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Use of simplified bowed string model in physics education: A laboratory experiment

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This paper proposes an educational science experiment on elastic string vibration where the string is excited by bowing. A custom-built monochord is used to study the resulting vibration experimentally. The monochord employs the Faraday's law of induction to measure the string displacement. The proposed experiment is accompanied by numerical simulations based on the integration of a nonlinear 1D damped wave equation with an external forcing term. The simulations give students the ability to compare the theory with practice. The study of bowed string dynamics aims to make the proposed science experiment more meaningful and engaging for students, since many string instruments, such as violin, cello, and other viol and viola family instruments, are played in such a manner. The proposed experiment can be used by teachers teaching BSc and MSc level students in acoustics, physics, applied mechanics, applied mathematics, and other related fields of science.

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1. INTRODUCTION

In this paper we propose an engaging discovery- and research-based science experiment on elastic string vibration excited by bowing. The study of bowed string vibration aims to make the experiments more meaningful and engaging to students. The proposed laboratory set-up combines several different physics disciplines such as electromagnetism, mechanics and acoustics. Also, by considering the bowed string we aim to introduce the notion of nonlinearity in a practical setting. The proposed experiment can be used by teachers teaching BSc and MSc level students.

Below, we combine practical laboratory experimentation with theoretical background that is intertwined with numerical simulations and frequency domain analysis of both the experimental and numerical data. The ability of students to combine numerical modelling with theoretical knowledge is of utmost importance in today's technologically minded society and is certainly of great benefit to their undergoing studies and future careers.

The bowed string vibration is usually not studied in most universities. Although, the laboratory equipment required to do so is not that complicated and most likely is already present in an average physics or acoustics laboratories. The laboratory experiments on elastic string vibration usually conducted in most universities, use some sort of a monochord with electrically conductive string that is excited electromagnetically with the aid of a signal generator and a magnet or magnets, see.[1–4] It is assumed that the resulting string vibration is harmonic, which implies that the relationship between fundamental frequency f_0, tension T and string speaking length L has the following form:

$$f_0 = \frac{c}{2L} = \frac{\sqrt{T/\mu}}{2L} = \frac{\sqrt{T/(\rho A)}}{2L}, \tag{1}$$

where c is the speed of the waves travelling on the string, ρ is the volumetric density of the string's homogeneous material, A is the cross-section area of a cylindrical string and μ is the linear mass density (mass per unit length) of the string. Additionally, it is assumed that the harmonics are related to the fundamental tone f_0 as follows

$$f_n = n f_0, \tag{2}$$

where n is a positive integer and it denotes the n-th harmonic. (1) and (2) are then used to design the experiments that aim to teach the notions of the harmonic content of standing waves.[1–4] As stated above, we propose to add to the usual approach by expanding it. The students graduate to a more involved and realistic excitations of the string. This especially should engage students who play relevant string instruments, such as, cello, contrabass, violin, viola, etc.

Historically, the problem of a vibrating elastic string such as that of a musical instrument was studied by d'Alembert, Euler, Bernoulli, and Lagrange.[5–7] More recently, numerical modelling of the strongly nonlinear friction force at the contact interface between a string and a rosin coated bow, and the bow–string interaction itself have generated a vast body of literature, starting with the observations of Helmholtz in the 19th century,[8] followed by the theoretical and experimental work of Raman in 1918.[9] A recent review paper by Woodhouse[10] offers a comprehensive history of the published literature on bowed string mechanics. The friction interaction between the string and the rosin-coated bow hair, in particular, remains an open problem. Recent work[11] has evaluated the state of the art in bow friction modelling, using both experimental and simulated results. Amongst aforementioned models, further reviewed by Desvages in her PhD thesis,[12] none were found to fit all experimental observations, although qualitative agreement was found with measurements in some aspects. In this paper the bow–string interaction is modelled using the model proposed by Bilbao in.[13] This nonlinear model is selected because of its relative simplicity in comparison to other more involved models, *cf.* [14–16]

