Wave Propagation Through Nonlinear Viscoelastic Felt

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Abstract

Felt is a non-woven fabric that is widely used in various acoustic applications. Mostly to dampen vibration and acoustic waves travelling through it. In this study the 1D nonlinear governing equation that describes deformation wave propagation through wool felt is studied both analytically and numerically. The equation was derived using experimentally obtained constitutive relation that features hysteretic damping. Dispersion analysis of the equation shows that a band gap (BG) and a negative group velocity (NGV) exist. An explanation of the influence of BG and NGV on the wave evolution is presented. It is speculated that BG and NGV influence spectral wave components with periods that are comparable to felt relaxation time. Additionally, a weakly nonlinear case is considered. The presented results shed light on the existence of waves with NGV in solids.

Keywords: Felt, material with memory, viscoelasticity, negative group velocity, nonlinear wave propagation.

1 Introduction

Felt is formed by intertwining fibres using heat, moisture, and pressure in a process called felting. Felt finds application in various industries, such as automotive, aeronautical, etc., where it performs noise reduction, vibration damping, and shock absorption functions. The governing equation describing the wave propagation in felt is obtained from the 1D equation of motion

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x}, \tag{1}$$

where ρ is the density, $u(x,t)$ is the displacement, and σ is the stress. The experimentally confirmed constitutive relation is the following:

$$\sigma(\varepsilon) = E_\mathrm{d}\Bigg[\varepsilon^p(t) - \frac{\gamma}{\tau_0}\int_0^t \varepsilon^p(\xi)\exp\left(\frac{\xi - t}{\tau_0}\right)\mathrm{d}\xi\Bigg], \tag{2}$$

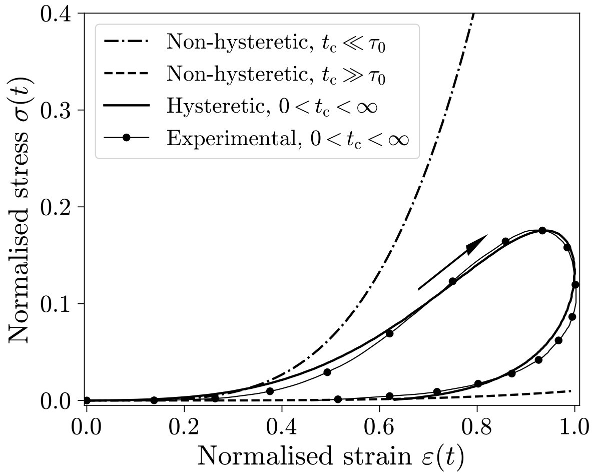

Figure 1: Typical dynamic stress-strain curves and loops describing the felt material. The arrow indicates the direction of the material loading.

where E_d is the dynamic Young's modulus, $\varepsilon = \partial u/\partial x$ is the strain, $p \geq 1$, $p \in \mathbb{R}$ is the compliance exponent introducing the nonlinearity, $0 \leqslant \gamma < 1$ is the hereditary amplitude, and τ_0 is the relaxation time [1–4].

Analysis of (2) indicates that under an extremely rapid loading-unloading cycle lasting for $t = t_\mathrm{c} \ll \tau_0$ the constitutive equation takes the following form:

$$\sigma(\varepsilon) = E_\mathrm{d}\varepsilon^p(t). \tag{3}$$

The material loading and unloading curves now follow the same path. Similarly, a relationship for a slow loading-unloading cycle, valid for low frequency waves and wave components, is found for $t = t_\mathrm{c} \gg \tau_0$

$$\sigma(\varepsilon) = E_\mathrm{s}\varepsilon^p(t), \tag{4}$$

where $E_\mathrm{s} = E_\mathrm{d}(1-\gamma)$ is the static Young's modulus. Figure 1 shows a typical stress-strain relationships given by (2), (3), and (4). For compacted wool felt p can be as high as 3.7 [2]. Additionally, the figure shows the fitting of parameters in (2) using an experimentally obtained curve.

