Algorithmic melody composition based on fractal geometry of music

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Brief history of fractal music

570 BCE Pythagoras believed that numbers are the source of music.

1026 Guido d'Arezzo created algorithmic music.

1815 - 52 Ada Lovelace worked with Charles Babbage the creator of the first programmable computer (difference engine).

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> "Supposing, for instance, that the fundamental relations of pitched sound in the signs of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent." – Ada Lovelace's notes 1851

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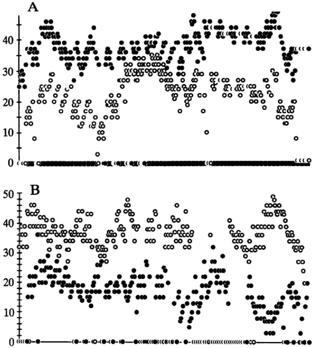
Self-similarity of music [1]



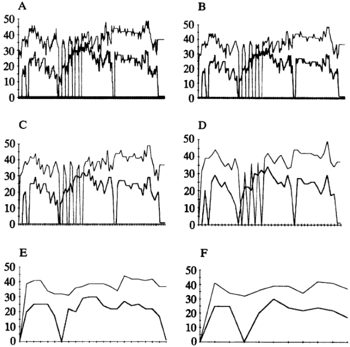


Figure: Scores of Bach's inventions no. 1 and 10. ○ right hand, • left hand. Fractal reduction of Bach's invention no. 1. The 1/2, 1/4. 1/8, 1/16, 1/32 reductions of the scores, respectively.

[1] K. J. Hsü, A. Hsü, "Self-similarity of the '1/f noise' called music," Proc. Natl. Acad. Sci., vol. 88, pp. 3507—3509, 1991.

Fractal geometry of musical scores [2]

Note interval i fluctuations

$$f_{n+1} = \sqrt[12]{2}\; f_n \qquad (1)$$

$$f(i) = \sqrt[12]{2^i} \qquad (2)$$

[2] K. J. Hsü, A. Hsü, "Fractal geometry of music," Proc. Natl. Acad. Sci., vol. 87, pp. 938—941, 1990.

Fractal geometry of musical scores [2]

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$$f_{n+1} = \sqrt[12]{2}\ f_n \qquad (1)$$

$$f(i) = \sqrt[12]{2^i} \qquad (2)$$

Occurrence frequency of interval i follows the inverse power law:

$$f(i) = \frac{c}{i^D} \qquad (3)$$

$$\log(f) = C - D\log(i) \qquad (4)$$

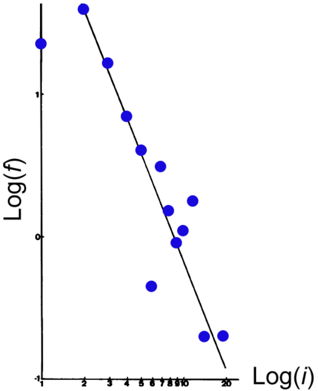

Figure: Fractal geometry of note frequency. Bach BWV 772.

[2] K. J. Hsü, A. Hsü, "Fractal geometry of music," Proc. Natl. Acad. Sci., vol. 87, pp. 938—941, 1990.

Fractal geometry of loudness fluctuations [3]

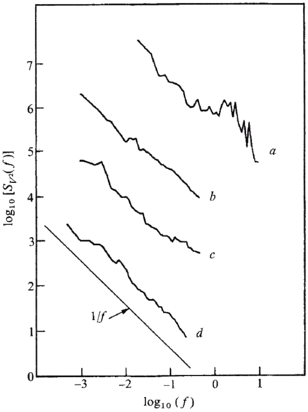

Figure: Loudness fluctuation spectra for a) Scott Joplin Piano Rags, b) classical radio station, c) rock station, d) news and talk station.

[3] R. V. Voss and J. Clarke, "'1/f noise' in music and speech," Nature, vol. 258, pp. 317—318, 1975.

Fractal geometry of pitch fluctuations [4] [5]

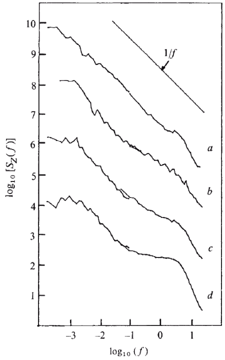

Figure: Pitch fluctuation spectra for a) classical, b) jazz, blues, c) rock, d) news and talk radio station.