This paper is organised as follows. Section 2 explains the used laboratory equipment/apparatus and the laboratory set-up. Section 3 lists the laboratory equipment used by the authors. Section 4 presents the theo-

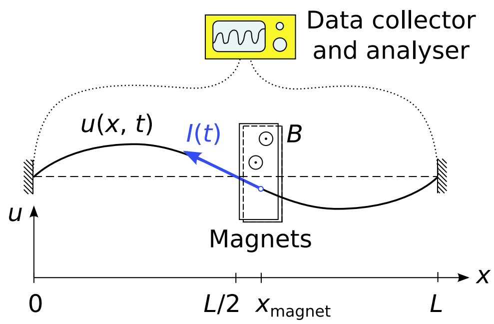

***Figure 1:** Schematic drawing of the laboratory set-up. The string with speaking length L and its displacement $u(x, t)$ are shown with the continuous solid curve. The string at its rest position is shown with the horizontal dashed line. The required electrical wires and cables are shown with the dotted curves. The time dependent and induced current $I(t)$ is shown with the blue arrow. The imaginary line connecting the north and south poles of each magnet are selected to be perpendicular to the string at its rest position. The magnets at $x = x_{\mathrm{magnet}}$ are movable along the x-axis. The magnetization vectors B of the magnets are parallel to each other and perpendicular to the string at its rest position and ux-plane.*

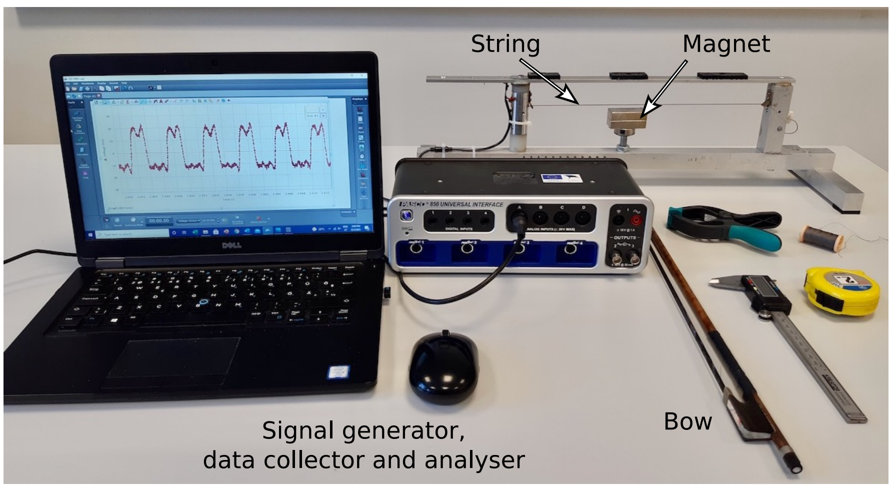

***Figure 2:** Photo of the proposed laboratory set-up.*

retical model used in the proposed laboratory experiment to predict the measured results. Section 5 presents the laboratory procedures and tasks that students are expected to put forward and carry out independently based on the provided questionnaire. Section 6 concludes the paper.

2. STUDY LABORATORY SET-UP

Figures 1 and 2 shows the schematic drawing and the photo of the experimental set-up and the custom-built monochord. The set-up employs the Faraday's law of induction to measure string vibration.[17] The vibration induces the current in the string. The resulting current is then measured using an off the shelf oscillogram. The set-up measures the time derivative signal of a vibrating point since the current produced by the set-up is proportional to the rate of change of the magnetic flux through the space between the magnets.[18,19] A similar set-up is described by Tsutsumanova and Russev in.[2] In the set-up the position of the two permanent magnets at $x = x_{\mathrm{magnet}}$ can be changed along the x-axis. While measuring the position of the magnets must not overlap the nodal points of the standing waves of interest, see Fig. 3. The set-up would fail to measure the vibration at the node since there is no displacement at the node. All measured data

