PIANO HAMMER-STRING INTERACTION: THE INFLUENCE OF THE ELASTIC PARAMETERS OF BASS HAMMERS ON THE CONTACT TIME DURATION

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Abstract: The influence of the elastic parameters of the bass hammers on the contact time duration is analyzed by physics-based mathematical modeling of the hammer string interaction. Two models that accomplish this with numerical calculation are described, and sample results are presented. The first model is based on the nonlinear hysteretic model of piano hammer that is in a good agreement with experimental data, and gives the possibility to calculate the motion of strings and hammers, and to simulate the piano hammer-string interaction. The second model takes into account that the mass of bass hammer is much less than the mass of the string, and it makes possible to consider the hammer-string interaction as a hammer strike against a rigid surface. According to these models (and in reality), the contact time duration of a hammer strike decreases with increasing of the hammer velocity. It means that the speed of a compression wave, traveling from the contact point to the hammer kernel and back, increases with the growth of its amplitude. Both models give similar predictions for the duration of the hammer strike.

1 Introduction

The process of the piano string excitation by striking with a hammer has been under investigation more than a hundred years. There are many studies devoted to this problem. This paper will concern only the contact time duration of the hammer-string interaction, which is one of the important characteristics of the sound formation. It was shown [1] that the mode energy spectrum of the vibrating string may be expressed entirely through the acting force $F(t)$ determined from the hammer compression, and the contact time t_0 according to

$$A_n + jB_n = j\left(\frac{2\sin n\pi\alpha}{n\pi c\mu}\right)\int_{-\infty}^{t_0} F(t)e^{j\omega_n t}dt, \quad (1)$$

where A_n and B_n are Fourier amplitudes, μ is the string density, c is the speed of transverse waves, α denotes the position of the striking point, and ω_n are the normal mode frequencies. This result gives the basis for the statement that the sound generated by an excited string strictly depends on the interaction time between the hammer and the string.

A central point of many papers in the past was the problem of the contact time duration between the hammer and the string, and the discussion on what can cause the hammer to rebound. In [2] this problem is clarified, and using the nonlinear hysteretic hammer felt model it was shown that the bass hammers, which are relatively light compared to the string, may lose string contact due to its elasticity, and without the assistance of waves traveling along the string and reflected from the agraffe.

In this paper we will consider the influence of the elastic parameters of the bass hammers on the contact time duration. For this purpose we will use and compare two different descriptions of the process of the hammer impact. The first method is based on the hysteretic hammer model, and the second one is based on the wave approach of the hammer compression.

2 Hysteretic model of piano hammer

The piano hammer felt made of wool is a unique and indispensable coating matter of wooden mallets. Experimental testing of piano hammers, which consist of a wood core covered by several layers of compressed wool felt demonstrates, that all hammers have a hysteretic type of the force-compression characteristics, which are essentially nonlinear [3, 4]. A main feature of hammers is that the slope of the force-compression characteristic is strongly dependent on the rate of loading (see Figure 1). These phenomena demonstrate that the piano hammer felt made of wool is a microstructured material possessing history-dependent properties, i. e. a material with memory. The nonlinear, hysteretic models of the piano hammer which are in a good agreement with experimental data were presented in [4, 5].

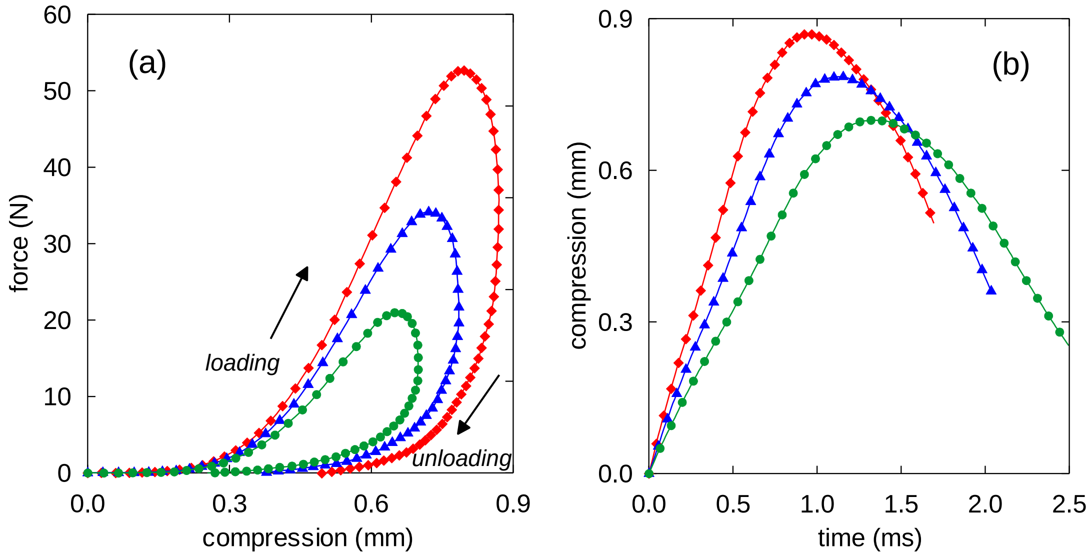

Figure 1. Numerical simulation of a hammer strike against a rigid surface [4]; (a) force-compression characteristics, and (b) compression histories. The symbols denote the data for hammer striking velocities 1.3m/s (diamonds), 1.0m/s (triangles), and 0.75m/s (bullets).

