaviors and illustrate the protrusion formation.

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Level set segmentation using non-negative matrix factorization with application to MRI

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Abstract. We present a new deformable model for image segmentation based on the level set method (LSM) and probabilistic non-negative matrix factorization (PNMF). The proposed model characterizes the histogram of the image, calculated over the image blocks, as nonnegative combinations of basic histograms computed using the PNMF algorithm. These basic histograms form a clustering of the image. Our model also takes into account the intensity inhomogeneity or bias field of medical images. In a level set formulation, this clustering criterion defines an energy in terms of the level set functions that represent a partition of the image domain. The image segmentation is achieved by minimizing this energy with respect to the level set functions and the bias field. Our method is compared, using brain MRI, to two other state-of-the-art level set methods that are based on k-means clustering [1] and local Gaussian distribution fitting [2]. It is shown that the proposed PNMF LSM is less sensitive to model parameters, more robust to noise in the image and, at the same time, has a higher convergence rate. These advantages are due to the fact that the proposed approach i) relies on the histogram for local clustering rather than image intensities, and ii) does not introduce additional model parameters to be simultaneously estimated with the bias field and the level set functions.

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Mathematical model of glioblastoma: Mechanistic insights of recurrence

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Abstract. Glioblastoma multiforme (GBM) is a malignant brain tumor with poor prognosis and high morbidity due to the tumors ability to invade the brain. Recent clinical trials of Bevacizumab, an anti-angiogenic drug, in newly-diagnosed GBM patients showed no beneficial effects on overall survival times, although progress-free survival times were better [1, 2]. The authors have recently reported a concise system of partial differential equations that models GBM biology at the scale of magnetic resonance imaging; the model includes replication, brain invasion, angiogenesis, and hypoxia [4, 5]. Here, we apply the model to replicate the patterns of recurrence of Bevacizumab-treated GBM [5, 3]. The results contribute mechanistic explanation that link tumor biology to the phenotypic behavior of the tumor.

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Perivascular glioma invasion is a VEGF-independent mechanism of tumor vascularization

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Abstract. Malignant glioma cells use various microanatomical brain structures to invade the brain. Cellular, and molecular mechanisms (and consequences) of these distinct patterns of tumor growth remain poorly understood. Perivascularly invading brain tumors become vascularized as they engulf preexisting normal brain microvessels over the course of tumor progression. This leads to an iterative phases by which glioma cells grow on vessels, and divide to fill in the space in between the blood vessels. We propose that this iterative arrangement of tumor growth is sufficient to explain the formation of a macroscopic tumor. We also propose that such growth mechanics allow the tumor to grow by replacement rather than by displacement of normal brain tissue. This explains many clinical and biological characteristics of glioma growth. We show that mouse, rat and human glioma stem cells, human glioblastoma biopsies (in situ), genetically-induced endogenous mouse brain tumors, and peripheral metastasis to the human brain also exhibit perivascular growth. As we also show that the clinically used antiangiogenic agents fail to block tumor growth, our data imply that gliomas can grow by a VEGF-independent mechanism of brain tumor vascularization. We conclude that growth and invasion throughout the brain perivascular space is sufficient to sustain long-term glioma growth independently of neoangiogenesis, and that this growth mechanism on its own can explain iterative tumor growth by replacement.

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SESSION 31
Wave propagation in complex media

Singlet lasing in supersymetric laser arrays

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Abstract. We present the notion of supersymmetric (SUSY) laser arrays that consists of a main optical array and its superpartner lattice [1] and we investigate the onset of their lasing oscillations. Due to the interaction between the degenerate optical modes of both arrays, their eigenvalues form doublets while the extra mode associated with unbroken SUSY

represents a singlet state. Single transverse mode lasing in this singlet state can be achieved for a wide range of design parameters either by introducing stronger loss in the partner array or by pumping only the main lattice. Surprisingly, our analysis also reveals that the complex interplay between supersymmetry and non-Hermiticity can lead to situations where one doublet state exhibits the lowest lasing threshold. We employ Brillouin-Wigner (BW) perturbation method [2] to examine this unexpected behavior and show that it arises as a result of the often neglected non-resonant interaction between the optical eigenmodes associated with the main lattice and its superpartner. Our findings suggest the possibility of building single-mode, high-power laser arrays and are also important for understanding light transport dynamics in multimode Parity-Time (PT) symmetric photonic structures.

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Selective excitation by active transformation optics in media with strong modal interactions

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Abstract. sing a nonlinear transformation of the active part of the dielectric constant, we propose a systematic method to achieve mode selection in incoherently driven optical systems with strong modal interactions, enabled by controlling modal interactions via the spatial pump profile.

Accelerating and abruptly autofocusing waves

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Abstract. Optical waves with self-accelerating properties have recently attracted a lot of attention [1, 2]. Such beams and pulses are associated with a variety of applications.

We are going to discuss about recent developments of such beams including abruptly autofocusing waves [3], Bessel-like beams with accelerating trajectories [4], nonparaxial accelerating beams, and linear and nonlinear dynamics of accelerating pulses in fibers.

