Title: Numerical analysis of weakly nonlinear acoustic wave propagation in felted material

Authors: Maria M. Vuin, Dmitri Kartofelev
Affiliation: Tallinn University of Technology, School of Science, Department of Cybernetics, Laboratory of Solid Mechanics, Tallinn, Estonia
Corresponding Author: Maria M. Vuin
Email: maria.vuin@taltech.ee

Preferred Presentation Format: Oral
Preferred Session Category: 1. General Nonlinear Acoustics

Abstract: Felt—a non-woven, fibrous material—plays a significant role in acoustic damping and vibration control across multiple industries. This study investigates the propagation of strain waves through wool felt by analysing a one-dimensional nonlinear viscoelastic model. Derived from an experimentally validated hysteretic constitutive relation, the model accommodates nonlinearity with hereditary effects. Both analytical and numerical approaches are employed to examine wave behaviour, with particular focus on the emergence and influence of a frequency band gap (BG) and a region of negative group velocity (NGV). Contrary to expectations, these phenomena do not produce a significant qualitative effect on the evolution or dispersion of wave pulses. Theoretical explanations are provided, and the results offer valuable insights into the use of felt in noise control and energy dissipation applications.

Keywords: [felt, nonlinear wave propagation, dispersion analysis, negative group velocity, band gap]

1. Introduction and model

Felt is created by the entanglement of fibres under the influence of heat, pressure, and moisture [1]. Its viscoelastic and dissipative nature makes it ideal for acoustic and vibration damping in various sectors like music instrument building, machinery, automotive and aerospace [2] [3] [4] [5]. This research addresses the dynamics of wave propagation in such materials using constitutive relation obtained from experimental piano hammer research [6] [7] [8] [9].

The governing 1D partial differential equation that describes the wave propagation in felt is the following:

$$\varepsilon_{tt} = (\varepsilon^p)_{xx} + (\varepsilon^p)_{xxt} - \delta\varepsilon_{ttt}, \tag{1}$$

where $\varepsilon(x, t)$ is the strain, $p \geq 1$ is a nonlinearity parameter, and $\delta \in [0,1]$ characterises hereditary effects. The derivation of (1) is grounded in the constitutive law:

$$\sigma(\varepsilon) = \mathrm{E}_d \left[\varepsilon^p(t) - (\gamma/\tau_0)\int_0^t \varepsilon^p(\xi)\exp\big((\xi - t)/\tau_0\big)\mathrm{d}\xi\right], \tag{2}$$

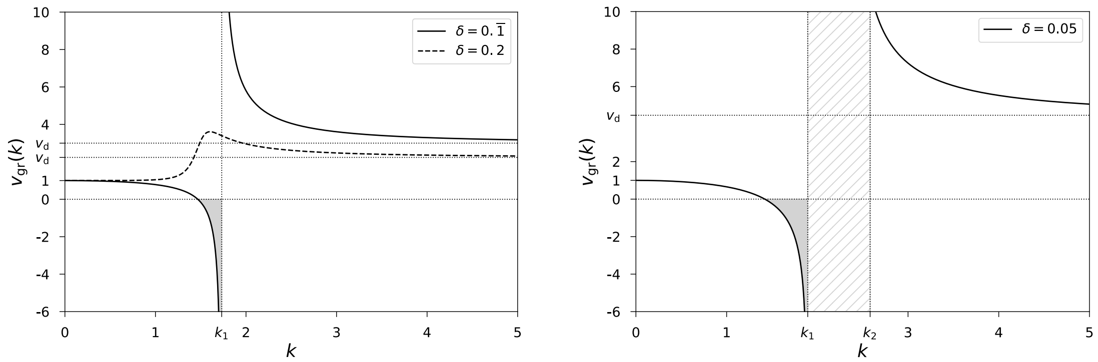

Figure 1: Group velocity curves for selected values of values of δ. *The BG appears in the wavenumber range* $k \in (k_1, k_2)$, *indicated by the dashed area. NGV region is shaded in grey. As* $k \to 0$, $v_{\text{gr}}(k) \to 1$, *and in the limit* $k \to \infty$, $v_{\text{gr}}(k) \to 1/\sqrt{\delta} = v_d$.

which describes hysteretic damping and accommodates different loading rates.

