Revision questions

Following are the questions and topics to consider while preparing for the final exam. Similar non-verbatim questions will be used in the exam.

Lecture 1

- 1. What is dynamics?
- 2. Name a dynamical system.
- 3. Define nonlinearity.
- 4. Determine if the following equations/systems are linear or nonlinear:

$$\dot{x} = \sin x, \tag{1}$$

$$\dot{x} = \ln x, \tag{2}$$

$$\begin{cases} \dot{x} = y, \\ \dot{y} = xy, \end{cases}$$
(3)

$$\ddot{x} + \dot{x} + x = 0. \tag{4}$$

- 5. What is ordinary homogeneous differential equation?
- 6. Define 1-D dynamical system. Name a 1-D problem.
- 7. What is phase space?
- 8. What is phase portrait?
- 9. Sketch 1-D phase portrait of the following systems:

$$\dot{x} = 3\cos x,\tag{5}$$

$$\dot{x} = 0.5x^2 - 1,\tag{6}$$

$$\dot{x} = x^3. \tag{7}$$

- 10. What is a fixed point?
- 11. How to a find fixed point of a differential equations?
- 12. Find the fixed point or points of the following system:

$$\ddot{x} + \dot{x} + x = 0. \tag{8}$$

- 13. Explain fixed point stability.
- 14. What is linear analysis of a system?
- Lecture 2 15. What does linearisation of a nonlinear system imply?
- 16. Linearise the following 1-D system

$$\dot{x} = x^3 - x \tag{9}$$

17. What is bifurcation?

- 18. What is bifurcation diagram?
- 19. What is saddle-node bifurcation?
- 20. What is transcritical bifurcation?
- 21. What is pitchfork bifurcation?
- 22. What is supercritical pitchfork bifurcation?
- 23. What is subcritical pitchfork bifurcation?
- 24. What is normal form in the context of bifurcations?
- 25. Are oscillation possible in 1-D systems?
- 26. Why are oscillations impossible in 1-D systems?
- 27. What does uniqueness of solutions imply in the context of phase space trajectories?

Lecture 3

- 28. What is symmetry-broken solution?
- 29. What is dimensional analysis of an equation of motion?
- 30. What is dimensionless form of an equation?
- 31. What is normalised form of an equation?
- 32. How many initial conditions does first-order ODE have?
- 33. How many initial conditions does secondorder ODE have?
- 34. Explain the notion of different time scales of a dynamical system.
- 35. Derive the dimensionless form of the following equation of motion:

$$m\frac{\mathrm{d}^2 u}{\mathrm{d}t^2} + b\frac{\mathrm{d}u}{\mathrm{d}t} + ku = 0, \qquad (10)$$

where u is the displacement, m is the mass and t is the time. Additionally, determine the dimensions of damping coefficient b and stiffness k.

36. Derive the dimensionless form of the following nonhomogeneous equation of motion:

$$m\frac{\mathrm{d}^2 u}{\mathrm{d}t^2} + b\frac{\mathrm{d}u}{\mathrm{d}t} + ku = F_0 \cos\omega_0 t, \quad (11)$$

where u is the displacement, m is the mass, F_0 and ω_0 are the driving force parameters, and t is the time. Additionally, determine the dimensions of damping coefficient b, stiffness k, driving force F_0 and driving force frequency ω_0 .

Lecture 4

- 37. How to plot a 2-D phase portrait of a system?
- 38. What are 2-D homogeneous linear systems?
- 39. What are non-homogeneous systems?
- 40. Classification of fixed points in 2-D systems.
- 41. Sketch a saddle node fixed point.
- 42. Sketch a stable node fixed point.
- 43. Sketch an unstable node fixed point.
- 44. Sketch a stable spiral (fixed point).
- 45. Sketch an unstable spiral (fixed point).
- 46. Sketch a center (fixed point).
- 47. Sketch a stable non-isolated fixed point.
- 48. Sketch an unstable non-isolated fixed point.
- 49. What are 2-D homogeneous nonlinear systems?
- 50. What does it mean that a fixed point is Lyapunov stable?
- 51. Give an example of Lyapunov stable fixed point.