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Nanoscale light manipulation by parity-time symmetry

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Abstract. Parity-time (PT) symmetry is a fundamental notion in quantum field theories and opens a new paradigm of non-Hermitian Hamiltonians ranging from quantum mechanics, electronics, to optics and photonics. By carefully exploiting the interplay among optical index, gain and loss in the complex dielectric permittivity plane, optics has become an ideal platform to study the fundamentals of PT symmetry. Instead of counteracting optical losses at micro and nano scales in integrated photonics, we started from an opposite viewpoint and developed a new paradigm of positively and strategically manipulating optical losses by the concept of PT symmetry to enrich fundamental optical physics and realized novel photonic synthetic matters with unique optical functionality. Based on the state-of-the-art nanophotonics technology, we have demonstrated unidirectional reflectionless light transport at exceptional points [1] and effective control of cavity resonant modes in a typical multi-mode laser cavity by the concept of PT symmetry [2].

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PT-symmetric Floquet lattices

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Abstract. We investigate spectral and dynamical properties of periodically driven PT-symmetric dimer systems and show that in the Floquet space they are described by Parity-Time symmetric lattices. The topology of the Floquet lattice depends on the complexity of the driving. For the simplest driving scheme associated with a sinusoidal coupling, we show that as the gain/loss parameter increases, the Floquet spectrum and the corresponding eigenvectors undergo a transition from an exact to a broken PT-symmetric phase via an Exceptional Point singularity. The phase transition is also reflected in the associated Floquet dynamics. This paves the way to experimentally investigate extended lattice dynamics in PT-systems. A realization in the RF domain is reported and it is shown to be in agreement with the theoretical analysis.

Beam dynamics in longitudinally modulated PT-symmetric lattices

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Abstract. We investigate the spectral properties and dynamical features of a beam propagating in complex PT-symmetric wave-guide arrays periodically modulated along direction of propagation. Paraxial beam propagation in such arrays can be described by the normalized Schrodinger equation:

$$i\frac{\partial\Psi}{\partial\xi}+\frac{\partial^2\psi}{\partial\eta^2}+[U\cos\eta+iV\sin\eta]\cdot[1+\epsilon\cos(\omega\xi)]\Psi=0,$$

where the amplitude of longitudinal modulations $\epsilon << 1$. The transmission spectrum of this system undergoes PT-symmetry breaking at $U = V$, that is identical to the phase transition of the unmodulated systems [1]. All Flouqet-Bloch modes remain stable up to this critical threshold value of V, however certain resonant frequencies of longitudinal modulations ω result in linear growth of the beam power. This linear power amplification is associated with the development of an intensity-locked region between the two beams formed through the double refraction of the unique incident beam. The intensity oscillations in this region conform to the periodicity of the lattice while the amplitude of oscillations remains constant in the space between the two beams. The linear growth of the power stems from linear increase of the span between the two beams that diverge at constant angle. We develop a theory of mode population dynamics and explain the intensity-locking phenomena through the spectral analysis of the resonantly driven modes.

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Wave packet evolution for non-Hermitian quantum systems in the semiclassical limit

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Abstract. In this talk I present some results on the quantum evolution of Gaussian wave packets generated by a non-Hermitian Hamiltonian in the semiclassical limit of small h-bar [1]. This yields the non-Hermitian analogue of the Ehrenfest theorem for the dynamics of observable expectation values. The resulting equations of motion for dynamical variables are coupled to an equation of motion for the phase-space metric—a phenomenon having no analogue in Hermitian theories. I shall discuss an illustrative example of a non-Hermitian and PT-symmetric harmonic oscillator in some detail [2].

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Time-asymmetric excitation of resonances by chirped laser pulses

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Abstract. When sufficiently long chirped laser pulses encompass the resonance frequency between two bound states, the well known effect of adiabatic rapid passage takes place resulting in a complete population exchange between the two states. In the case that the excited state is a resonance, such as atomic Feshbach resonances, the problem is much more complicated and interesting as well. The adiabatic view on the problem would suggest that the state exchange should take place if the contour of the chirped laser pulse encircles the

complex degeneracy (exceptional point) in the frequency-laser-strength plane $[\omega - \epsilon_0]$. No difference should be observed whether the pulse chirping is positive or negative. The fact that this is not what actually happens in the dynamics is explained by a non-hermitian non-adiabatic effect, which is engaged. The peculiarity of the non-hermitian non-adiabatic effect is that it does not prevail in short pulses as usual, but determines the process in the limiting case of infinitely long pulses.

In this paper, we derive analytical formulas for quantum dynamical amplitudes for infinitely long chirped pulses characterized by a constant radius from the exceptional point, where the radius is defined by the real part of the quasienergy split between two instantaneous Floquet states, which correspond to the two strongly coupled field-free states. Rather counterintuitively, the time-asymmetric state exchange takes place exclusively for a large radius exceeding the field-free resonance half-width, while if the radius is smaller, wherefore the quasienergy split is governed by linear terms below a square root, no state exchange occurs. The conclusions will be illustrated on an excitation of helium Feshbach resonances in a full-dimensional quantum dynamical simulation.

Control of light transport via non-local wave interference effects in random media

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Abstract. The concept of diffusion is widely used to study the propagation of light through multiple scattering media such as clouds, colloidal solutions, paint, and biological tissues. Diffusion, however, is an approximation as it neglects wave interference effects. Most of the scattered waves follow independent paths and have uncorrelated phases, so their interference is averaged out. Notwithstanding, a wave may return to a position it has previously visited after multiple scattering events, and there always exists the time-reversed path which yields identical phase delay. Contributions due to constructive interference between these pairs of paths to transport coefficients, in particular second order quantities such as fluctuations and correlations, do not average out to zero.

