

Analytical and Numerical Insights into Deep Water Waves and Waves Over a Variable Bottom

Päivo Simson^{*}, Dmitri Kartofelev^{*} and Andrus Salupere^{*}

We study two-dimensional, time-dependent deep water waves and waves over a variable bottom using the conformal mapping method, along with approximate equations formulated in physical variables.^{1 2} A major challenge in water wave theory is that the shape of the free surface evolves dynamically, making direct analysis difficult. The conformal mapping approach simplifies this problem by transforming the physical domain into a strip of constant depth, allowing for efficient computations and deeper theoretical insights.

Beyond the numerical applications commonly explored in previous studies,^{3 4 5} we derive and analyze approximate equations that effectively describe complex deep water wave behavior, including the formation of long-lived nonlinear structures such as breathers.⁶ Our results highlight differences between deep and shallow water waves, where deep water dynamics are governed by more intricate interactions that influence extreme wave formation and energy concentration mechanisms that are fundamentally different from that of shallow water waves.

To complement our analytical findings, we develop a numerical solver that combines fast Fourier transforms with finite difference methods. The solver is stable and accurately captures key wave phenomena, confirming the validity of our theoretical approach. Additionally, our study establishes new connections to recent findings on deep water waves, particularly in relation to breathers, providing further insight into the mechanisms underlying energy transport and rogue wave formation.

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^{*}Laboratory of Solid Mechanics, Department of Cybernetics, School of Science, Tallinn University of Technology, Ehitajate Rd. 5, 19086 Tallinn, Estonia

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