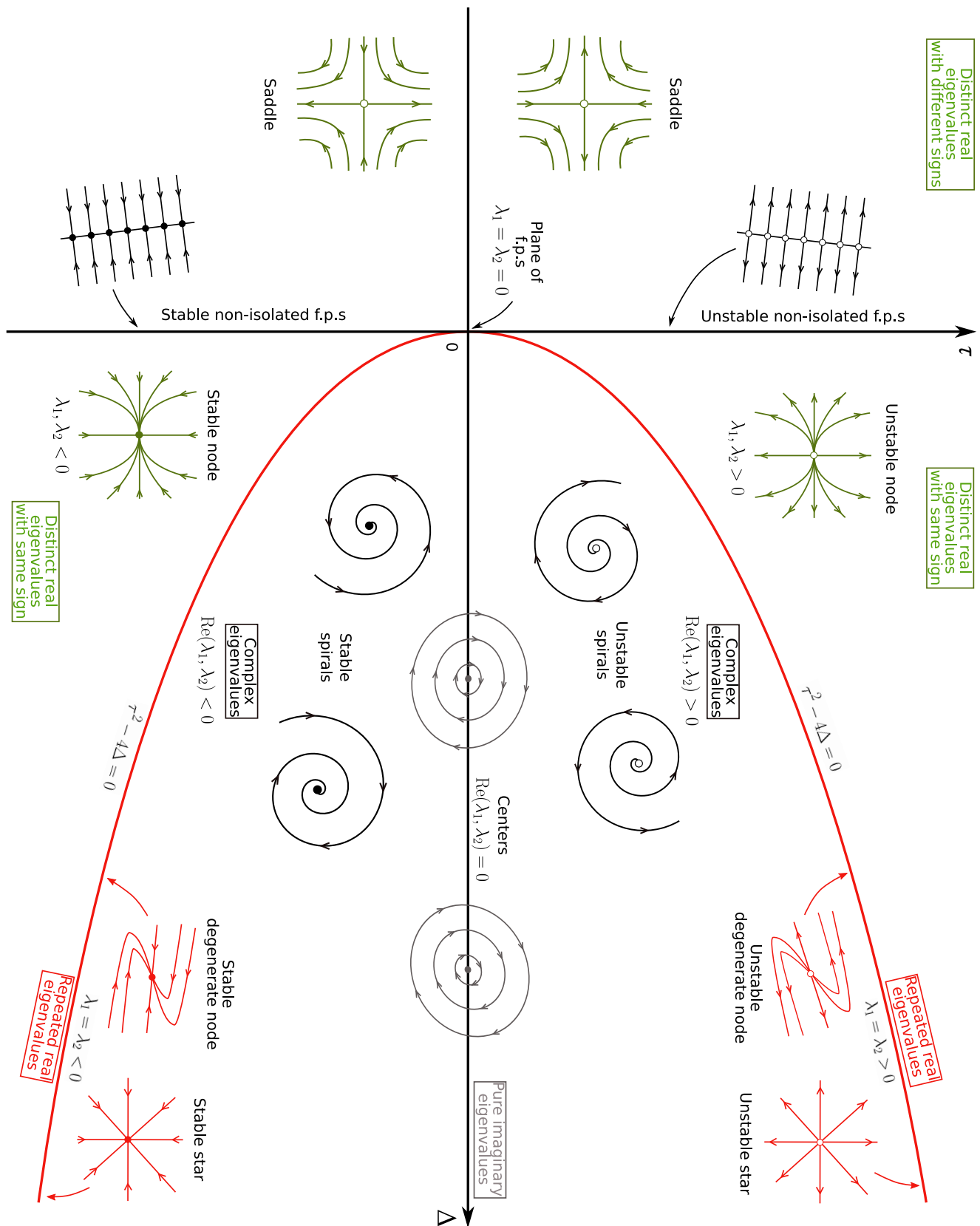


Classification of fixed points in linear homogeneous 2-D systems  $\dot{\vec{x}} = A\vec{x}$  where trace  $\tau = \lambda_1 + \lambda_2$  and determinant  $\Delta = \lambda_1\lambda_2$  are determined by the  $2 \times 2$  system matrix  $A$ .



The classification chart presented above can also be summarised in a concise flowchart:

- ▶ if  $\Delta < 0$ :  
Isolated fixed point  
CASE 1: **Saddle point**<sup>1</sup>
- ▶ if  $\Delta = 0$ :  
Non-isolated fixed points
  - if  $\tau < 0$ :  
CASE 5a: **Line of stable fixed points**<sup>2</sup>
  - if  $\tau = 0$ :  
CASE 5b: **Plane of fixed points**<sup>3</sup>
  - if  $\tau > 0$ :  
CASE 5a: **Line of unstable fixed points**<sup>4</sup>
- ▶ if  $\Delta > 0$ :  
Isolated fixed point
  - if  $\tau < -\sqrt{4\Delta}$ :  
CASE 2a: **Stable node**<sup>5</sup>
  - if  $\tau = -\sqrt{4\Delta}$ :
    - if there is one uniquely determined eigenvector (the other is non-unique):  
CASE 4a: **Stable degenerate node**<sup>6</sup>
    - if there are no uniquely determined eigenvectors (both are non-unique):  
CASE 4b: **Stable star**<sup>7</sup>
  - if  $-\sqrt{4\Delta} < \tau < 0$ :  
CASE 2b: **Stable spiral**<sup>8</sup>
  - if  $\tau = 0$ :  
CASE 3: **Centre**<sup>9</sup>
  - if  $0 < \tau < \sqrt{4\Delta}$ :  
CASE 2b: **Unstable spiral**<sup>10</sup>
  - if  $\tau = \sqrt{4\Delta}$ :
    - if there is one uniquely determined eigenvector (the other is non-unique):  
CASE 4a: **Unstable degenerate node**<sup>11</sup>
    - if there are no uniquely determined eigenvectors (both are non-unique):  
CASE 4b: **Unstable star**<sup>12</sup>
  - if  $\sqrt{4\Delta} < \tau$ :  
CASE 2a: **Unstable node**<sup>13</sup>

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General notes:

- ▶ For 2-D linear systems, the above predictions are always accurate.
- ▶ For 2-D nonlinear systems, when the above are used as predictions of nonlinear dynamics:
  - The descriptions are always correct for cases 1, 5, 8, 10, and 13 but can be inaccurate for cases 2, 3, 4, 6, 7, 9, 11, and 12.
  - Ambiguous cases 6, 7, 11, and 12 at least have their stability correctly determined.
  - If the system is conservative, a prediction of case 9 is accurate.