Lecture 5

- 52. Provide an example of nonlinear 2-D system.
- 53. Explain linearisation of 2-D systems about fixed points.
- 54. Can all nonlinear systems be linearised with the aim of identifying their fixed point type?
- 55. Linearise the following system

$$\begin{cases} \dot{x} = 4x - 4xy, \\ \dot{y} = -9y + 18xy. \end{cases}$$
(12)

56. Without taking derivatives, linearise the following systems:

$$\begin{cases} \dot{x} = -y + xy, \\ \dot{y} = x, \end{cases}$$
(13)

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = x + y^2. \end{cases}$$
(14)

57. Define the Jacobian matrix of a system.

- 58. Sketch a homoclinic orbit.
- 59. Define conservative dynamical system.

Lecture 6

- 60. Expand on the connection between 2-D conservative systems and centres.
- 61. Sketch a heteroclinic orbit.
- 62. What is limit-cycle?
- 63. Sketch a stable limit-cycle.
- 64. Sketch an unstable limit-cycle.
- 65. Sketch a half-stable (stable from outside) limit-cycle.
- 66. Sketch a half-stable (stable from inside) limit-cycle.
- 67. Define and sketch a null-cline.
- 68. What is the Dulac's criterion?
- 69. State the Poincaré-Bendixson theorem.
- 70. Does the Poincaré-Bendixson theorem apply to 3-D systems?
- 71. Can chaos occur in 2-D systems?

Lecture 7

- 72. Classification of bifurcations in 2-D.
- 73. What is the Hopf bifurcation?
- 74. What is the supercritical Hopf bifurcation?
- 75. What is the subcritical Hopf bifurcation?
- 76. What are global bifurcations of closed orbits?
- 77. Name some global bifurcations of closed limit-cycles.
- 78. What is a saddle-node coalescence (or bifurcation) of limit-cycles?
- 79. What is hysteresis on level of cycles?
- 80. Name dangers associated with the *Hopf* bifurcation.
- 81. What is a saddle-node infinite period bifurcation?
- 82. What is a (saddle-loop or) homoclinic bifurcation?
- 83. Name examples of dynamical instabilities.

Lecture 8

- 84. What is quasi-periodicity?
- 85. Can quasi-periodic system generate a chaotic solution? Why?
- 86. Do limit-cycles exist in 3-D phase spaces? Sketch an example.
- 87. What are 3-D and higher order systems?
- 88. What is chaos in the context of dynamical systems (deterministic chaos, chaos the-

ory)?

- 89. Name properties of chaotic systems.
- 90. What does it mean that a chaotic system has a SRB measure (the Sinai-Ruelle-Bowen measure)?
- 91. What is chaotic water wheel?
- 92. What is the Lorenz attractor?

Lecture 9

- 93. Define attractor.
- 94. Define strange attractor.
- 95. What is the difference between a strange attractor and an attractor?
- 96. Name properties of the Lorenz attractor.
- 97. What are the Lyapunov exponents?
- 98. What is the Lyapunov exponent?
- 99. What determines the number of Lyapunov exponents?
- 100. What is the Kolmogorov entropy?
- 101. What is predictability horizon?
- 102. What is the Lyapunov time?
- 103. Can a long-term solution to a chaotic system be predicted? Explain.
- 104. List some examples of chaos in nature.
- 105. What is final-state sensitivity?
- 106. What is chaos?
- 107. What is intermittent chaos?
- 108. What is transient chaos?
- 109. What is crisis?
- 110. What is strange non-chaotic attractor?

