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Frequency-dependent dissipation in dispersive wool felt

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Abstract

Felt is a non-woven fabric (textile) that is produced by matting, condensing and pressing natural or synthetic fibres by a process called wet felting. Felt is the oldest form of fabric known to humankind by predating weaving and knitting. There are many different types of felts for industrial, technical, designer and craft applications. While some types of felt are very soft, some are tough enough to form construction materials. Felt can vary in terms of fibre content, dimensions, density and more factors depending on the use of the material. Not many physical properties of felt are well known or actively studied. The purpose of this study is to investigate compressional strain wave propagation through natural wool felt. In this paper a frequency-dependent dissipation and dispersion of acoustic waves propagating through felt are analysed in the one-dimensional setting. The presented model is based on a experimentally obtained constitutive relation that takes into account the elastic and hereditary properties of the microstructured felt. The numerical solutions of the linear problem are used to estimate a strain pulse amplitude decay and they are analysed in the context of complex dispersion curves. It is shown that in the linear case the exponential decay rates for different frequencies may be obtained rather accurately by using dispersion analysis. It is concluded that intertwined and anisotropically oriented fibres in porous felt give rise to frequency-dependent attenuation of acoustic waves propagating through the material. Presented results are useful for various acoustical applications of felt material.

Keywords: felt, viscoelastic media, dissipative media, porous material, dispersion analysis.

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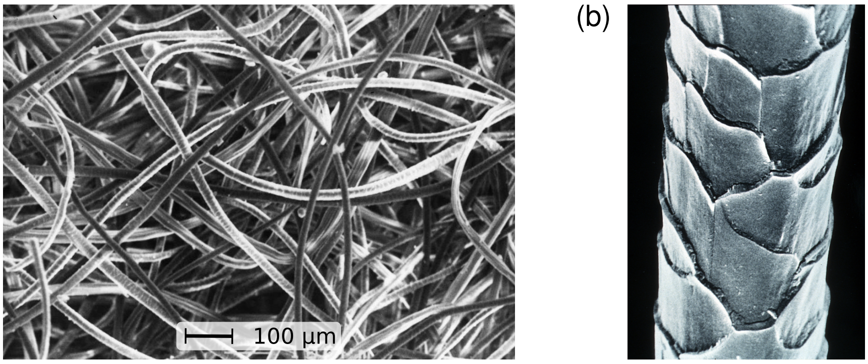
1 Introduction

Felt is a truly unique and interesting material that is used for a wide variety of applications: vibration isolation, sound absorption, noise reduction, filtering, construction etc. Felt as a non-woven fabric (textile) that is produced by matting, condensing and pressing natural or synthetic fibres by a process called wet felting. While some types of felt are very soft, some are tough enough to form construction materials. Felts can vary in terms of fibre content, dimensions, density and more factors depending on the intended use of the material. Felt is highly dissipative and dispersive material and the properties of felts are directly related to their internal structures [1, 2].

(a) (b)



Source: ((a) Davies, C.; Hardy, C., 2004. (b) CSIRO science image, 2016.)

Figure 1: **(a) Micrograph of internal structure of wool felt. (b) Electron microscope image of an individual mechanically and chemically untreated wool fibre. Fibre diameters range from 10 to 55 µm. Fibre cuticles are clearly visible.**

The anisotropically oriented fibres, shown in Fig. 1 a, of the porous microstructured felt provide an elastic and/or viscoelastic frame that composes the mechanically active volume of the medium. In addition, the wet felting process ensures that individual fibres are under randomly distributed (positive and negative) tension. In the case of natural felt, e.g. wool felt, the individual fibres are covered with layers of overlapping shingle-like scales (cells) called cuticles and with oily/greasy wax called lanolin secreted by the sebaceous glands of domestic sheep. This residue and other naturally occurring oils, fats, waxes and moisture can also be found between the fibre cuticles. Mechanical properties including surface roughness of natural fibres depend among other things on temperature and air moisture [3, 4, 5]. Figure 1 b shows an electron microscope image of a wool fibre. Deformation wave propagation through such fibre mass is a complicated multifaceted phenomenon. The shape of a diffused and rapidly dissipating initial disturbance is determined by number of simultaneous effects: elastic and viscous stretching of fibres; collision of fibres against each others; porosity; friction of fibres and fibre scales against each other [6, 7]; various viscous effects induced by lanolin wax; internal friction of the wax and hierarchically microstructured (cellular structure) fibres [8]; etc. The cellular structure of felt fibres means, in essence, that a single fibre could be modelled as a microstructured material with several characteristic scales [9, 10, 11].

