Lecture № (i): An application of nonlinear dynamics and chaos theory: **fractal music**

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TAL TECH
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Lecture outline

1. History, introduction, definitions
2. Pre-fractal geometry of human composed music
3. Composition of fractal music
4. Musical fractals, maps, sequences, etc.
5. Properties of musical fractals
6. Conclusions, inferences, and speculations

1. History

570_b BC Pythagoras believed that numbers are connected to music (formalism: numbers $\rightarrow$ music).

90_b CE Ptolemy believed that music is describable with equations.

1026 Guido d'Arezzo devised a method for creating algorithmic music based on religious literature (liturgies).

15th cent. Canonical music presented in monasteries.

18th cent. Mechanical music box is invented. Box can be *programmed* to play back different music compositions.

1. History

1756_b Mozart and his *Musikalisches Wurfelspiel* – musical dice game.

1815_b Ada Lovelace's journal (considered to be the first computer programmer. She worked with Charles Babbage).

"Supposing, for instance, that the fundamental relations of pitched sound in the signs of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent." – Ada Lovelace's notes 1851

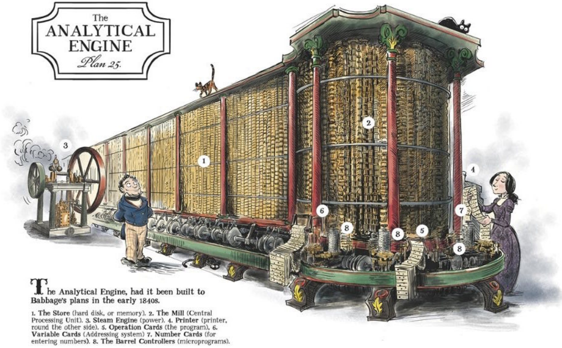


Figure: Ada Lovelace and Charles Babbage's Turing-complete *Analytical Engine* (mechanical computer).

Credit: Sydney Padua (engine drawing).

1. History

- *1944* First electronic digital computer is built (Colossus Mark 1, Colossus Mark 2).
- 1945 Twelve-tone technique also known as dodecaphony or twelve-tone serialism (Arnold Schöenberg).
- 1950 Computer CSIRAC plays back music.
- 1955–57 Computer ILLIAC composes algorithmic music. Result was performed by humans.
- 1990 PC's are able to compute and present complicated musical algorithms.
- 2000 Smartphones are able to compute and present complicated musical algorithms.

1. History

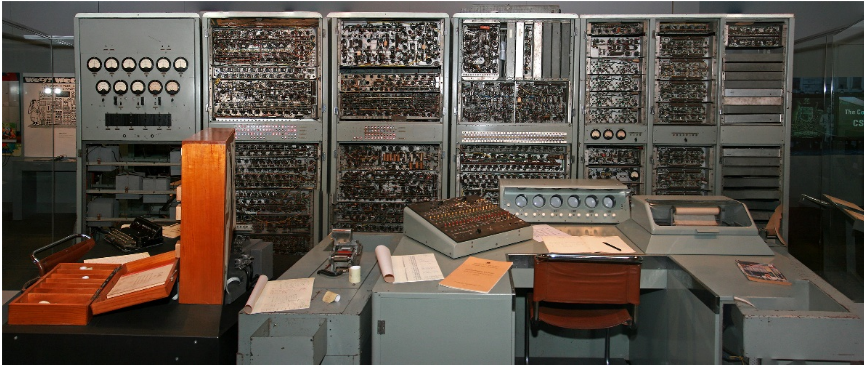


Figure: CSIRAC (Council for Scientific and Industrial Research Automatic Computer) or CSIR Mk 1.

1. Algorithmic score and music composition

- Fractal music is a subset of a machine generated or algorithmic music.
- In this lecture we'll discuss algorithmic music that is generated solely by using fractals or pre-fractals.

1. Fractal

Fractals and pre-fractals feature self-similarity on all or on a finite range of scales...

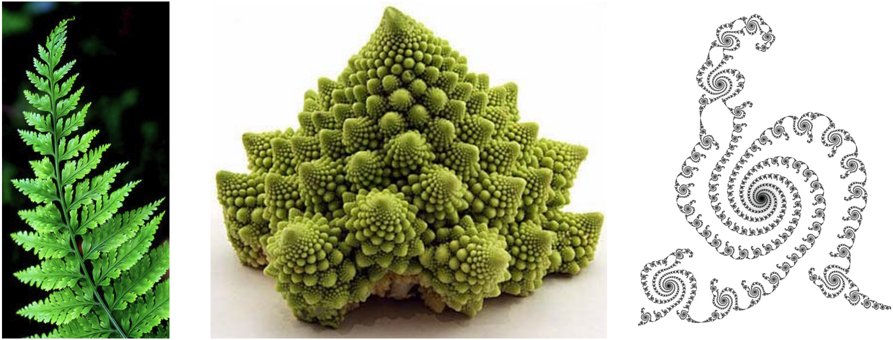


Figure: Fern, Romanesco broccoli, Julia set $z_{n+1} = \dfrac{2 - 5z_n^3}{6z_n^2 - 6z_n - 1}$.

