Influence of the Edge of the Cast Iron Frame Curvature on the Spectrum of the Piano String Vibration

Dmitri Kartofelev and Anatoli Stulov

Institute of Cybernetics at Tallinn University of Technology, Tallinn, Estonia
e-mail: dima@cs.ioc.ee

Abstract

Investigation of the boundary condition of vibrating string is a very important problem in musical acoustics. It is well known that the fundamental frequency of piano string is strictly determined by the type of the string termination. The types of the string support in the piano are different for the bass and treble notes. All the far ends of the piano strings are terminated on the bass and treble bridges, which are the rather complicated resonant systems. The nearest ends of the bass and long treble strings begin from the agraffe that can be considered as an absolutely rigid clamp termination. But the most part of the treble strings starts from the edge of the cast iron frame. These strings turn the rigid edge, and its vibration tone depends on the curvature of this termination. We investigate the vibrations of the ideal flexible string, which one end is rigidly clamped, and another one is terminated on the curved contact surface. The vibrating string touches repeatedly this termination, and this, in turn, causes the modulation of fundamental frequency of the string, and the train of high frequency oscillations is generated. The problem is studied both analytically, and numerically. The effect of the contact nonlinearity and of the shape of the contact surface on of the spectral structure of the string vibration is considered. The influence of the impact amplitude on the vibration spectra of struck string is discussed.

Models of Nonlinear String Support

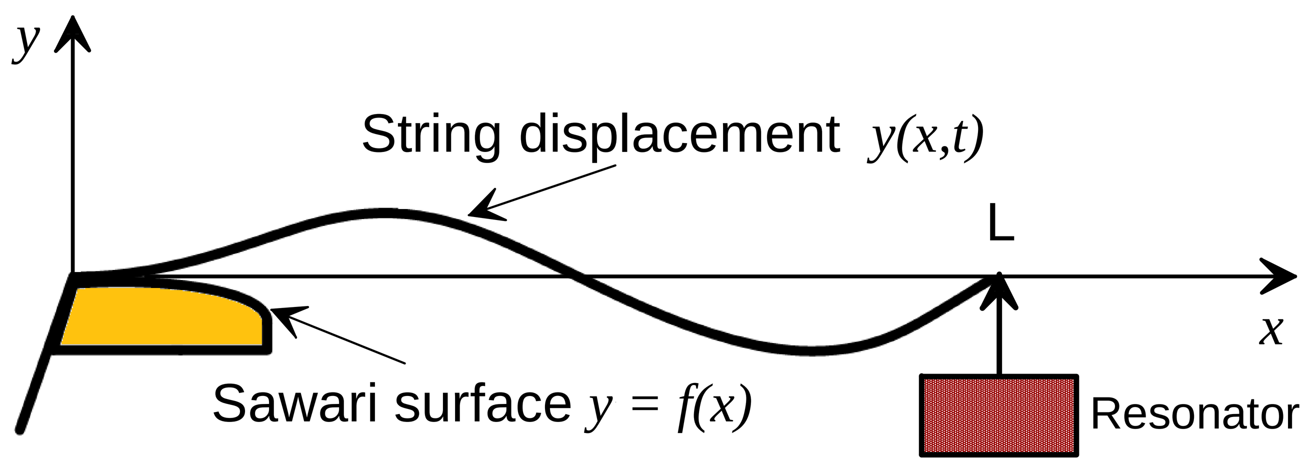

Figure 1: Scheme of sawari model

Taguti, T. and Tohnai, Y. K., Acoustical Science and Technology, 22, 199 – 207, 2001.
Taguti, T., 18th International Congress on Acoustics, Kyoto, 2119 – 2122, 2004.
Taguti, T., 12th International Congress on Sound and Vibration, Portugal, Lisbon, 2005.
Taguti, T., 19th International Congress on Acoustics, Spain, Madrid, 1 – 6, 2007.