In this talk, we describe a novel scheme of manipulating light transport in random media via coherent effects. Changing the waveguide geometry allows us to control the crossing probability of scattering paths as a function of position. We illustrate our approach with several experiments demonstrating how the spatial dependence of the average intensity as well as the long-range correlations can be effectively modified.

This work opens the possibility of using geometry to control non-local effects in mesoscopic transport without changing structural disorder. In addition to the fundamental importance, understanding and manipulating the spatial correlations of light inside the random system is useful for imaging and focusing of light in multiply scattering media using wavefront shaping techniques. The degree of such coherent control is limited by the number of modes that can be controlled. Our results suggest that the overall geometry can provide an additional degree of freedom and can be used along with wavefront shaping techniques to more efficiently control the light transport through the random media.

Fluctuating hydrodynamics for a discrete Gross-Pitaevskii equation: Mapping to Kardar-Parisi-Zhang universality class

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Abstract. We show [1] that several aspects of the low-temperature hydrodynamics of a discrete nonlinear Schrodinger equation (discrete-NLS) can be understood by mapping it to a nonlinear version of fluctuating hydrodynamics. This is achieved by first writing the discrete-NLS in a hydrodynamic form of a continuity and an Euler equation. Respecting conservation laws, dissipation and noise due to the system's chaos are added, thus giving us a nonlinear stochastic field theory in general and the Kardar-Parisi-Zhang (KPZ) equation in our particular case. This mapping to KPZ is benchmarked against exact Hamiltonian numerics [2] on discrete-NLS by investigating the non-zero temperature dynamical structure factor and its scaling form and exponent. Given the ubiquity of the discrete-NLS (a.k.a. Gross–Pitaevskii equation), ranging from nonlinear optics to cold gases, we expect this remarkable mapping to the KPZ equation to be of paramount importance and far reaching consequences.

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PT-symmetric microring lasers

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Abstract. The concept of parity-time (PT) symmetry has recently attracted considerable attention in optics. In general, PT-symmetric structures utilize gain and loss in a balanced fashion in otherwise symmetric cavities or waveguide arrangements. As recently demonstrated, such systems can exhibit altogether new properties and functionalities which are unattainable in Hermitian platforms. One such phenomenon is the transition from all real to partially complex spectra at the so-called PT-symmetry breaking threshold. This abrupt transition is associated with an exceptional point in non-Hermitian systems. Here, we present our recent findings in the demonstration of stable single-mode operation in on-chip PT-symmetric micro-ring lasers. Along these lines, an active ring is paired with a lossy but otherwise identical partner. By choosing an appropriate coupling strength between them, one may withhold amplification from all undesired modes by keeping them below the PT-symmetry breaking. Therefore, the one mode which exceeds the symmetry-breaking threshold dominates the emission spectrum with high fidelity and enhanced extraction efficiency. In this talk, the utilization of exceptional points as a new tool for designing novel functionalities in photonic platform will be discussed.

Linear and nonlinear propagation effects in periodic and random photonic lattices

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Abstract. The nonlinear propagation of ultrashort laser pulses in the form of solitons, filaments and light bullets is an exciting research field [1]. Beyond the basic studies on the complex physical phenomena involved, the field is driven significantly by the numerous applications.

Here we present experimental and theoretical results on the spatio-temporal properties of ultrashort laser beams propagating in structured media. We show that an appropriate periodic modulation of the index of refraction of the medium offers a very robust way of tailoring the propagation of high intensity laser beams [2, 3]. We also explore tailor-made media presenting strong non-uniformity as well as scattering properties. In the latter case we find conditions under which extreme events are observed. The role of linear and nonlinear contributions is studied in depth, resulting in some very interesting findings on the respective roles [4].

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Tunable photonic oscillators

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Abstract. Limit Cycle oscillators are used to model a broad range of periodic nonlinear phenomena. Using the optically injected semiconductor oscillator as a *paradigm*, we have demonstrated that at specific islands in the optical frequency detuning and injection level map, *Period One Limit Cycle* oscillation frequency is simultaneously insensitive to multiple perturbation sources. In our system these include the temperature fluctuations experienced by the master and slave lasers as well as fluctuations in the bias current applied to the slave laser. Tuning of the oscillation frequency then depends only on the injected optical field amplitude. Experimental measurements are in good quantitative agreement with numerical modeling and analysis based on a reduced *Adler* phase dynamics type equations. These special operating regions should prove valuable for developing ultra-stable nonlinear oscillators, such as sharp linewidth, frequency tunable photonic microwave oscillators. Finally the concept of an *Isochron* originally developed in mathematical biology will be reviewed and placed on context for efficient design of stable frequency sources in systems of optically coupled limit cycles oscillators.

Singular amplification and constant intensity waves in non-hermitian Photonics

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Abstract. It was recently proposed that the concept of parity-time (PT)-symmetry [1] can be physically realized within the framework of classical wave optics [2]. As a result, the first experimental observation of PT-symmetry breaking, was demonstrated in two coupled waveguides with gain and loss [3]. Such synthetic structures can utilize loss as an advantage and have been proven to be important in engineering new composite systems with novel functionalities.