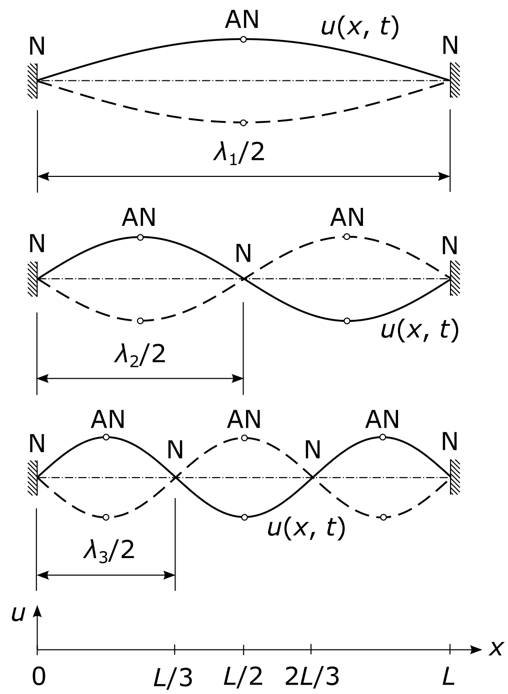

Figure 3: First three vibration modes of a stretched elastic string with the speaking length L featuring the half wavelengths of the respective standing waves $\lambda_i/2$ where $i = 1, 2, 3$. The nodes are indicated with "N" and the antinodes are indicated with the hollow bullets and "AN." String displacement $u(x, t)$ is shown with the solid curves and the displacement after a half of the period $u(x, t \pm P_i/2)$ where P_i is the period of i-th harmonic is shown with the dashed curve. (Top:) The first harmonic corresponding to the fundamental tone. (Middle:) The second harmonic corresponding to the first overtone. (Bottom:) The third harmonic corresponding to the second overtone.

are collected and stored in a digital form for the ease of data analysis and visualisation.

3. EXAMPLE LABORATORY EQUIPMENT

Monochord: Thin conductive string fixed at both ends; Suitable wires or cables connected to string ends, see Figures 1 and 2; Neodymium magnets (grade N52) with dimensions 50×25×25 mm; Violin bow; Rosin.
Experimental apparatus: Personal computer; PASCO™ 850 Universal Interface (power supply, electric signal generation up to 100 kHz, analog input, analog to digital converter with sampling rate up to 10 MHz); PASCO Capstone™ 2.0 software (signal analysis, data capture, data export and visualisation).
Other equipment: Meter stick; Software for integrating the equation of motion (Python 3.8 interpreter, NumPy 1.20.1, SciPy 1.6.2, matplotlib 3.3.4, Spyder v5.1.1 IDE software).

4. BOWED STRING VIBRATION MODEL

In this section we consider the uniplanar vibration of a bowed ideal string. The oscillations of the string excited by bowing belong to a class of relaxation oscillations which depend upon the fact that dry sliding friction decreases with the velocity of the sliding surfaces. Similarly to Bilbao[13] the system describing the bowed string vibration may be selected as follows:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - 2\gamma \frac{\partial u}{\partial t} + \delta(x - x_{\mathrm{bow}}) F_{\mathrm{bow}} \Psi(v_{\mathrm{rel}}), \tag{3}$$