The dissipation in solids may be produced by several different mechanisms, and although ultimately these all result in the mechanical energy being transformed into heat, two main dissipative processes are involved. The first type of process is known as “static hysteresis”, where the effect is that the energy loss per loading cycle is independent on frequency. The principal cause may be associated simply with the “static” nonlinear stress–strain behaviour of the materials [12, 13]. The other type is attributed to “viscosity”, according to which many materials show losses which are associated with the velocity gradients set up by the vibrations, and hence losses are dependant on the frequency. The forces producing these losses may be considered to be of a viscous nature and imply that the mechanical behaviour will depend upon the rate of straining. This is the subject of the “linear viscoelasticity” [13].

In this paper the dissipation is studied as the amplitude attenuation of harmonic Fourier spectral components in the case of a single pulse propagation. The problem is considered in a one-dimensional setting and for the linear case only. A damped one-dimensional harmonic strain wave in a linear solid travelling in a positive direction of the x-axis can be described with the aid of a complex valued wave number $\kappa(\omega) = k(\omega) + i\lambda(\omega)$ (where $k = \mathrm{Re}(\kappa)$, $\lambda = \mathrm{Im}(\kappa)$) as follows:

$$\varepsilon(x,t) = \hat{\varepsilon} e^{i\kappa x - i\omega t} = \hat{\varepsilon} e^{i(k+i\lambda)x - i\omega t} = e^{-\lambda x} \hat{\varepsilon} e^{ikx - i\omega t}, \tag{1}$$

where $\varepsilon(x,t)$ is the strain, $\hat{\varepsilon}$ is an amplitude, ω is the circular frequency and i is the imaginary unit. It is clear that for positive values of $\lambda(\omega)$ it acts as an exponential decay function for the harmonic spectral components of the strain wave $\varepsilon(x,t)$. In other words spectral components decay exponentially as $x,t \to \infty$ for $\lambda(\omega) > 0$.

The organisation of the paper is as follows. In Sect. 2 the problem description and theoretical model are presented. The corresponding dispersion analysis is performed. In Sect. 3 we discuss the nature of loss mechanism present in felt. Section 4 compares the result of the numerical solution with the theoretical result derived from the dispersion analysis. And finally, Sect. 5 presents the main results and conclusions.

2 Viscoelastic felt model and dispersion relation

Based on the experimentally obtained constitutive relation (nonlinear modification of the Kelvin-Voigt model) [14, 15, 16] a wave equation for felt type material was derived in the following form:

$$\rho \frac{\partial^2 u}{\partial t^2} + \rho \tau_0 \frac{\partial^3 u}{\partial t^3} - E_d \left\{ (1-\gamma) \frac{\partial}{\partial x} \left[\left(\frac{\partial u}{\partial x} \right)^p \right] + \tau_0 \frac{\partial^2}{\partial x \partial t} \left[\left(\frac{\partial u}{\partial x} \right)^p \right] \right\} = 0, \tag{2}$$

where $u(x,t)$ is the displacement, ρ is felt density, τ_0 is the relaxation time, E_d is the dynamic Young’s modulus, γ is the hereditary amplitude and p is the nonlinearity exponent [1, 2].

Dimensionless and normalised form of the Eq. (2) for the strain variable $\varepsilon(x,t) = \partial u / \partial x$ is obtained by using following non-dimensional variables that are introduced by relations

$$u \Rightarrow \frac{u}{l_0}, \quad x \Rightarrow \frac{x}{l_0}, \quad t \Rightarrow \frac{t}{\alpha_0}, \tag{3}$$

where the characteristic length l_0 and time duration α_0 are defined as $\alpha_0 = \tau_0/\delta$, $\delta = 1-\gamma$, $l_0 = c_d\alpha_0\sqrt{\delta}$, $c_d = \sqrt{E_d/\rho}$, $c_s = c_d\sqrt{\delta}$. Thus, Eq. (2) in terms of the strain variable takes the following form:

$$\frac{\partial^2(\varepsilon^p)}{\partial x^2} - \frac{\partial^2\varepsilon}{\partial t^2} + \frac{\partial^3(\varepsilon^p)}{\partial x^2\partial t} - \delta\frac{\partial^3\varepsilon}{\partial t^3} = 0. \tag{4}$$