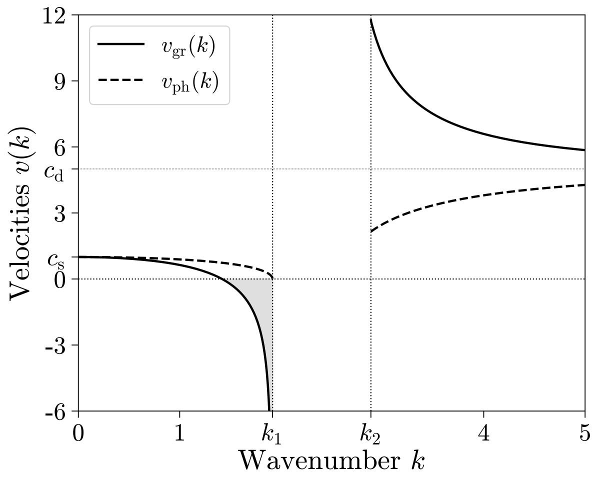

Figure 2: Phase and group velocities define by (10) for $\delta = 0.04$. BG exist for $k \in (k_1, k_2)$ where $k_1 = 1.9$ and $k_2 = 2.9$. NGV region exist for $k \in [1.4, k_1]$ and it is shown with the grey coloured region. When $k \to 0$, $\{v_{\mathrm{ph}}(k), v_{\mathrm{gr}}(k)\} \to 1 = c_{\mathrm{s}}$ and at the limit $k \to \infty$, $\{v_{\mathrm{ph}}(k), v_{\mathrm{gr}}(k)\} \to 1/\sqrt{\delta} = c_{\mathrm{d}}$.

Substituting (2) into (1) and elimination of the integral term yield the following equation:

$$\rho \frac{\partial^2 u}{\partial t^2} + \rho \tau_0 \frac{\partial^3 u}{\partial t^3} - E_{\mathrm{d}} \left[(1-\gamma) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)^p + \tau_0 \frac{\partial^2}{\partial x \partial t} \left(\frac{\partial u}{\partial x} \right)^p \right] = 0. \quad (5)$$

The dimensionless form of the above equation in terms of displacement variable $u(x,t)$ was derived in [3]

$$u_{tt} = [(u_x)^p]_x + [(u_x)^p]_{xt} - \delta u_{ttt}, \quad (6)$$

where $\delta = 1 - \gamma$ is the hereditary parameter. For $\delta \to 0$, i.e., $\gamma \to 1$ the hereditary properties approach maximum levels and the dissipative properties approach minimum levels. Here, the subscripted indices denote the differentiation with respect to the indicated variables.

The most suitable form of equation for studying the wave propagation through the felt material is obtained for the strain variable $\varepsilon(x,t)$

$$\varepsilon_{tt} = (\varepsilon^p)_{xx} + (\varepsilon^p)_{xxt} - \delta \varepsilon_{ttt}. \quad (7)$$

In the linear case for $p = 1$ the equation is referred to as the Moore-Gibson-Thompson equation [5]. The corresponding equations of (7) for extremely fast and slow loading cycles that correspond to constitutive relationships (3) and (4) are the following:

$$\varepsilon_{tt} = (\varepsilon^p)_{xx}, \quad (8)$$

$$\varepsilon_{tt} = \delta^2 (\varepsilon^p)_{xx}. \quad (9)$$

Equation (8) describes the propagation of a fully formed shock wave's front.

1.1 Negative group velocity in solids

Dispersion relation corresponding to (7) has the form

$$k^2 - \omega^2 - \mathrm{i}k^2\omega + \mathrm{i}\delta\omega^3 = 0, \quad (10)$$

where k is the wavenumber, ω is the angular frequency, and i is the imaginary unit. Analysis of (10) shows that felt can feature a band gap (BG) for certain wavenumbers and a region with negative group velocity (NGV) shown in Fig. 2.

Both properties are surprising for the material that is comprised of randomly oriented elastic lossy fibres. The BG and NGV influence only low frequency wave components. It can be shown that for realistic parameters the waves with wavenumbers located in the BG and/or NGV region correspond to the case where $t_{\mathrm{c}} \gg \tau_0$. Meaning that a *long* wave or a pulse corresponding to the BG and/or NGV region is successfully described by (9) in addition to full model (7). For realistic parameters, no dispersive effects driven by NGV on wave shape evolution exist. [4] The presented results shed light on the existence of waves with NGV in solids.

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