In the first part of this talk, we will present a class of non-hermitian materials that are on average lossy, even though they

contain gain. These systems can transiently amplify light, the magnitude of which is related to the singular values and the pseudospectra of the underlying non-normal propagator [4]. The second part is devoted to the new concept of constant intensity waves [5] that exist only in non-hermitian environments. The diffraction properties of these generalized plane waves, as well as, their modulation instability in Kerr nonlinear waveguides are studied in detail.

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Ultra-sensitivity using higher order exceptional points

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Abstract. Exceptional points (EPs) are by nature interesting mathematical objects associated with non-Hermitian systems. Quite recently they have received considerable attention within the framework of microwaves and optics[1]. One important property of these points arises from the eigenvalue bifurcations resulting from a perturbation of the system matrix in the parameter space. In an n-dimensional system, the eigenvalues split as the n-th root of the perturbation thereby having major implications upon the sensing potential of such a system. Structures utilizing optical mode splitting similar to an EP-2 have been demonstrated as sensors for nanoparticles[2]. Here we propose an optical system exhibiting higher order exceptional points leading to a manifold enhancement of the sensitivity[3]. A slight perturbation at the n-th decimal digit of the loss can be detected as a change of another variable in the first decimal digit. This effect can be detected through the location of minima in the transmission spectrum of a PT-symmetric system based on high-Q coupled microcavities

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On-chip PT-symmetric microring lasers

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Abstract. Microring resonators represent one of the most promising structures for on-chip laser sources. Due to their high quality factors, small footprints, and simplicity of fabrication, microrings are already widely popular for passive applications such as sensing and optical communications. However, utilization of such resonators in designing laser cavities is still rather challenging, because of the fact that they support several longitudinal and transverse modes within the gain bandwidth afforded by semiconductors. This multimode operation has a critical effect on the spectral purity and beam quality of these lasers, causing spatial and temporal fluctuations in the emitted power. Thus enforcing single mode laser operation is a necessary step in utilizing microring resonators as an integrated light source. Although several methods have been proposed in the literature to address this issue, there is still a need for a novel and versatile approach for laser design without imposing extra demands in terms of complexity and fabrication. Using a PT-symmetric arrangement, here, we present the first experimental demonstration of stable single-mode operation of a micro-ring cavity, which normally tends to support several transverse and longitudinal modes for the same amount of pump power. Along these lines, an identical but lossy partner has been coupled to the active ring in order to form the PT-symmetric arrangement. By engineering the distance between the pair, all undesired modes have been kept below the PT-symmetry breaking threshold. Therefore, only the fundamental mode oscillates in the symmetry-broken regime, dominating the emission spectrum with high stability and enhanced efficiency.

Deep penetration of light needles through soft-matter nanosuspensions

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Abstract. In the past decade, the development of artificial materials exhibiting novel optical properties has become one of the major scientific endeavors. Of particular interest are soft-matter systems, which play a central role in numerous fields ranging from life sciences to chemistry and physics. The ability to control light transportation in these systems is an interesting problem for a variety of scientific and technological fields. Here, we present a brief review of our recent work on a few types of soft-matter systems with tunable optical nonlinearities. These include the dielectric colloidal suspensions with negative polarizability, the plasmonic type of metallic colloidal suspensions, and the biological suspensions of algal cells. In all these systems, by appropriately engineering the soft-matter suspensions, we can alter at will the nonlinear light-matter interactions in order to overcome the effects from diffraction and scattering, hence to achieve deep beam penetration through the colloid. In particular, in the plasmonic nanosuspensions, we have demonstrated self-trapping of light beam and its robust soliton-like propagation over distances up to 25 diffraction lengths, which would have been otherwise impossible in conventional dielectric settings. Our findings may bring about solutions to overcome large scattering loss in various soft matter systems, promising for various applications including optical trapping and manipulation as well as control of chemical and biological processes.

Supersymmetry in optics

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Abstract. The concepts of symmetry and symmetry breaking play a crucial role in physics. Discrete symmetries like that of charge-parity-time and of supersymmetry are central elements in formulating the fundamental laws of modern physics. By respecting the underlying analogies between the mathematical formalism of quantum mechanics and that of classical electrodynamics, we show that optics can provide a fertile ground for studying, observing, and fruitfully utilizing supersummetry (SUSY), one of the fundamental symmetries that is currently out of reach in other areas of physics. Optical SUSY demands that any one-dimensional optical structure can be paired with a superpartner with similar electromagnetic properties. In particular, two partner waveguides can share the exact same guided mode eigenvalue spectra except for that of the fundamental mode of the original waveguide which may lack a counterpart in the partner waveguide. We show that this interesting property can be utilized to design new types of mode filters and mode multiplexers. In addition, we show that supersymmetric partner structures share similar scattering characteristics, as a result a given scatterer can be replaced with its superpartner which has an entirely different shape while preserves the same reflection and transmission coefficients. Based on this we introduce a novel type of transformation optics.

PT-symmetric diffraction gratings

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Abstract. Parity-time (PT) symmetric optical structures have recently attracted considerable attention. By utilizing balanced regions of gain and loss in conjunction with their refractive index distributions, such non-Hermitian arrangements can exhibit a range of interesting properties which do not have a counterpart what so ever in traditional Hermitian settings. Of interest would be to explore scattering properties of optical diffraction gratings in the presence of PT-symmetric gain/loss profiles.