Such a form of the governing equation depends only on two parameters. Parameter p is the nonlinearity parameter and parameter δ, which is defined in the interval $(0,1)$ is related to the hereditary and dissipative properties of the felt material. If parameter $\delta \to 0$ then the hereditary properties approach maximum levels. The last term of this equation is the dissipation term. The linear form of the Eq. (4) for $p = 1$ is

$$\frac{\partial^2\varepsilon}{\partial x^2} - \frac{\partial^2\varepsilon}{\partial t^2} + \frac{\partial^3\varepsilon}{\partial x^2\partial t} - \delta\frac{\partial^3\varepsilon}{\partial t^3} = 0. \tag{5}$$

The corresponding dispersion relation of Eq. (5) is in the form

$$i\delta\omega^3 - \omega^2 - i\kappa^2\omega + \kappa^2 = 0, \tag{6}$$

where κ is the wave number and ω is the circular frequency. It can be show that solution of this dispersion law for κ indeed results in a complex valued solution, $\kappa(\omega) \in \mathbb{C}$ [1]. Since $\kappa = k + i\lambda$, dispersion relation (6) can be rewritten for real values of k and λ in the following form:

$$k^2 + 2ik\lambda - \lambda^2 - ik^2\omega + 2k\lambda\omega + i\lambda^2\omega - \omega^2 + i\delta\omega^3 = 0. \tag{7}$$

In order to study real and imaginary parts separately, the system of equations based on relation (7) is derived

$$\begin{cases} k^2 - \lambda^2 + 2k\lambda\omega - \omega^2 = 0, \\ 2k\lambda - \omega(k^2 - \lambda^2) + \delta\omega^3 = 0. \end{cases} \tag{8}$$

Solution of this system is

$$k(\omega) = LM\left(\sqrt{1+M^2} - 1\right)^{-1/2}, \tag{9}$$

$$\lambda(\omega) = L\left(\sqrt{1+M^2} - 1\right)^{1/2}, \tag{10}$$

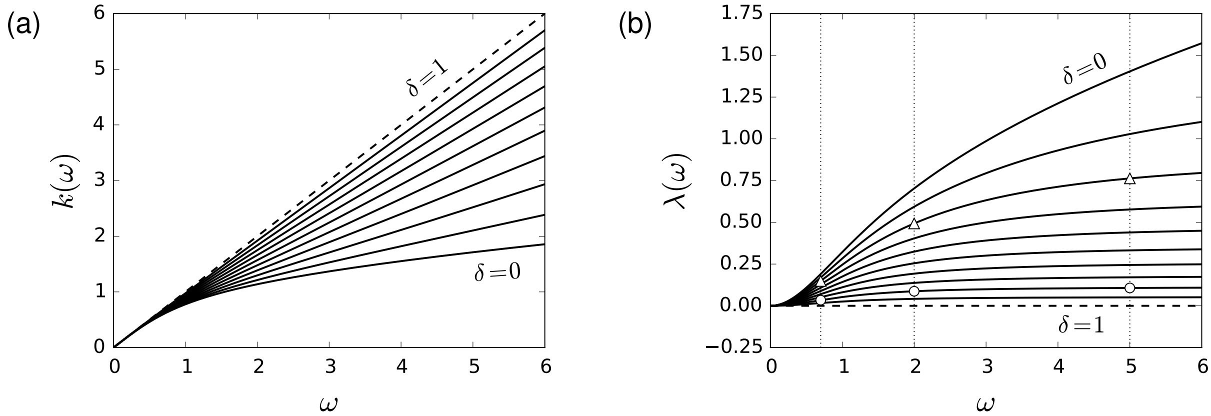
where

$$L = \omega\sqrt{\frac{1+\delta\omega^2}{2(1+\omega^2)}}, \quad M = \frac{(1-\delta)\omega}{1+\delta\omega^2}. \tag{11}$$