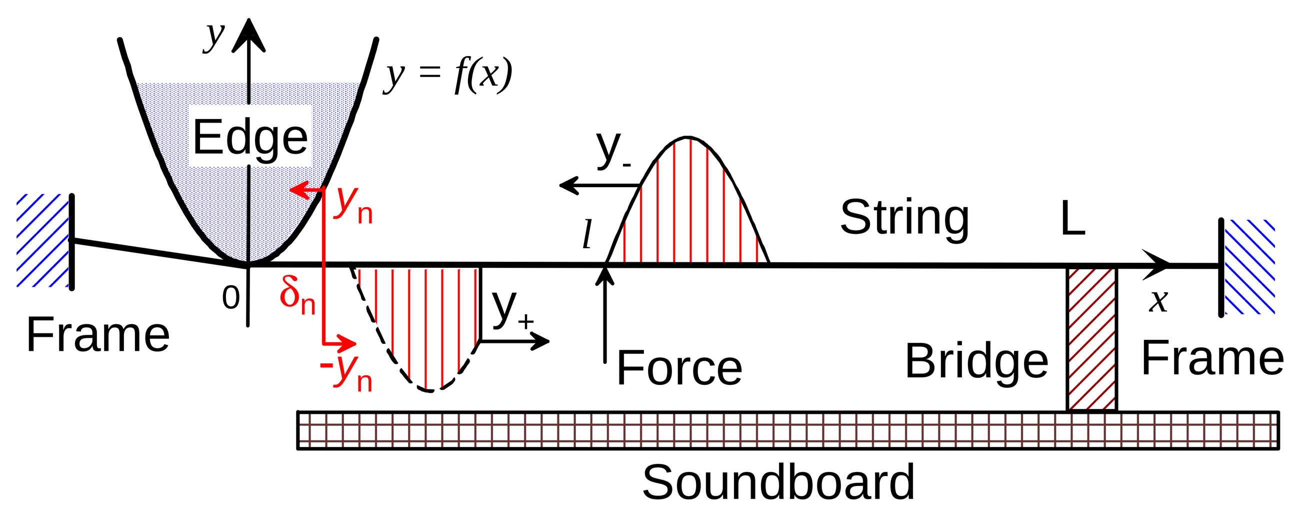

Figure 2: Scheme of piano string model

$$y_-(t) = \sum_{n=-\infty}^{+\infty} y(t_n)\frac{\sin\omega_{max}(t-t_n)}{\omega_{max}(t-t_n)} \qquad y_+(t) = -\sum_{n=-\infty}^{+\infty} y(t_n)\frac{\sin\omega_{max}(t-t_n+t_n^*)}{\omega_{max}(t-t_n+t_n^*)}$$

Numerical Results

For numerical simulation of the piano string with nonlinear support we chose here the string number $n=70$ (note $F_6^\sharp$; frequency f=1480 Hz; L=119 mm; l=7.2 mm; string tension T=644.8N; hammer mass m=2.1 g; static stiffness Q_0=4270 N/mm^p; stiffness exponent p=4.75; hereditary parameter α=0.395 ms). The form of the rigid edge of the frame is described here by the function $y=(2R)^{-1}x^2$, where R is the frame curvature at x=0.