2. Dispersion and Numerical Analysis

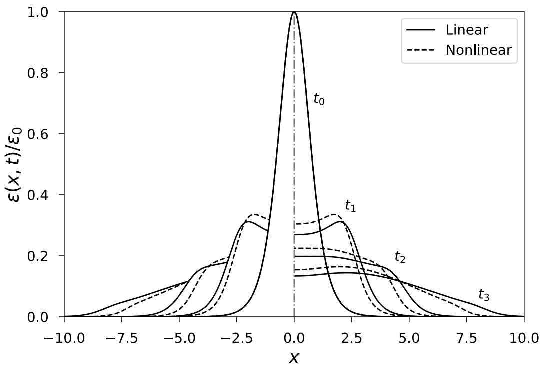

Figure 2: Linear and weakly nonlinear solutions with $\delta = 0.05$; $p = 1.0$ *for the linear case and* $p = 1.1$ *for the nonlinear case. Solutions are shown at time instances* $t_0 = 0$, $t_1 = 2.4$, $t_2 = 4.4$ *and* $t_3 = 8.0$.

Dispersion analysis of the linearised model reveals critical features such as a band gap (BG) and a region of negative group velocity (NGV), the latter occurring at low wavenumbers k, see Fig. 1. The linearised form leads to a characteristic equation:

$$\delta\sigma^3 + \sigma^2 + k^2\sigma + k^2 = 0, \tag{3}$$

The imaginary component of the solution determines wave attenuation. Three regimes emerge depending on the value of δ:

- For $\delta = 1$, the material behaves non-dispersive and without dissipation.
- For $\delta \in [1/9, 1)$, dispersion is present, but continuous.
- For $\delta < 1/9$, a band gap appears, with NGV observed when $\delta \lesssim 0.134$.

However, computational experiments involving pulses with frequencies in the BG and NGV regions revealed no substantial alteration in waveform dispersion or shape. This remains true in both the linear and weakly nonlinear cases, see Fig. 2.

3. Conclusions

This work presents an exploration of wave dynamics in nonlinear viscoelastic felted materials. Despite the theoretical presence of a frequency BG and a region of NGV, their impact on wave propagation is negligible under realistic parameter ranges. The findings refine our understanding of acoustic energy transmission in complex media and reinforce the applicability of felts for noise suppression without unexpected dispersive effects. Future research could build upon these results by exploring alternative excitation regimes or variations in material composition, such as fibre orientation, void sizes and constituent materials. A deeper comprehension of the NGV phenomenon holds promise for significant advancements in the engineering and application of nonwoven fibrous felted materials, with potential uses in vibration and noise control, and even in wave manipulation technologies.

Acknowledgements

Supported by the Estonian Ministry of Education and Research (PRG1227).

References

[1] T. J. Gillick, "Natural and Synthetic Fiber Felts," *Ind. Eng. Chem.,* vol. 51, no. 8, pp. 904-907, 1959.

[2] Y. Tian, G. Lin, J. Guo and Q. Zhang, "Permeability of porous microstructure in thermal protection system under high-temperature reactive gas conditions," *Phys. Fluids,* vol. 37, no. 1, p. 012008, 2025.

[3] H. Reda, K. Zaydana, N. Ghayth, M. Hammoud, J. F. G. M and H. Lakiss, "Wave propagation in viscoelastic composite materials with long-memory effects," *Continuum Mech. Thermodyn.,* vol. 33, no. 1, pp. 269-287, 2021.

[4] V. Naik, M. Kumar and V. Kaup, "A review on natural fiber composite material in automotive applications," *Eng. Sci.,* vol. 18, pp. 1-10, 2021.

[5] K. Kobiela-Mendrek, M. Baczek, J. Broda, M. Rom, I. Espelien and I. Klepp, "Acoustic Performance of Sound Absorbing Materials Produced from Wool of Local Mountain Sheep," *Mater.,* vol. 15, no. 9, p. 3139, 2022.

[6] D. Kartofelev and A. Stulov, "Wave propagation and dispersion in microstructured wool felt," *Wave Motion,* vol. 57, pp. 23-33, 2015.

[7] D. Kartofelev and A. Stulov, "Propagation of deformation waves in wool felt," *Acta Mech.,* vol. 225, pp. 3103-3113, 2014.

[8] A. Stulov, "Experimental and computational studies of piano hammers," *Acta Acust United Acust,* vol. 91, pp. 1086-1097, 2005.

[9] A. Stulov, "Hysteretic model of the grand piano hammer felt," *J. Acoust. Soc. Am.,* vol. 97, no. 4, pp. 2577-2585, 1995.