The frequency dependencies $k(\omega) = \mathrm{Re}(\kappa)$ and $\lambda(\omega) = \mathrm{Im}(\kappa)$ of dispersion relation (6) are shown in Fig. 2 for the selected values of the material parameter δ. From (9) and (10) one can see that if $\omega \to \infty$, then $k(\omega) \to \omega\sqrt{\delta}$, and

$$\lambda_\infty = \lim_{\omega\to\infty} \lambda(\omega) = \frac{1-\delta}{2\sqrt{\delta}}. \tag{12}$$

For large frequencies the exponential decay function $\lambda(\omega)$ depends only on parameter δ by asymptotically approaching the limit value λ_∞.



Source: (Kartofelev, D., 2016)

Figure 2: **(a) Dispersion curves for various values of parameter δ in range [0.0, 1.0] with step 0.1. Dashed lines shows the dispersionless limiting case, where $\delta = 1$. Felt with maximum heredity and dissipative properties corresponds to $\delta = 0$. (b) Fourier spectral component decay function $\lambda(\omega)$ for the same values of δ. Limiting dispersionless and lossless case shown for $\delta = 1$. The hollow symbols denote frequency values used in Tab. 1. The triangles correspond to the case where $\delta = 0.2$ and circles correspond to $\delta = 0.8$.**

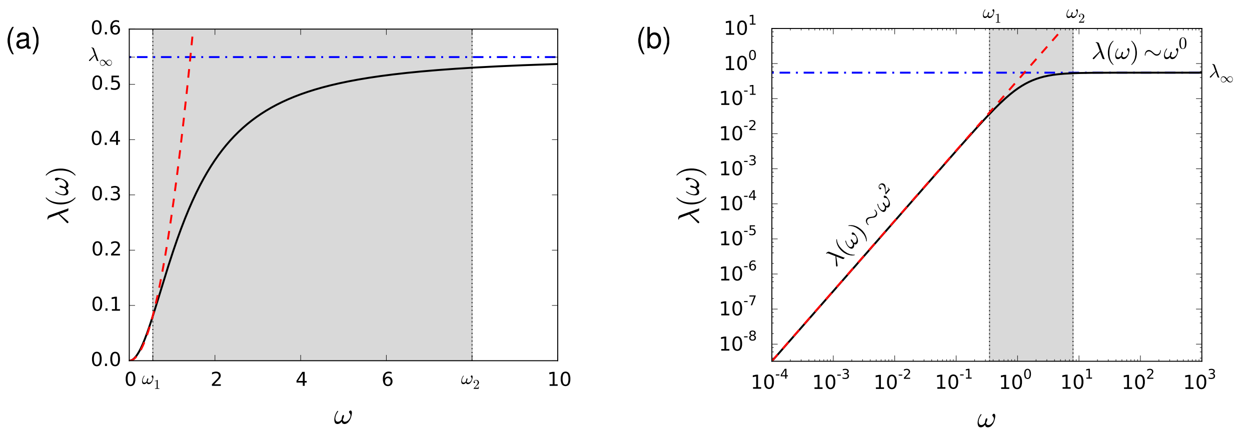
If $\delta = 1$ then from (9) and (10) one can find that $k(\omega) = \omega$ and $\lambda(\omega) = 0$. This real valued non-dispersive limiting case is shown in Fig. 2 by dashed lines. In fact, this case describes a usual elastic material in which the wave propagates without attenuation.

3 Dissipation in felt-type material

A closer examination of the exponential decay function $\lambda(\omega)$ suggests that the wave attenuation may be attributed to three different loss mechanisms which depend on the frequency ω. Figures 3 a and 3 b show these three frequency regions: low-, transitional midrange- and high frequency region.

For the low frequencies $\omega \leq \omega_1$ the wave component decay depends on the frequency as ω^2. Longer wavelength, i.e. low frequency waves interact with the internal structure of felt at a larger scale by elastically deforming the intertwined fibre mass and by introducing fibre-to-fibre friction and collision. It has been shown earlier for granular viscoelastic media that viscous dissipation which yields an attenuation that scales as the square of frequency at low frequencies corresponds to a frictional stress that is proportional to the particle (fibre) velocity (*cf.* [17, 18, 19, 20]).