Lecture 10

- 111. What is cobweb diagram?
- 112. What is recurrence map or recurrence relation?
- 113. What is 1-D map?
- 114. How to find fixed points of 1-D maps?
- 115. What is the Lorenz map?
- 116. What is the logistic map?
- 117. What is sine map?
- 118. What is period doubling?
- 119. What is period doubling bifurcation?
- 120. What is tangent bifurcation?
- 121. Do odd number periods (period-p orbits) exist in chaotic systems?
- 122. Do even number periods (period-p orbits) exist in chaotic systems?
- 123. Can maps produce transient chaos?
- 124. Can maps produce intermittency?
- 125. Can maps produce intermittent chaos?
- 126. What is orbit diagram (or the Feigenbaum

diagram)?

127. What are the Feigenbaum constants?

Lecture 11

- 128. What are the values of the Feigenbaum constants?
- 129. What are the Feigenbaum constants (more in-depth answer)?
- 130. Define superstable fixed point of a map.
- 131. Define superstable period-p point (or period-p cycle) of a map.
- 132. What are the universals of unimodal maps?
- 133. What is the universal route to chaos?
- 134. Idea behind renormalisation?
- 135. What are the universal limiting functions in the context of maps?
- 136. Name discrete-time dynamics analysis methods.
- 137. What is the Poincaré section?
- 138. What is the Poincaré map (return map)?
- 139. What is the Lorenz section?

Lecture 12

- 140. How is it possible for two trajectories with almost equal initial conditions to deviate exponentially and remain attracted to a strange attractor (remain in the basin of attraction)?
- 141. Give an example of a dynamics that features global stability and local instability.
- 142. Explain fractal microstructure of strange attractors.
- 143. Define fractal.
- 144. What is pre-fractal?
- 145. Construct a simple fractal (general idea).
- 146. What is self-similarity?
- 147. What is scale-invariance?
- 148. Are all fractals self-similar?
- 149. What is fractal geometry?
- 150. What is fractal dimension?
- 151. What are similarity and box counting dimensions?
- 152. What is a power law?
- 153. What is the Cantor set?
- 154. What is the von Koch curve?
- 155. What is a 2-D map?
- 156. How to find fixed points of 2-D maps (period-1 point)?
- 157. What is the Hénon map, its significance?

Lecture 13

- 158. Is it possible to see stretching–folding–reinjection dynamics in cobweb plots?
- 159. What does linearisation of a nonlinear 2-D map imply?
- 160. Define sensitive dependence on initial conditions in maps.
- 161. Define basin of attraction of a map.
- 162. Sketch a saddle fixed point.
- 163. Sketch a stable node (sink) fixed point.
- 164. Sketch an unstable node (source) fixed point.
- 165. What are improper oscillations of map iterates?
- 166. What is the cause of improper oscillation of map iterates in terms of eigenvalues?
- 167. What is the video feedback effect?

Lecture 14

- 168. Define fractal (technical definition).
- 169. Define pre-fractal.
- 170. Explain the coastline paradox.
- 171. Can a coastline be described with Euclidean geometry?
- 172. What determines spectral characteristics of dynamical systems?

- 173. What is a 1-D complex valued map?
- 174. What are the Mandelbrot set and the Fatou sets?
- 175. What is the Julia set?
- 176. Assuming z = x + iy, c = r + is, and $z, c \in \mathbb{C}$, show that map in the form

$$\begin{cases} x_{n+1} = x_n^2 - y_n^2 + r, \\ y_{n+1} = 2x_n y_n + s, \end{cases}$$
(15)

is the real counterpart of the Mandelbrot set.

- 177. What is the physical meaning of the Mandelbrot set?
- 178. What is the physical meaning of the Fatou sets?
- 179. What is the generalised Mandelbrot set also known as the Multibrot set?
- 180. Name an example of self-similar phenomena in nature.

Lecture i

- 181. Name some applications of fractal geometry and chaos theory.
- 182. What is fractal music composition?
- 183. Why does fractal music sound similar to human composed music?

NB! Positively graded coursework is a prerequisite for taking the exam.

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