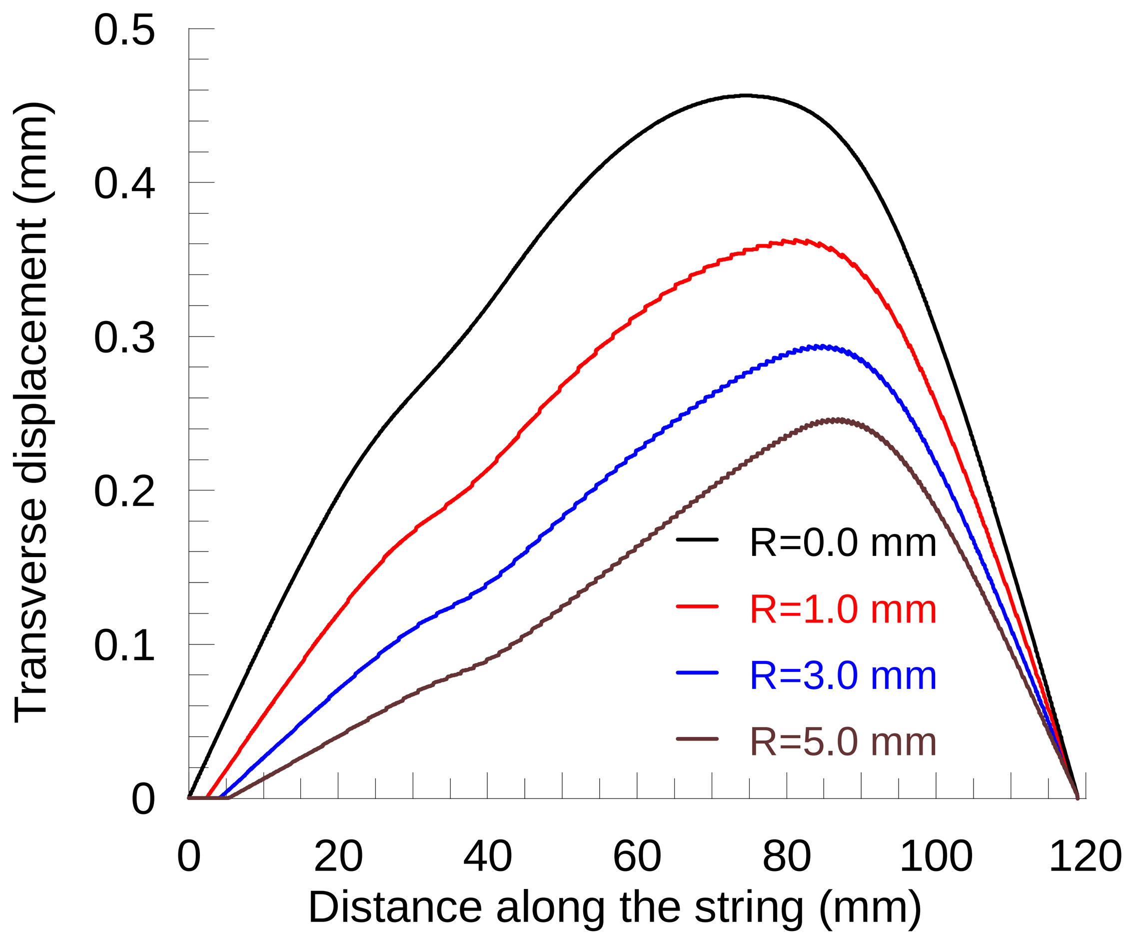

Figure 3: String displacement for string n=70. Varying the edge curvature R with fixed hammer velocity V=3m/s.

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$$

$$\frac{dz}{dt} = -\frac{2T}{cm}g(t) + V$$

$$\frac{dg}{dt} = \frac{c}{2T}F(t)$$

$$y(l,t) = g(t) + 2\sum_{i=1}^{\infty} g\left(t - \frac{2iL}{c}\right) - \sum_{i=0}^{\infty} g\left(t - \frac{2(i+a)L}{c}\right) - \sum_{i=0}^{\infty} g\left(t - \frac{2(i+b)L}{c}\right)$$

$$F(u(t)) = Q_0\left[u^p + \alpha\frac{d(u^p)}{dt}\right]$$

$$u(t) = z(t) - y(l,t)$$

The initial conditions at the moment when the hammer first contacts the string, are taken as $g(0) = z(0) = 0$, and $dz(0)/dt = V$.