Here, we investigate the diffraction properties of periodic PT-symmetric gratings. We show that when light encounters such a grating, the diffraction pattern is in general asymmetric. In particular, at the PT symmetry breaking threshold, all the negative diffraction orders can be in principle eliminated at once while the remaining positive diffraction orders can be enhanced. We demonstrate this effect in polymer-based metamaterials.

Controlling the radiation and lifetime of relativistic particles by shaping their wavepackets

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Abstract. We show that shaping the wavepackets of Dirac particles can alter fundamental relativistic effects such as length contraction and time dilation, or even change the spectrum of the radiation they emit. As an example, we shape decaying particles as self-accelerating Dirac wavepackets and show how it extends their lifetime. This arises from their modified Bessel profile, exhibiting an effective event-horizon singularity, and boost-invariant dynamics.

Furthermore, we show that the quantum wavepacket nature of a charged particle creates new phenomena in the Cerenkov Effect (which is the radiation emitted from a charged particle when it travels faster than the phase velocity of light in a medium). Specifically, with proper design of particle wavepacket as a Bessel beam, we predict the traditional Cerenkov radiation angle splits into two distinctive cones of photonic shockwaves. More importantly, the spectral response reveals an upper frequency cutoff at which the photon emission rate is diverging, manifesting a new resonant light-matter interaction. Our findings are observable for electron beams with realistic parameters, offering new applications including monochromatic x-ray sources and open a new realm for Cerenkov detectors.

Nonlinear eigenvalues in a generalized PT-symmetric problem

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Abstract. We prove the persistence of real linear eigenvalues to real non-linear eigenvalues for a non self-adjoint eigenvalue problem with a general symmetry including the $PT-$symmetry. In particular, we consider in a Hilbert space H the problem

$$A\psi - gf(\psi) = \mu\psi, \quad \|\psi\|_H = 1$$

with $g \in \mathbb{R}$ (small), a densely defined, closed linear operator A with a non-empty resolvent set and with a Lipschitz continuous f. Moreover, we assume that A and f commute with an antilinear symmetry operator C, *e.g.* the PT-symmetry or a partial PT-symmetry operator. Using a fixed point argument, we show that real eigenvalues of A persist to real nonlinear eigenvalues $\mu(g)$ for g small enough and the corresponding eigenfunctions are C-symmetric. An example is the PT-symmetric nonlinear Schrödinger problem

$$-\Delta\psi + V(x)\psi - g|\psi|^p\psi = \mu\psi, \ x \in \mathbb{R}^d$$

with $V(-x) = \overline{V(-x)}, p, d \in \mathbb{N}$. We also present numerical examples of eigenvalue continuation in g.

CONTRIBUTED PAPERS

Prediction of extreme events in nonlinear dispersive wave equations

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Abstract. The aim of this work is the prediction of extreme ocean waves. These freak waves demand attention due to their catastrophic impact on ships, coastal structures, and human life. To investigate these waves, we use a family of nonlinear Schrodinger-type equations, which govern the evolution of the wave envelope on the surface of deep water. We show how chance energy localization due to dispersion triggers the excitation of large amplitude coherent structures with finite lifetime. Via analysis of the governing equations, we determine which types of localized wave packets are likely to trigger these extreme events. We then describe a simple technique for detecting these packets using a set of localized Gabor modes, which allow us to predict extreme events for low computational cost.

New WENO-θ scheme and its application to the 2D Riemann problem

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Abstract. A new adaptive weighted essentially non-oscillatory WENO-θ scheme in the context of finite difference is proposed. Depending on the smoothness of the large stencil used in the reconstruction of the numerical flux, a parameter θ is set adaptively to switch the scheme between a 5th-order upwind and 6th-order central discretization. A new indicator τ^θ measuring the smoothness of the large stencil is chosen among two candidates which are devised based on the possible highest-order variations of the reconstruction polynomials in L^2 sense. In addition, a new set of smoothness indicators $\tilde{\beta}_k$'s of the substencils is introduced. These are constructed in a central sense with respect to the Taylor expansions around the point x_j.

Numerical applications to the 2D Riemann problem of the new scheme are presented. Depending on the combination of elementary waves, namely, shock-, rarefaction-, and contact-wave, there are 19 configurations of the initial data. Numerical results show the outperformance of the WENO-θ scheme over

other comparing methods, especially in capturing small-scaled vortices along contact curves.

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Creation and amplification of electro-magnon solitons by electric field in nanostructured multiferroics

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Abstract. We develop a theoretical description of electro-magnon solitons in a coupled ferroelectric-ferromagnetic heterostructure. The solitons are considered in the weakly nonlinear limit as a modulation of plane waves corresponding to two, electric- and magnetic-like branches in the spectrum. Emphasis is put on magnetic-like envelope solitons that can be created by an alternating electric field. It is shown also that the magnetic pulses can be amplified by an electric field with a frequency close to the band edge of the magnetic branch.

On the existence of maximizers for Airy-Strichartz inequalities

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Abstract. Recently, in a joint work with Ademir Pastor [1], we give a simple proof of the classical Kenig, Ponce and Vega well-posedness result for the generalized KdV equation [2]

$$\begin{cases}\partial_t u + \partial_x^3 u + \partial_x(u^{k+1}) = 0, \ \ x \in \mathbb{R}, \ \ t > 0, \ \ k \geq 4, \\ u(x,0) = u_0(x).\end{cases}$$

The key ingredient in the proof is the following Airy-Strichartz estimate

$$\|U(t)u_0\|_{L_x^{5k/4}L_t^{5k/2}} \leq C_k \|u_0\|_{\dot{H}_x^{s_k}},$$

where $k > 4$, $s_k = (k-4)/2k$ and $U(t)$ denotes the linear propagator for the KdV equation.