For the transitional midrange frequencies $\omega_1 < \omega < \omega_2$ the attenuation depends on the frequency as ω^a with a in the interval (2, 0). This smooth continues transitioning of the power a is shown in Fig. 3 b. Probably the fibre-to-fibre frictional forces are still the most contributing/dominating in the midrange frequency region. It is possible that the viscous effects of lanolin wax (if present) and moisture start contributing more at this stage. In addition, porosity of felt might become a significant loss mechanism. It is well-known that a simple porous solid with a rigid frame gives rise to an attenuation that is constant at high frequencies and scales as the square root of frequency below a critical frequency (*cf.* [17, 18, 19, 20]).

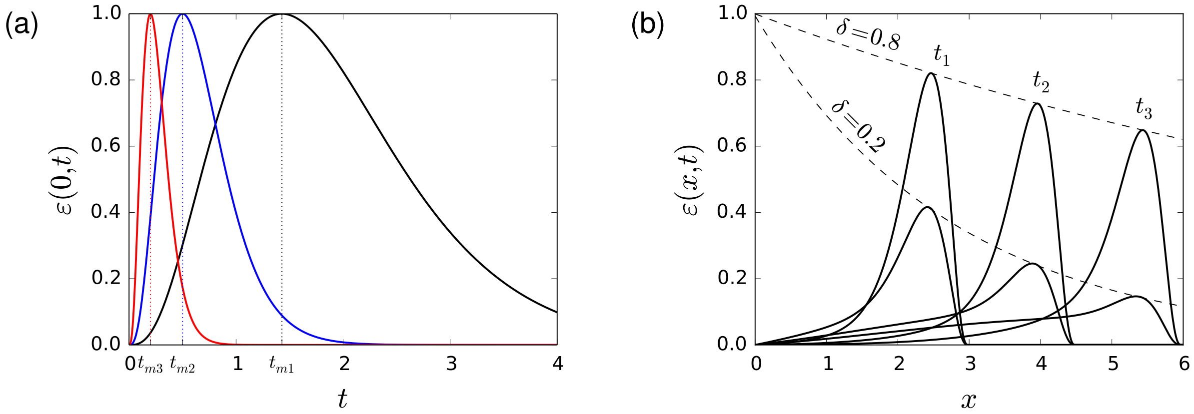


Source: (Kartofelev, D., 2016)

Figure 3: **(a) Spectral component decay function $\lambda(\omega)$ for $\delta = 0.35$ shown by the solid line. The low frequency region, where $\omega \leq \omega_1$. Parabolic dependency is demonstrated with parabola shown by red dashed line. The transitional midrange frequency $\omega_1 < \omega < \omega_2$ region shown by grey background. The high frequency region $\omega \geq \omega_2$, where the decay depends approximately as a constant of frequency, is shown by a horizontal blue dash-dotted line at $\lambda(\omega) = \lambda_\infty$. (b) Same result shown on a log-log plot. The transitional midrange frequency region is shown by the grey background. The dashed and dash-dotted lines have the same meaning as in Fig. 3 a.**

For the higher frequencies $\omega \geq \omega_2$ the attenuation depends on the frequency approximately as ω^0 (constant). For higher frequencies the characteristic wavelength starts to be of the order of fibre diameters or smaller. At this stage viscoelastic friction of fibre mass may be accompanied by internal frictional forces corresponding to internal microstructure of fibres, cuticles and wax. Similarly to the previous case porosity may also play a significant role (*cf.* [19, 20, 21]).

More precise and detailed understanding of the nature of dissipation regimes and the underlying loss mechanisms requires additional theoretical and experimental investigations. It must be stressed that results presented here are valid for the linear case only. The real felts are described by a strong nonlinear behaviour [1, 2]. The value of the nonlinearity parameter p in Eqs (2) and (4) can be rather high, usually $2.0 \leq p \leq 3.5$. One can only hope that for the realistic nonlinear case the frequency dependence of dissipation behaves in a similar manner to the linear case discussed here.