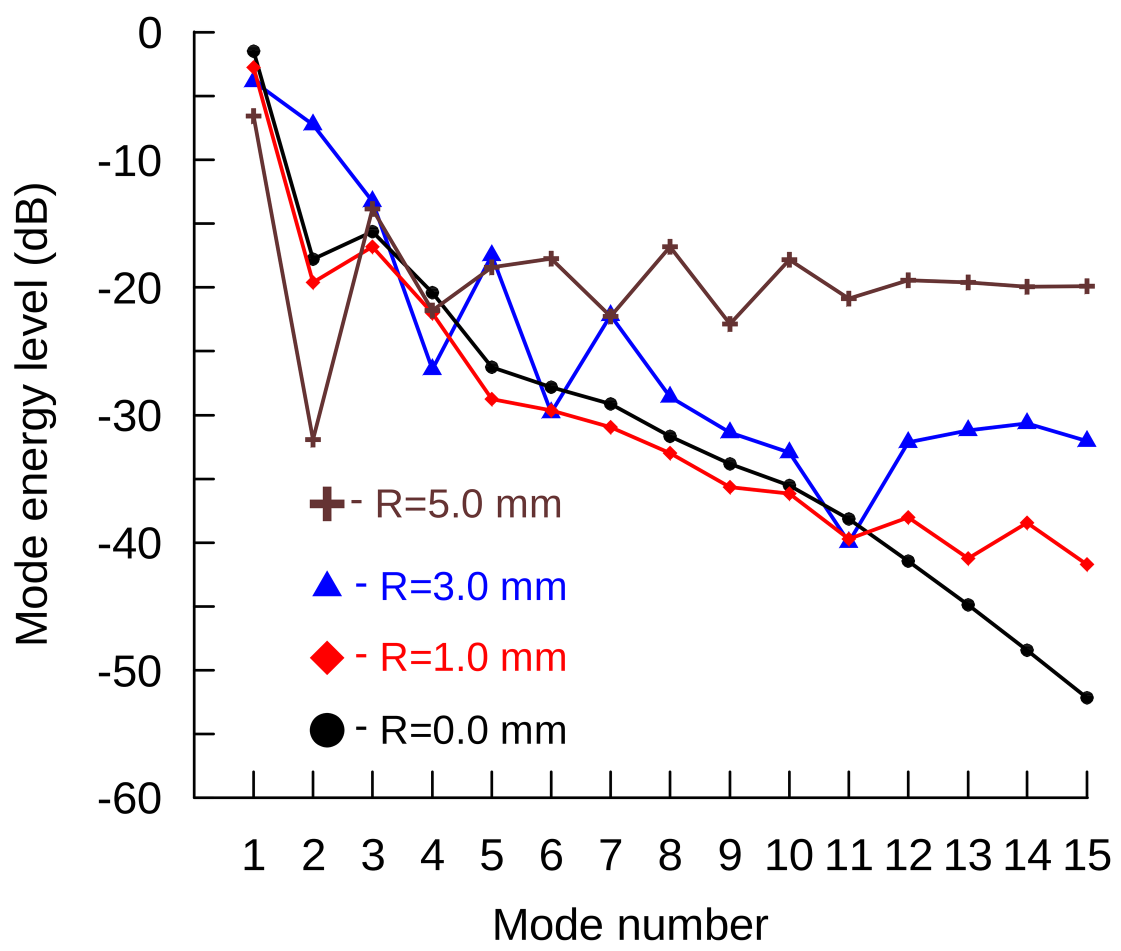

Figure 4: Spectral envelopes for string n=70. Varying the edge curvature R with fixed hammer velocity V=3m/s.

$$y(x,t) = \sum_{i=0}^{\infty} g\left(t - T_{1i}\right) - \sum_{i=1}^{\infty} g\left(t - T_{2i}\right), \text{ if } x \leqslant l$$

$$y(x,t) = \sum_{i=0}^{\infty} g\left(t - T_{3i}\right) - \sum_{i=1}^{\infty} g\left(t - T_{4i}\right), \text{ if } x > l$$

$$T_{1i} = c^{-1}[l - x + 2i(L-\delta_{1i})]$$

$$T_{3i} = c^{-1}[x - l + 2i(L-\delta_{3i})]$$

$$T_{2i} = c^{-1}[l + x - 2L + 2i(L-\delta_{2i})]$$

$$T_{4i} = c^{-1}[2\delta_{4i} - x - l + 2i(L-\delta_{4i})]$$

For the case $\delta_{ij} \ll L$ we can find $\delta_{ij} = f^{-1}(G_{ij})$, where