Our goal here is to prove the existence of maximizers for the above inequality. The main tool we use is a linear profile decomposition for the Airy equation with initial data in $\dot{H}_x^{s_k}(\mathbb{R})$. As a consequence, we also establish the existence of maximizers for a more general class of Strichartz type inequalities associated to the Airy equation.

This is a joint work with Henrique Versieux (UFRJ). The author was partially supported by CNPq/Brazil and FAPEMIG/Brazil.

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An integrable Hamiltonian hierarchy based on so$(3,\mathbb{R})$ with three potentials

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Abstract. Many concrete examples of soliton hierarchies, such as the KdV hiearchy, AKNS hierarchy, the Dirac hierarchy, etc., can be generated from matrix spectral problems (or Lax pairs) based on specified matrix loop algebras. Analysis by means of trace identity or variational identity shows that these soliton hierarchies possess Hamiltonian or bi-Hamiltonian structures, and they are Liouville integrable.

Based on a recent work reporting a counterpart of the Dirac soliton hierarchy[1], we investigated a matrix spectral problem associated with the Lie algebra so$(3,\mathbb{R})$ (which has a derived algebra that is identical to itself and therefore also 3-dimensional), and generated from the zero curvature equation an integrable soliton hierarchy with three independent potentials. The analysis on its Hamiltonian structures by means of the trace identity shows that all the recursively created members in the soliton hierarchy are Hamiltonian and Liouville integrable, and they possess infinitely many commuting symmetries and conservation laws.

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Stability of waves with vorticity

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Abstract. Euler's equations describe the dynamics of gravity waves on the surface of an ideal fluid with arbitrary depth. In this talk, we discuss the stability of periodic traveling wave solutions for the full set of Euler's equations with constant vorticity via a generalization of a non-local formulation of the water wave problem due to Ablowitz, *et al.* [1], and Ashton & Fokas [2]. We determine the spectral stability for the periodic traveling wave solution by extending Fourier-Floquet analysis to apply to the non-local problem. We will discuss some interesting and new relationships between the stability of the traveling wave with respect to long-wave perturbations and the structure of the bifurcation curve for small amplitude solutions.

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On the Hankel operator approach to completely integrable systems

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Abstract. As well-known, many problems in the theory of completely integrable systems can be formulated in terms of Riemann-Hilbert boundary problems. This has been used (explicitly or implicitly) since the late 1980s. On the other hand, it is also well-known that the Riemann-Hilbert problem is closely related to the theory of Hankel and Toeplitz operators. Moreover, since the 1960s (and implicitly even earlier) the former has stimulated the latter. But, surprisingly enough, while having experienced a boom at the same time, soliton theory and the theory of Hankel and Toeplitz operators have not shown much of direct interaction.

In the KdV context, we construct a Hankel operator which symbol is conveniently represented in terms of the scattering data for the Schrodinger operator associated with the initial data. Thus the spectral properties of this Schrodinger operator can be directly translated into the spectral properties of the Hankel operator. The latter then yield properties of the solutions to the KdV equation through explicit formulas. This allows us to recover and improve on many already known results as well as a variety of new ones. The main feature of this approach is that it applies to large classes of initial data far beyond the classical realm. For instance, we can handle low regularity initial data, lift any decay assumption at minus infinity, and significantly relax the decay at plus infinity. In this talk we discuss some representative results in this context focusing on well-posedness issues and basic properties of underlying solutions.

Our approach is not restricted to the KdV. Moreover, we believe that the interplay between soliton theory and Hankel operators may be even more interesting and fruitful for some other integrable systems with richer than KdV structures.

The talk is based on joint work with Sergei Grudsky.

Interaction of waves, sea-bottom second-order pressure and microseisms

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Abstract. The phenomenon of the second order pressure independent of the depth has been supposed to be at the origin of microseisms by Longuet-Higgins [1]. According to first order wave theory, the pressure variations in the sea decrease exponentially with the depth, while in the second approximation to the standing wave there is a second-order pressure that is not attenuated with the depth. In this work are presented different cases to have this second-order pressure independent of the depth due to the interaction of surface waves with nearly equal frequencies but nearly opposite directions. These microseisms, which can be recorded, are localized in time and space and their records are not continuous. The phenomenon is explained for each case considered through the expressions of the velocity potential, the free surface and the pressure. Conditions are also established in order to consider an expression, independent of the depth, for the second-order pressure and the occurrence of resonance in both two- and three- dimensional regimes.

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Long time behaviour of dispersive PDEs with generalized damping

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Abstract. We present some results about the long-time behavior of the solution of the generalized Korteweg-de Vries equation [5]

$$u_t + u_x + u_{xxx} + u^p u_x + L_\gamma(u) = 0,$$

and the generalized Benjamin-Bona-Mahony equation [2]

$$u_t + u_x - u_{xxt} + u^p u_x + L_\gamma(u) = 0.$$

The proposed dampings L_γ satisfy

$$\int L_\gamma(u(x))u(x)dx \geq 0,$$

and generalize standards ones, as parabolic $(L_\gamma(u) = \Delta u)$ or weak damping $(L_\gamma(u) = \gamma u)$. In comparison with the conservative case, i.e. $L_\gamma(u) = 0$, the second conservation law is decreasing as follows

$$\frac{d}{dt}||u(t)||^2 = -||u(t)||^2_{H^\gamma} \leq 0.$$

After establishing the well-posedness in the energy space H^γ, some dampings, e.g. band limited ones, are numerically build to preserve the decay of solutions.