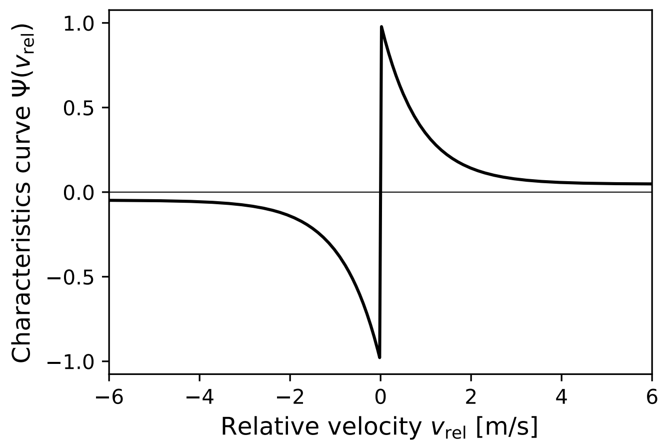

Figure 4: Dimensionless friction characteristics curve $\Psi(v_{\text{rel}})$ where $\beta = 1.16$ s/m and $\psi_{\text{min}} = 0.05$.

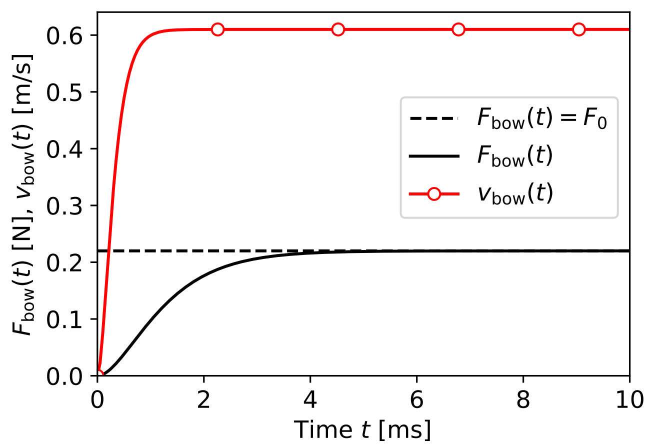

Figure 5: Bowing force $F_{\text{bow}}(t)$ acting at $x = x_{\text{bow}}$ is shown for $F_0 = 0.22$ N and $\alpha_F = 1500$ s^{-1}. The bowing velocity curve is shown for $V_0 = 0.61$ m/s and $\alpha_v = 6000$ s^{-1}.

$$u(x, 0) = u(0, t) = u(L, t) = 0, \tag{4}$$

where the second term on the right-hand side of (3) introduces a frequency-independent loss. It is easy to show that for $\gamma > 0$ all spectral frequency components of simulated waves will decay $\sim \exp(-\gamma t)$. The system assumes that the string is at rest prior to bowing. $F_{\text{bow}} = F_{\text{bow}}(t)$ is the external downward bowing force, and the dimensionless friction characteristics curve $\Psi(v_{\text{rel}})$, shown in Fig. 4, is selected in the following form:

$$\Psi(v_{\text{rel}}) = \text{sgn}(v_{\text{rel}}) \frac{\exp(-\beta|v_{\text{rel}}|) + \psi_{\text{min}}}{1 + \psi_{\text{min}}} \tag{5}$$

where $\beta \geq 0$ and ψ_{min} are the free parameters that control the curve shape. Relative velocity v_{rel} between the string at the bow location $x = x_{\text{bow}}$ and the externally applied bowing velocity $v_{\text{bow}} = v_{\text{bow}}(t)$ is defined as[13]

$$v_{\text{rel}} = \frac{\partial}{\partial t} u(x_{\text{bow}}, t) - v_{\text{bow}}. \tag{6}$$

Initial bowing velocity $v_{\text{bow}}(t)$ and bowing force $F(t)$ histories are, in principle, mutually independent which means that they can be selected arbitrarily, independent of each other. For simplicity we may select