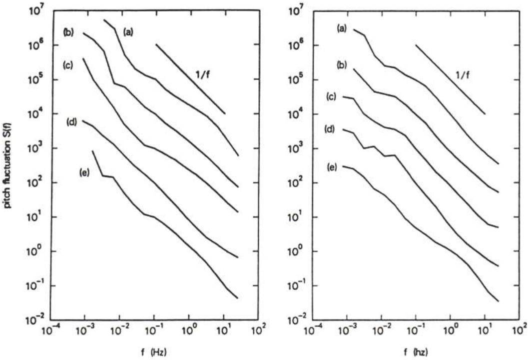

Figure: **Left:** a) Babenzele Pygmies b) Japanese traditional c) Indian classical. d) Russian folklore e) USA blues **Right:** a) Medieval music b) Beethoven's 3. symphony c) Debussy piano d) Strauss, Ein Heldenleben e) The Beatles, Stage Pepper

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[4] R. V. Voss and J. Clarke, "'1/f noise' in music and speech," Nature, vol. 258, pp. 317—318, 1975.

[5] K. J. Hsü, "Applications of fractals and chaos," Springer-Verlag, pp. 21—39, 1993.

Fractal geometry of musical rhythm [6]

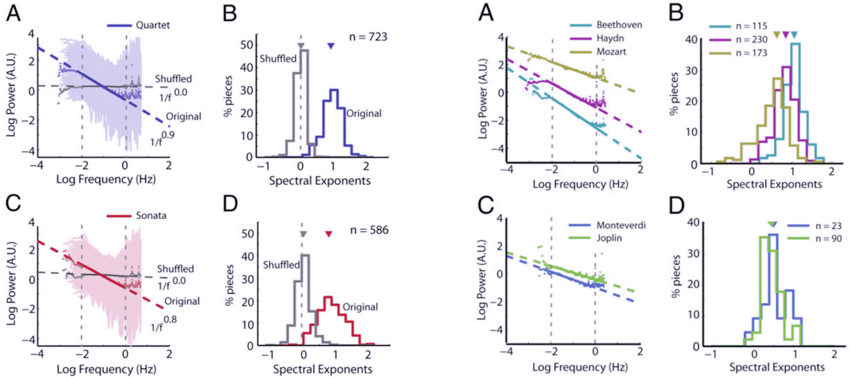

Figure: The $1/f^D$ rhythm spectra are ubiquitous across genres. Analysis of 558 compositions spanning over a period of 4 centuries.

[6] D. J. Levitina, P. Chordiab, and V. Menonc, "Musical rhythm spectra from Bach to Joplin obey a 1/f power law," PNAS, vol. 109, no. 10, pp. 3716—3720, 2012.

Music as fractal

- All components of human composed music (melody, harmony, rhythm, loudness) have a fractal geometry.
- Fractal dimension D of musical time series can have values in the interval $(0.5, 2)$.[7]
- Fractal geometry of human music can be exploited for the purpose of the algorithmic music composition.

[7] M. Bigerelle, A. Iost, "Fractal dimension and classification of music," Chaos, Solitons and Fractals, vol. 11, pp. 2179—2192, 2000.

Fractal music composition: main idea

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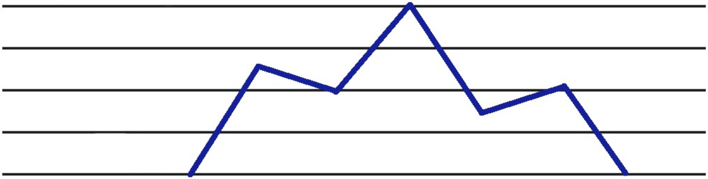
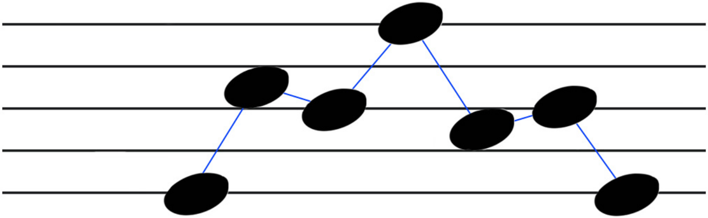
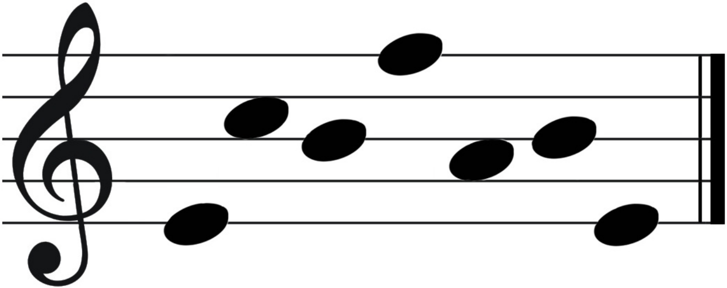