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The minimum uncertainty squeezed states for quantum harmonic oscillators

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Abstract. We describe a multi-parameter family of the minimum-uncertainty squeezed states for the harmonic oscillator in non-relativistic quantum mechanics. They are derived by the action of the corresponding maximal kinematical invariance group on the standard ground state solution. We show that the product of the variances attains the required minimum value 1/ 4 only at the instances that one variance is a minimum and the other is a maximum, when the squeezing of one of the variances occurs. The generalized coherent states are explicitly constructed and their Wigner function is studied. The overlap coefficients between the squeezed, or generalized harmonic, and the Fock states are explicitly evaluated in terms of hypergeometric functions and the corresponding photon statistics are discussed. Some applications to quantum optics, cavity quantum electrodynamics and superfocusing in channelling scattering are discussed.

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Spectral analysis of operators that approach the Dirichlet-Neumann operator on the water waves problem with variable depth

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Abstract. I will focus on the Hamiltonian formulation of the water waves problem with variable depth given by Zakharov in [3] in which the nonlocal Dirichlet Neumann (DN) operator, appears explicitly on the Hamiltonian. We consider different approaches of this operator which have been made for the Boussinesq regime for water wave equations derived by Craig and others by a systematic approximation of this DN operator in [2]. We also consider the exact linear dispersion of the Euler equations as it is proposed in, Aceves, Minzoni, Panayotaros (AMP) model's for small and smooth depth variations in [1].

Specifically, we propose the spectral study of three Hamiltonians that obey different approaches to the problem: a) The Hamiltonian for the system with flat bottom, b) The Hamiltonian that arises from smooth variations of the bottom, which is the model (AMP) and finally c) the Hamiltonian in which we consider a pseudodifferential operator for explicitly capture the effect of the depth.

Once we make the discretization in the Fourier space of each one of the operators and once we do a Galerkin truncation of them we show the spectral analysis of the matrices result of the three quadratic forms that comprise the three Hamiltonians listed above, it means, to compute the eigenvalues and eigenvectors of those matrices for different bottoms. We also analyze the asymptotic behavior of those matrices. We perform this analysis on uniparametrics families of functions that approximate to the flat bottom, in order to compare the normal modes of the problem of water waves with variable depth with the known normal modes of the flat bottom problem.

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Closed solutions for the degenerate parametric oscillator and inhomogeneous nonlinear Schrödinger equation

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Abstract. Using Ermakov and Riccati type systems, we present how we can construct explicit solutions for the propagator for a generalized harmonic oscillator. This has applications describing the process of degenerate parametric amplication in quantum optics as well as light propagation in a nonlinear anisotropic waveguide. We also present explicit solutions of the inhomogeneous paraxial wave equation in a linear and quadratic approximation, showing the existence of oscillating laser beams in a parabolic waveguide and spiral light beams in varying media. Finally, using a similar approach we show how we can construct soliton solutions for the nonautonomous nonlinear Schroedinger equation.

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Stokes phenomena in discrete Painlevé I

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Abstract. In this study, we consider the asymptotic behaviour of the first discrete Painlevé equation in the limit as the independent variable becomes large. Using an asymptotic series expansion, we identify two types of solutions which are pole-free within some sector of the complex plane containing the positive real axis, similar to the tronquée and tritronquée solutions obtained for the first continuous Painlevé equation [1]. Using exponential asymptotic techniques developed in [2], we determine the Stokes Phenomena effects present within these solutions, and hence the regions in which the asymptotic series expression is valid. From a careful analysis of the switching behaviour across Stokes lines, we find that the first type of solution is uniquely defined, while the second type contains two free parameters, and that the region of validity may be extended for appropriate choice of these parameters.

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New nonlinear regimes of pulse generation in mode-locked fiber lasers

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Abstract. Mode-locked fiber lasers provide convenient and reproducible experimental settings for the study of a variety of nonlinear dynamical processes. The complex interplay among the effects of gain/loss, dispersion and nonlinearity in a fiber cavity can be used to shape the pulses and manipulate and control the light dynamics and, hence, lead to different mode-locking regimes. Major steps forward in pulse energy and peak power performance of passively mode-locked fiber lasers have been made with the recent discovery of new nonlinear regimes of pulse generation, namely, dissipative solitons in all-normal-dispersion cavities and parabolic self-similar pulses (similaritons) in passive and active fibers [1, 2]. Despite substantial research in this field, qualitatively new phenomena are still being discovered.

In this talk, we review recent progress in the research on nonlinear mechanisms of pulse generation in passively mode-locked fiber lasers. These include similariton mode-locking, a mode-locking regime featuring pulses with a triangular distribution of the intensity [3], and spectral compression arising from nonlinear pulse propagation [4]. We also report on the possibility of achieving various regimes of advanced temporal waveform generation in a mode-locked fiber laser by inclusion of a spectral filter into the laser cavity [5].