According to a four-parameter hereditary model the nonlinear force $F(t)$ exerted by the hammer is related to the felt compression $u(t)$ by the following expression:

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right]. \tag{2}$$

Here instantaneous hammer stiffness F_0 and compliance nonlinearity exponent p are the elastic parameters of the felt, and ε and τ are the hereditary parameters. Continuous variations in the hammer parameters vs. key number n were obtained by numerical simulation of the experimental data. In short, the impact of the hammer can be described by the equation of motion

$$m\frac{d^2u}{dt^2} + F(u) = 0\,, \tag{3}$$

with the initial conditions $u(0) = 0$, $du/dt\,(0) = V$. Here m and V are the hammer mass and the striking velocity, respectively, and $F(u)$ is defined by formula (2). The contact time durations of hammer strike calculated for hammer numbers n = 1, 5, 10, and for various hammer striking velocities are presented in Figure 2.

3 Compression model of hammer strike

For the estimation of a contact time of a hammer strike, we must take into account two important features of an impact. Usually the striking velocity of piano hammer V is not greater than 5 m/s, which is much less than the speed of compression wave c in the felt material. Besides, the mass of bass hammers is significantly smaller than the mass of the corresponding string. For these reasons we can propose another model of the hammer-string interaction.

For slow loading the nonlinear force $F(t)$ exerted by the hammer, as it was shown in [4], instead of formula (2), is given by

$$F(u(t)) = F_0(1-\varepsilon)\, u^p(t) = Q_0\, u^p(t)\,, \tag{4}$$

where Q_0 is the static hammer stiffness.

During the strike the initial energy of the hammer is equal to the sum of kinetic and potential energy of the deformed hammer. According to the energy conservation law, we have

$$\frac{mV^2}{2} = \frac{m}{2}\left(\frac{du}{dt}\right)^2 + \frac{Q_0}{p+1}u^{p+1}. \tag{5}$$

The maximum compression corresponds to the moment, when $du/dt = 0$, and that yields

$$u_{\max} = \left(\frac{p+1}{2}\frac{m}{Q_0}V^2\right)^{\frac{1}{p+1}}. \tag{6}$$

The contact time can be found by integration of equation (5), and is given by

$$t_0 = \frac{2u_{\max}}{V}\int_0^1 \frac{dx}{\sqrt{1-x^{p+1}}} = \frac{2b\sqrt{\pi}}{V}\frac{\Gamma(1+a)}{\Gamma(1+b)}u_{\max}\,. \tag{7}$$

Here $a = (p+1)^{-1}$, $b = a + ½$, and $\Gamma(x)$ is the Gamma function.

In case of linear hammer ($p = 1$), we have

$$u_{\max} = V\sqrt{\frac{m}{Q_0}}\,; \qquad t_0 = \pi\sqrt{\frac{m}{Q_0}}\,, \tag{8}$$

thus the maximum hammer compression is proportional to the hammer velocity, but the contact time duration does not depend on the rate of loading.

4 Two models comparison and conclusions

The contact time of strike as function of hammer velocity calculated in according to formula (7) for various bass hammers is also presented in Figure 2. The set of hammer parameters was approximated using [4]

$$\begin{aligned} &m = 11.074 - 0.074n + 10^{-4}n^2\,; \quad p = 3.7 + 0.015n\,; \quad Q_0 = 183\exp(0.045n); \\ &\varepsilon = 0.9894 + 8.8\times10^{-5}n^2; \quad \tau = 2.72 - 0.02n + 9\times10^{-5}n^2, \end{aligned} \tag{9}$$

for all hammer numbers $1 \le n \le 88$. Here the dimension of parameter τ is [μs], for m is [g], and for Q_0 is [N/mm^p].

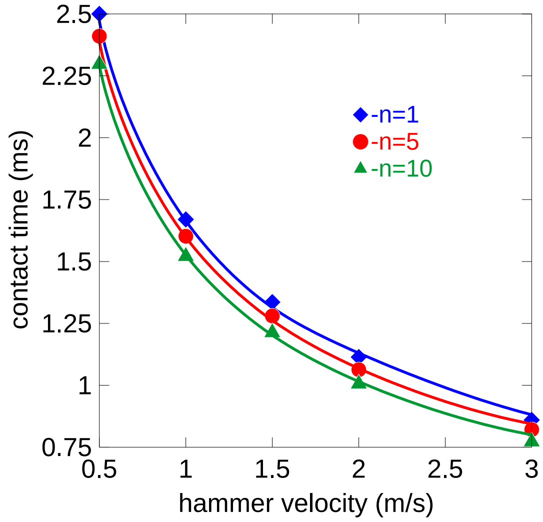

Figure 2. Contact times as functions of hammer velocity. The symbols denote the data obtained for hysteretic hammer, using equations (2, 3). The solid lines are obtained for compression model, using equations (6, 7).

It is evident that two different models give similar results for duration of the hammer strike. Moreover, the maximum hammer compression predicted by both models is also almost the same.

According to these models (and in reality), the contact time duration of a hammer strike decreases with increasing of the hammer velocity. It is evident that the speed of a compression wave, traveling from the contact point to the hammer kernel and back, increases with the growth of its amplitude, and this dependence is proportional to $V^{2/p+1}$.

The results obtained give a possibility to estimate the speed of compression wave in the hammer felt material for slow loading. Taking into account the thickness of the bass hammer felt, which is equal to 15 - 17 mm approximately, we can say that the value of the compression wave speed may be in interval 25 - 75 m/s. This is a very small speed for an elastic material, but this is one of many amazing features the piano hammer felt that is a miraculous matter indeed.

Acknowledgements

This project was supported by Arcimedes Foundation and DoRa Scholarship.

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