Fractal music composition: main idea



Fractal music composition: main idea



Logistic map

Logistic map is in the form

$$y_{n+1} = r y_n (1 - y_n), \qquad (5)$$

where r is parameter that has values in the interval $(0, 4]$

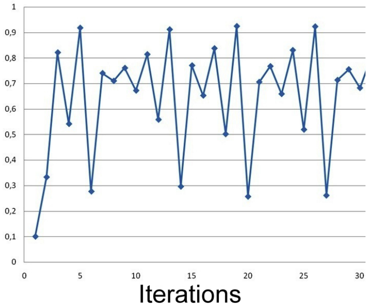


Figure: Logistic map, where $y_0 = 0.1$ and $r = 3.71$

Audio example of the logistic map (15 s)

$1/f$ noise generator (pink noise)

$1/f$ noise generator is in the form

$$y_{n+1} = my_n + k\sqrt{1 - m^2}, \quad (6)$$

where m is in the interval $[0, 1]$ and k is random number.
For music generation value of k is taken from logistic map, where parameter $r = 4$.

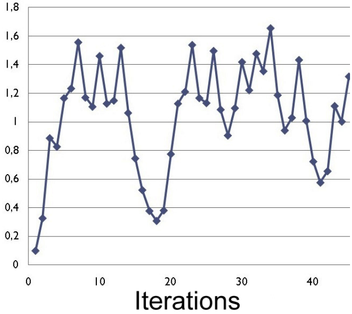


Figure: $1/f$ noise where $x_0 = 0.1$ and $m = 0.7$

Audio example of the pink noise generator (15 s)

Lorenz fractal

Lorenz fractal is in the form

$$y_{n+1} = a(3y_n - 4y_n^3), \quad (7)$$

where parameter a is in the interval $[0, 1]$.
For the music generation best vales of the parameter a are $[0.65, 1]$.

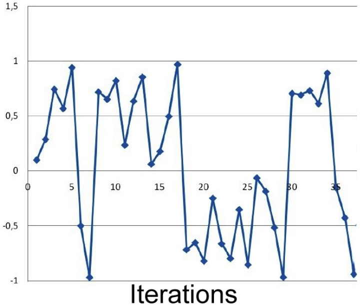


Figure: Lorenz fractal where $a = 0.97$ and $y_0 = 0.1$

Audio example of the Lorenz fractal (15 s)

Henoni fraktal

Henon fractal is in the form

$$\begin{cases} x_{n+1} = 1 + y_n - a x_n^2 \\ y_{n+1} = b x_n \end{cases} \quad (8)$$

For music generation, $a = 1.4$ and $b = 0.3$

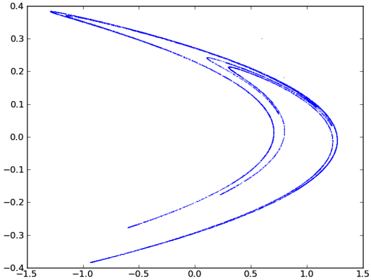


Figure: Henon fractal where $a = 1.4$, $b = 0.3$, $x_0 = y_0 = 1$, 10^4 iterations

Audio example of the Henon fractal (15 s)

Hopalongi fraktal

Hopalong fractal is in the form

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sqrt{|bx_n - c|} \\ y_{n+1} = a - x_n, \end{cases} \tag{9}$$

where a, b ja c are the control parameters.

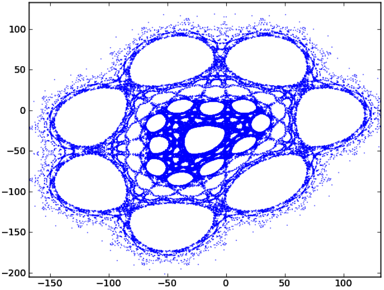


Figure: Hopalong fractal where $a = -55$, $b = 17$, $c = -21$, $x_0 = y_0 = 0$, $5 \cdot 10^4$ iterations

Audio example of the Hopalong fractal (15 s)

Gingerbread man fractal

Gingerbread man fractal is in the form

$$\begin{cases} x_{n+1} = 1 - y_n + |x_n| \\ y_{n+1} = x_n \end{cases} \quad (10)$$

For music generation we select $x_0 = -0.1$ and $y_0 = 0$.