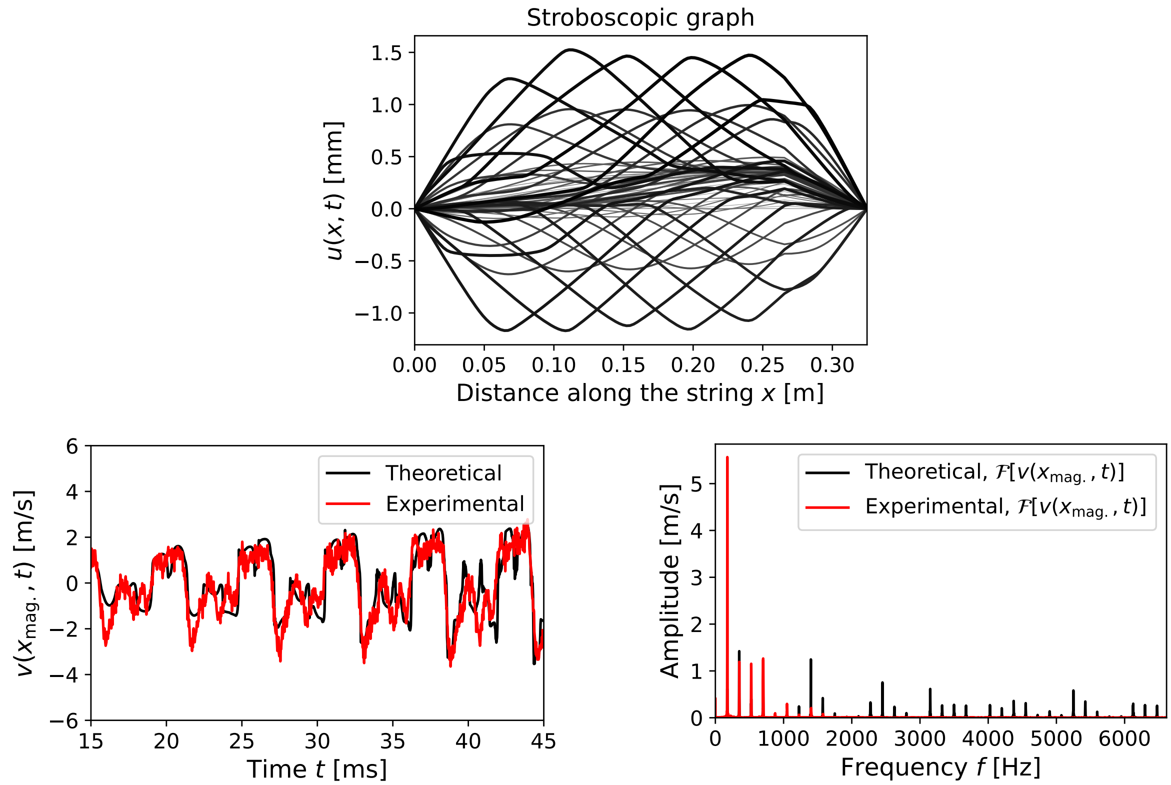

***Figure 6:** A typical simulation result and a comparison of a simulated time-series $v(x_{\mathbf{magnet}}, t)$ with an experimental measurement obtained using the set-up. The theoretical stroboscopic graph shows four periods of string displacement $u(x, t)$ being divided into 60 frames. The line thickness is used to indicate the flow of time. The parameter values used in the simulation: string length $L = 0.325$ m, string tension $T = 30$ N, string linear density $\mu = 2.32 \cdot 10^{-3}$ kg/m, the friction characteristics curve parameter $\beta = 0.66$ s^2/m^2, bowing force parameter $\alpha_F = 1500$ s^{-1}, maximum bowing force $F_0 = 0.22$ N, bowing velocity parameter $\alpha_v = 6000$ s^{-1}, maximum bowing velocity $V_0 = 0.61$ m/s, bowing point $x_{\mathbf{bow}} = 0.815L$, attenuation parameter $\gamma = 2.0$ s^{-1}, integration time $t_{\mathbf{max}} = 1.1$ s, position of the magnets $x_{\mathbf{magnet}} = 0.415L$ and temporal sampling rate of 44100 Hz.*

the bowing force to be constant in time

$$F_{\text{bow}}(t) = F_0 = \text{const.}, \tag{7}$$

or if more control is desired, then we may prefer the following form:

$$F_{\text{bow}}(t) = F_0 \left[1 - (1 + \alpha_F t) \exp(-\alpha_F t)\right], \tag{8}$$

where F_0 is the maximum force magnitude reached during bowing and α_F is the parameter controlling the initial steepness of the curve. We select the bowing velocity time-series in the following form:

$$v_{\text{bow}}(t) = V_0 \left[1 - (1 + \alpha_v t) \exp(-\alpha_v t)\right], \tag{9}$$

where V_0 is the maximum bowing velocity reached during bowing and α_v is the parameter controlling the curve shape. Initial conditions (8) and (9) are both monotonically and asymptotically approaching force magnitude F_0 and velocity V_0 values, respectively. Figure 5 shows an example of the selected bowing force and velocity. At this point we integrate system (3) directly using a suitable numerical integration method such as the finite difference method (FDM).[13] Figure 6 shows a typical simulation result.

5. LABORATORY PROCEDURES, TASKS AND PROBLEMS

Prior to arriving at the lab students are encouraged to independently familiarise themselves with the experimental set-up and all the necessary tools including software required to conduct the measurements. The set-up is pre-tuned and prepared for measurements. The string is excited with an off the shelf violin bow. The student is instructed to bow the string with constant bowing velocity and above moderate bowing force, see initial conditions (7) or (8) and (9). The measurement apparatus is then used to investigate and record the resulting string vibration and its Fourier spectra (if possible).

The student compares the numerically simulated result with the obtained measurements, see Fig. 6. A comparison of experimentally obtained Fourier spectrum with a theoretically calculated counterpart is performed. The effects of the bowing point at $x = x_{\mathrm{bow}}$, and the position of the permanent magnets at $x = x_{\mathrm{magnet}}$ on the obtained measurement results are investigated.

It can be argued that it will be hard for students to reproduce the bowed experimental data that coincide well with the simulated data, see Fig. 6. But, that in itself can/will teach students about the nature of modelling of nonlinear physical phenomena. It should also force them to think about the presented/used initial conditions and characteristic time scales of various phenomena taking place during the string excitation events. Student may use the following questions to guide their exploration of the set-up and its dynamics. Example problems and questions for students:

1. What is the Helmholtz motion? Can you reproduce it on the provided set-up?
2. Is frictional force higher for faster sliding speeds in dry sliding friction experiments?
3. Sketch the relationship of the friction force against the sliding speed.
4. Does the spectra of the measured and simulated signals meet your expectations? Explain.
5. Does vibration timbre (spectrum) depend on the selection of initial bowing velocity and force?
6. What happens to the vibration spectra (the Fourier amplitude or power spectra) for bowing points $x_{\mathrm{bow}} = L/2$, $L/3$, $L/4$, etc.?
7. How is the observed Helmholtz motion related to the friction between the bow and the string?

6. CONCLUSIONS

In this paper a novel educational laboratory science experiment that combines laboratory measurements with theoretical and numerical analysis of the measured results was proposed. This experiment measures elastic string vibration excited by violin bow.

Sections 2 and 3 presented the laboratory set-up, shown schematically in Fig. 1. The required laboratory set-up can be built using existing equipment/apparatus already present in most well-stocked university study laboratories. Section 5 presented explanation of the proposed experiment and gave some practical guidelines and instructions for teachers and students. The elastic string vibration is described using a damped wave equation with external forcing term. Students are introduced to numerical integration. Amongst other tasks students perform time-domain and frequency-domain analysis of the measured and simulated vibration data that are based on the theoretical model introduced in Section 4.

In conclusion, we believe that the introduction of the bowed string excitation method makes the proposed laboratory experiments more meaningful and engaging for students, since many string instruments, such as violin, cello and other viola and viol family instruments, are played in such a manner. Additionally, students' ability to combine numerical modelling, theory and independent discovery-based and research-based laboratory experimentation will surely help students to excel at learning and understanding.

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