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POSTERS

Stability and convergence analysis for a fully discrete spectral scheme for Boussinesq systems

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Abstract. The family of (a,b,c,d) Boussinesq systems was derived and analyzed in [1] as an asymptotic model based on the Euler equations under a long wavelength and small-amplitude assumption. These systems model the propagation of nonlinear small amplitude water waves. As far as we know, most of the numerical results for these systems are concentrated in specific choices of parameters (for example in [2]). Such results address a very important question raised in [1] concerning the construction of accurate and efficient numerical methods for approximating solutions of IBVPs related to those systems.

In this paper we perform the stability analysis for the linear family of Boussinesq systems in order to determine the influence of its parameters on the efficiency and accuracy of the numerical scheme. The systems are discretized in space by the standard Fourier-Galerkin spectral method and in time by the explicit fourth-order Runge-Kutta scheme. We identify which regions of parameter space are most appropriate for obtaining a consistent numerical solution. For this nonlinear family of systems, we also prove an H^s-error bound of spectral accuracy in space and we discuss the same type of bound of fourth-order accuracy in time.

Numerical experiments are shown in order to verify the stability of the linear solution in each region of parameters and to confirm the theoretical temporal order of accuracy.

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Effects of thermoregulation on human sleep patterns

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Abstract. Experimental work and prior models suggest that changes in ambient temperature can affect sleep patterns in humans. This is further supported by the observation that the neurons responsible for sleep behavior and many temperature-sensitive neuron populations are both located in the hypothalamus. We have constructed a mathematical model of human sleep–wake behavior with thermoregulation and temperature effects using the Morris-Lecar equations to model the principal groups of neurons in the hypothalamus that affect sleep–wake and REM–Non-REM cycles. Simulations of this model show features previously presented in experimental data such as elongation of duration of REM bouts and number of REM bouts during the night, REM latency at the onset of sleep, and the appearance of awakenings due to deviations in body temperature from thermoneutrality. This model qualitatively agrees with experimental data which suggests that humans experience more awakenings during the night when sleeping at extreme ambient temperatures.

Dynamics of soliton parameters in coupled NLS equations

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Abstract. We develop a direct perturbation theory for a two-component setting of dark-bright solitons and derive the equation of motion for the soliton parameters. In this method, we solve the linearized wave equation around the solitons by expanding its solution into a set of complete eigenfunctions of the linearization operator. Suppression of secular growth in the linearized solution gives the evolution equations of soliton parameters. We use the resulting equations in a number of case examples motivated by recent consideration of such waves in atomic Bose-Einstein condensates. In particular, we consider the effect of finite-temperature perturbations, as well as the impact of δ-function external potentials. In these cases, the examination of small-amplitude perturbations of the solitary wave from an equilibrium position enable us to explore its temporal evolution, revealing good agreement with results previously obtained via alternative methods including energy-based methods, as well as numerical linear stability analysis and/or time-dynamics of the dark-bright solitary waves.

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Numerical studies of the KP line solitons

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Abstract. The Kadomtsev-Petviashvili (KP) equation describes the motion of shallow water waves in a two-dimensional region [2]. It admits a class of solitary wave solutions, called line-soliton solutions, which are localized along distinct lines in the xy-plane. These types of solutions have been studied extensively in recent years [1, 3]. Using a variety of initial conditions, the corresponding soliton solutions will be simulated numerically, and how these solitons interact will be studied. The goal is to determine which of the many exact solutions of the KP equation the soliton solutions converge given the initial conditions.

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A higher-order generalization of the dissipative NLS equation

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Abstract. In 1979, Dysthe [2] derived the modified nonlinear Schrödinger equation from the Euler equations by extending the perturbation analysis from $O(\epsilon^3)$ to $O(\epsilon^4)$. In 2008, Dias, Dyachenko and Zakharov [1] added a viscous correction to the Euler equations in order to derive the dissipative NLS equation (DNLS). In this poster, we derive the $O(\epsilon^4)$ generalization of the DNLS equation and present some of its properties.

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Modeling Manakov soliton trains: Effects of external potentials and inter-channel interactions

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Abstract. By now it is well known that the Complex Toda chain (CTC) models adequately the N-soliton train dynamics for the perturbed (scalar) NLS equations for various choices of the perturbation $iR[u]$ [1], including the case $iR[u] = V(x)u$, $V(x)$ being an external potentials, see [2] and the references therein.

Here we intend to prove that the CTC models also the interactions of the Manakov soliton trains in external potentials or the one-dimensional Gross-Pitaevsky eq. [3]:

$$i\vec{u}_t + \frac{1}{2}\vec{u}_{xx} + (\vec{u}^{\dagger}, \vec{u})\vec{u} = V(x)\vec{u}(x,t) + c_1\sigma_1\vec{u}(x,t), \tag{22}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{23}$$

in the presence of inter-channel interaction $c_1 \neq 0$. We have analyzed the effects of several types of potentials: a) harmonic $V(x) = v_2x^2 + v_1x + v_0$, b) periodic $V(x) = A\cos(\Omega x + \Omega_0)$ and c) shallow potential wells $V(x) = c_0(\tanh(x - x_{\mathrm{f}}) - \tanh(x - x_{\mathrm{in}}))$, $c_0 \ll 1$ and $x_{\mathrm{in}} < x_{\mathrm{f}}$. Here we will pay more attention that the perturbed CTC adequately models the soliton train dynamics for a wide region of the initial soliton parameters also for $c_1 \neq 0$

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