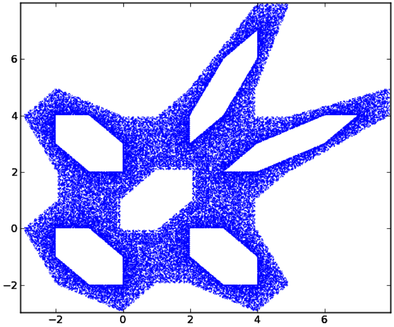


Figure: Gingerbread man fractal where $x_0 = -0.1$, $y_0 = 0$, $5 \cdot 10^4$ iterations

Audio example of the gingerbread man fractal (15 s)

L-system (Lindenmayer system)

Formal grammar developed by Aristid Lindenmayer.

Rules: P1: $a \to ab$

P2: $b \to a$

Axiom: b

$$\begin{aligned} n = 0 :& \quad b \\ n = 1 :& \quad a \\ n = 2 :& \quad ab \\ n = 3 :& \quad aba \\ n = 4 :& \quad abaab \\ n = 5 :& \quad \underline{abaababa} \end{aligned}$$

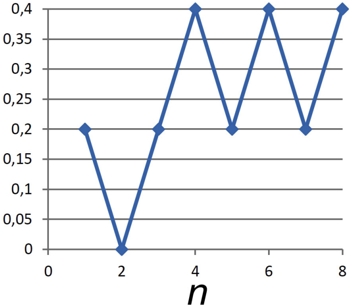
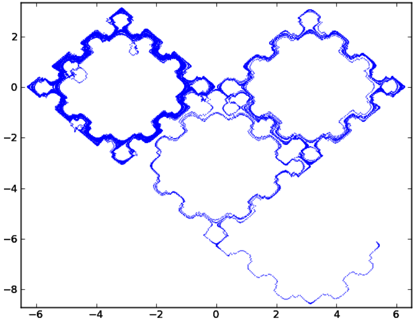


Figure: Interpretation: $a = 0.2$ and $b = -0.2$

Audio example of the L-system fractal (15 s)

Other musical fractals: Popcorn fractal

$$\begin{cases} x_{n+1} = x_n - h\sin(y_n + \tan 3y_n) \\ y_{n+1} = y_n - h\sin(x_n + \tan 3x_n) \end{cases} \tag{11}$$



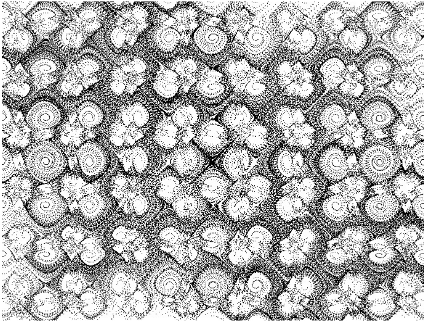


Figure: Popcorn fractal where $h = 0.05$, $x_0 = -0.1$, $y_0 = 0$, $5 \cdot 10^4$ iterations.

Other musical fractals: Quadrup-Two

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sin(\ln |b(x_n - c)|) \tan^{-1} |c(x_n - b)|^2 \\ y_{n+1} = a - x_n \end{cases} \quad (12)$$

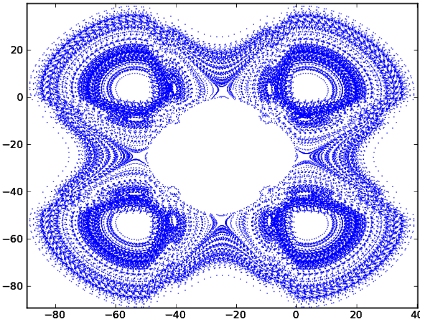


Figure: Quadrup-Two fractal where $a = -50$, $b = -1$, $c = -41$, $x_0 = -1$, $y_0 = 1$, $5 \cdot 10^4$ iterations

Other musical fractals: Mira fractal

$$\begin{cases} x_{n+1} = by_n + ax_n + \frac{2(1-a)x_n^2}{1+x_n^2} \\ y_{n+1} = -x_n + ax_{n+1} + \frac{2(1-a)x_{n+1}^2}{1+x_{n+1}^2} \end{cases} \quad (13)$$