$$G_{1i} = g(t - c^{-1}(l - x + 2iL))$$

$$G_{2i} = g(t - c^{-1}(l + x - 2L + 2iL))$$

$$G_{3i} = g(t - c^{-1}(x - l + 2iL))$$

$$G_{4i} = g(t - c^{-1}(-x - l + 2iL))$$

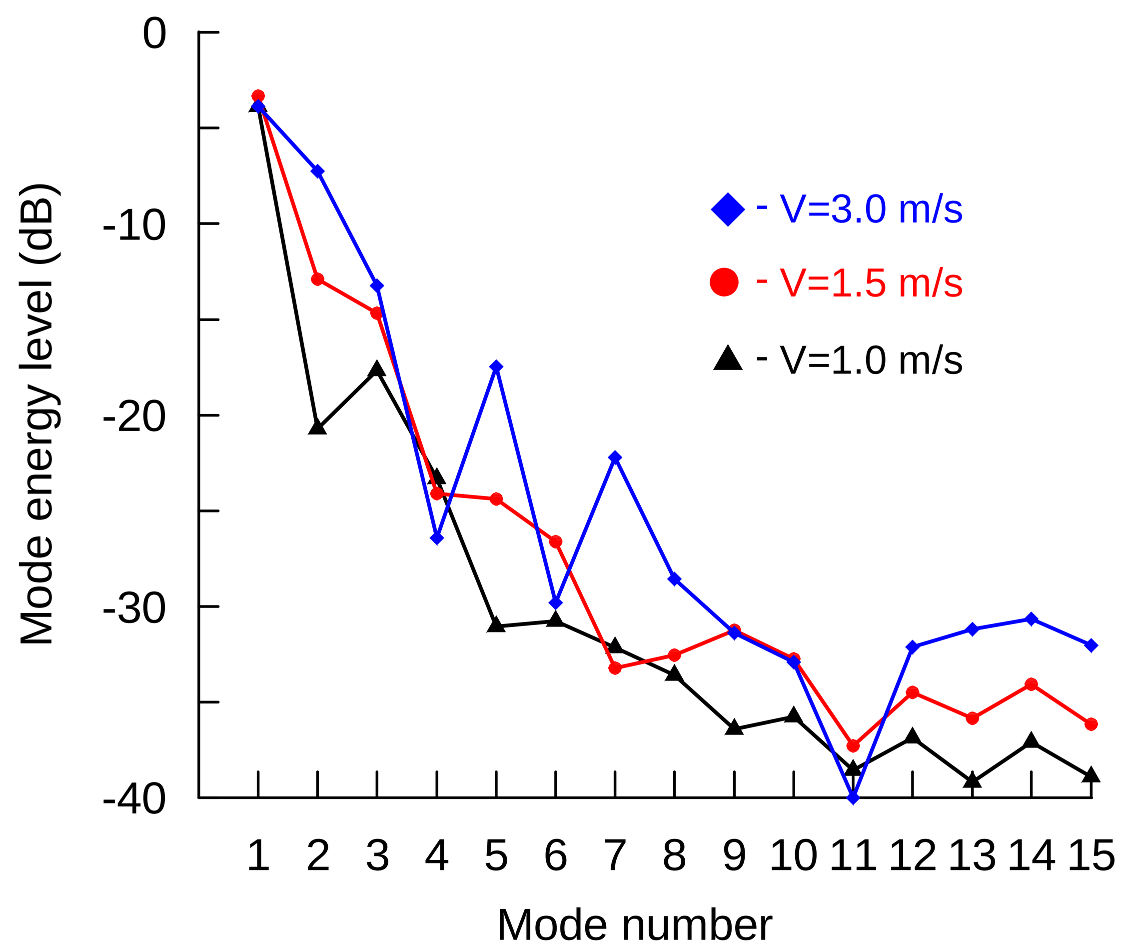

Figure 5: Spectral envelopes for string n=70. Varying hammer velocity V with fixed edge curvature R=3mm.

$$y(x,t) = \sum_i (A_i\cos\omega_i t + B_i\sin\omega_i t)\sin\left(\frac{i\pi x}{L}\right)$$

$$v(x,t_0) = \frac{\partial y}{\partial t}\bigg|_{t=t_0} = \frac{y(x,t_0) - y(x,t_0-\Delta)}{\Delta}$$

$$E_i = 10\log\left[\frac{2M\omega_i^2}{mV^2}\left(A_i^2 + B_i^2\right)\right]$$

$$A_i = \frac{2}{L}\int_0^L y(x,t_0)\sin\left(\frac{i\pi x}{L}\right)dx$$

$$B_i = \frac{2}{L\omega_i}\int_0^L v(x,t_0)\sin\left(\frac{i\pi x}{L}\right)dx$$

References

1. Skudrzyk, E., The Foundation of Acoustics, Springer-Verlag, Wien, New-York, 790 p. 1971.
2. Hall, D., E., “Piano string excitation VI: Nonlinear Modeling”, J. Acoust. Soc. Am., 92, 95 – 105, 1992.
3. Stulov, A., “Experimental and computational studies of piano hammers”, Acta Acustica united with Acustica, 91, 1086–1097, 2005.