Source: (Kartofelev, D., 2016)

Figure 4: **(a) Shape of BV defined by (13) for three t_m values used in Tab. 1. (b) Snapshots of the pulses profiles, where $t_m = t_{m2} =$ 0.5, shown for time moments $t_1 =$ 3.0, $t_2 =$ 4.5 and $t_3 =$ 6.0 and for parameters $\delta =$ 0.2 (more realistic case) and $\delta =$ 0.8 (less realistic case). The dashed trace-lines show the amplitude decay. For $\delta =$ 0.2 case the fitted amplitude decay function is $e^{\lambda_{\text{num}} x} = e^{-0.36x}$; for $\delta =$ 0.8 the amplitude decay function is $e^{\lambda_{\text{num}} x} = e^{-0.082x}$.**

4 A pulse amplitude attenuation

In this section a boundary value problem (BVP) is considered, and the numerical solution of Eq. (5) describing the strain wave amplitude attenuation is analysed. The equation is solved using the finite difference method. A boundary value (BV) of the strain prescribed at $x = 0$ is selected in the following form:

$$\varepsilon(0,t) = A\left(\frac{t}{t_m}\right)^3 e^{3\left(1-\frac{t}{t_m}\right)}, \tag{13}$$

$$\frac{\partial}{\partial t}\varepsilon(0,t) = 0, \tag{14}$$

$$\frac{\partial^2}{\partial t^2}\varepsilon(0,t) = 0, \tag{15}$$

where parameter t_m defines the time corresponding to the maximum of the pulse amplitude. Figure 4 a shows the shape of the BV (13) for selected t_m values. Here we suppose that the *fundamental* spectral component ω of the pulse (13) is estimated from relationship $\omega t_m \simeq 1$. In reality any pulse with a finite width has a wide continuous spectrum. This is a rough approximation, but below it is shown that the resulting numerical calculations are in agreement with the dispersion analysis. For the sake of clarity the effect of dispersion on a pulse amplitude decay is not considered in this paper. Figure 4 b shows the numerical solution of the BVP (5), (13), (14), (15) with corresponding trace-lines of a pulse amplitude decay.

Table 1 displays parameter δ, BV parameter t_m, the corresponding value of *fundamental* frequency ω, value of $\lambda(\omega)$, the value of numerically obtained exponential decay constant λ_{num}

(corresponding to the trace-lines shown in Fig. 4 b), and the absolute difference between λ and λ_{num}. We conclude that results presented in Tab. 1 for selected values of δ are sufficient to confirm that the values of numerically calculated decay constants λ_{num} and the theoretical values of λ defined by relation (10) are approximately equal. The result for $\delta = 0.2$ deviates more, this is due to higher dispersion that makes our assumption $\omega t_m \simeq 1$ less accurate for $x, t \to \infty$. In principle, this approach can be used to verify the amplitude decay rates for any specific value of t_m and δ rather accurately.

Table 1: **Comparison of exponential decay constants λ for different values of material parameter δ and pulse frequency ω. The values of ω used here are also indicated in Fig. 2 b by the hollow symbols.**

δ	t_m	ω	$\lambda(\omega)$	λ_{num}	$\lvert\lambda(\omega) - \lambda_{\text{num}}\rvert$
0.2	1.40	0.7	0.15	0.31	0.16
	0.50	2.0	0.49	0.36	0.13
	0.20	5.0	0.76	0.39	0.38
0.8	1.40	0.7	0.034	0.069	0.062
	0.50	2.0	0.087	0.082	0.005
	0.20	5.0	0.107	0.091	0.016

5 Conclusions

In this paper a novel viscoelastic wave equation of felt type material was presented. The equation was derived on the basis of experimental research conducted earlier [1, 2, 14, 15, 16]. Using the linear form of this wave equation, the BVP that describes the propagation of strain waves in the felt type material was studied.

With the aid of complex valued dispersion relation it was shown that the loss mechanism, associated with the decay of harmonic Fourier spectral components, in linear felt may entail up to three different dissipation regimes. It was concluded that more precise and detailed understanding of the dissipation and underlying loss mechanisms requires additional theoretical and experimental investigations.

The numerical solution of the linear BVP was used to estimate a strain pulse amplitude decay during its propagation through the felt. It is shown that in the linear case the exponential decay rates may be obtained rather accurately by use of dispersion analysis. In conclusion, we state that the wool felt is a strongly dissipative and dispersive medium, with a strong damping effect on any wave propagating through it.

Acknowledgements

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