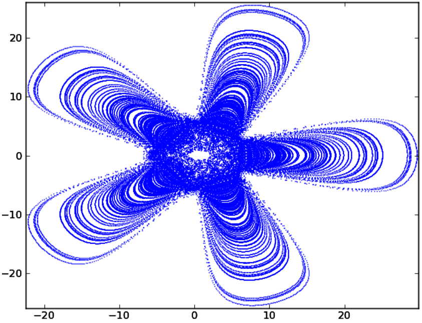


Figure: Mira fractal where $a = 0.31$, $b = 1$, $x_0 = 12$, $y_0 = 0$, $5 \cdot 10^4$ iterations

Other musical fractals: Hopalong 2 fractal

$$\begin{cases} x_{n+1} = y_n + \operatorname{sgn} x_n |b x_n - c| \\ y_{n+1} = a - x_n \end{cases} \tag{14}$$

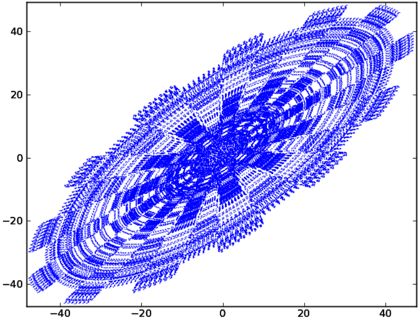


Figure: Hopalong 2 fractal where $a = 0.6$, $b = 1.5$, $c = -2.5$, $x_0 = y_0 = 1$, $5 \cdot 10^4$ iterations

Other musical fractals: Hopalong 3 fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sqrt{|x_n \sin a - \cos a|} \\ y_{n+1} = a - x_n \end{cases} \tag{15}$$

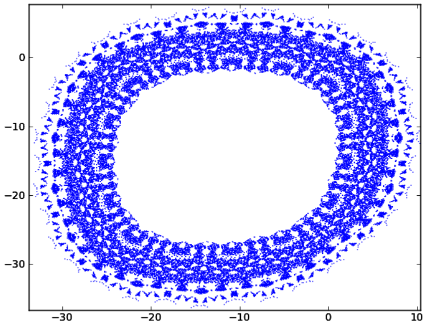


Figure: Hopalong 3 fractal where $a = -26$, $x_0 = y_0 = 0$, $5 \cdot 10^4$ iterations

Examples of fractal music

Using aforementioned knowledge and methods *the engine* (computer) *might compose elaborate and scientific pieces of music of any degree of complexity or extent.*

Example composition no. 1 (duration 40 s)

Example composition no. 2 (duration 40 s)

Example composition no. 3 (duration 40 s)

Why it works? (possible connections)

- Natural phenomena, Richardson Effect.[8]

[8] K. J. Hsü, 1983.

[9] Y. Yu, R. Romero, and T. S. Lee, 2005.

[10] M. A. Schmuckler, D. L. Gilden, 1993.

[11] T. Musha, 1997.

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- Human behaviour (clapping of hands).[11]
- Biological processes (heart rate fluctuations, pupil diameter and focal accommodation, instantaneous period fluctuations of the organ Alpha-rhythms, self-discharge and action potential impulses of neurons, etc.).[11]

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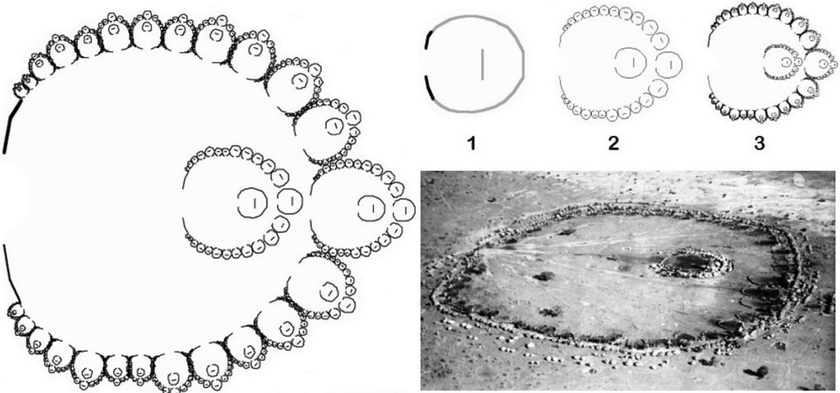
Why it works? (possible connections)

Self-similarity on different scales. Social behaviour.



Fractal generation of Ba-ila simulation. First iteration is similar to single house, second is similar to family ring, third to village as whole.

Figure: Ba-Ila fractal village plan.

Conclusions

- Music composed by humans has a fractal geometry (can be described by a fractional dimension).
- Main ideas and methods behind the fractal and algorithmic music composition were presented.
- Some examples of the musical fractals where presented and discussed.

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