- **Melody**
- **Harmony**
- **Rhythm**
- **Timbre**
- Form
- Tempo
- Dynamics
- Texture
- **Loudness** *fluctuation*
- Lyrics (the Zipf law)

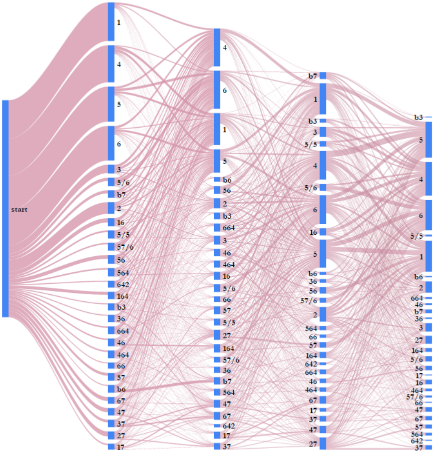


Figure: Chord progression of 5000 compositions (human music).

1. Golden ratio and the Fibonacci sequence

$$\frac{a+b}{a} = \frac{a}{b} = \varphi \approx 1.618 \qquad (1)$$

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144,\ \ldots$$

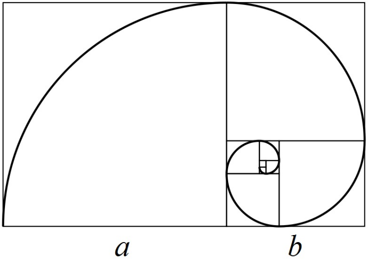


Figure: Golden ratio as present in the Fibonacci spiral.

Spectra of white, pink, and the Brownian (black) noises.

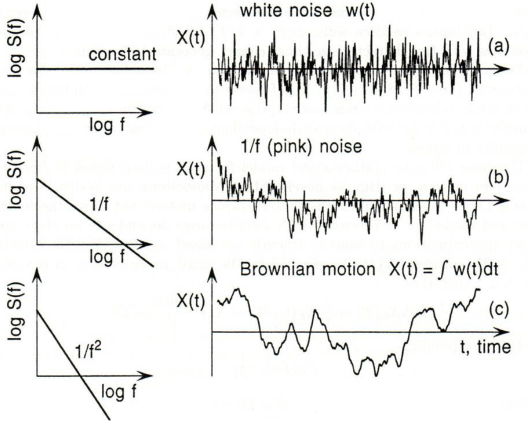
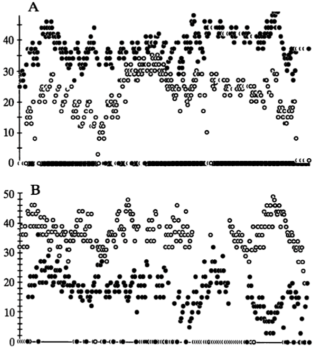


Figure: Signals with different dynamics and fractal dimensions (statistical fractals).

2. Self-similarity of melody[1]



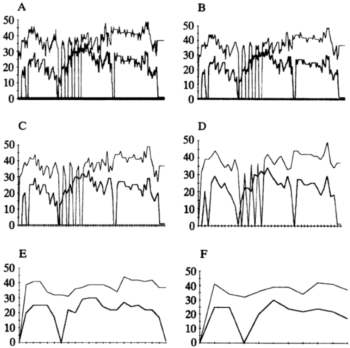


Figure: Scores of Bach's inventions no. 1 and 10. ○ right hand, • left hand. Fractal reduction of Bach's invention no. 1. The 1/2, 1/4. 1/8, 1/16, 1/32 reductions of the scores, respectively.

[1] K. J. Hsü, A. Hsü, "Self-similarity of the '$1/f$ noise' called music," Proc. Natl. Acad. Sci., vol. 88, pp. 3507–3509, 1991.

2. Fractal geometry of melody[2]

Frequency interval i

$$f_{n+1} = \sqrt[N]{2}\ f_n, \tag{2}$$

$$f(i) = \sqrt[N]{2^i}, \tag{3}$$

where N is the number of tones (notes) per octave.

[2] K. J. Hsü, A. Hsü, "Fractal geometry of music," Proc. Natl. Acad. Sci., vol. 87, pp. 938–941, 1990.

2. Fractal geometry of melody[2]

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where N is the number of tones (notes) per octave.

The distribution of i is described by the following power law:

$$f(i) = \frac{c^*}{i^D} \qquad (4)$$

$$\log(f) = C^* - D\log(i) \qquad (5)$$

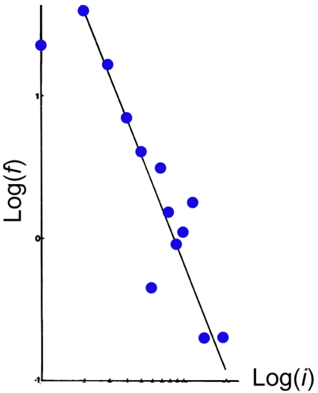


Figure: i distribution in Bach BWV 772. Evidence of the fractal geometry.

[2]K. J. Hsü, A. Hsü, "Fractal geometry of music," Proc. Natl. Acad. Sci., vol. 87, pp. 938–941, 1990.

2. Fractal geometry of pitch evolution[3] [4]

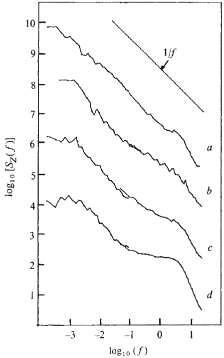


Figure: Pitch fluctuation spectra for a) classical, b) jazz, blues, c) rock, d) news and talk radio station.

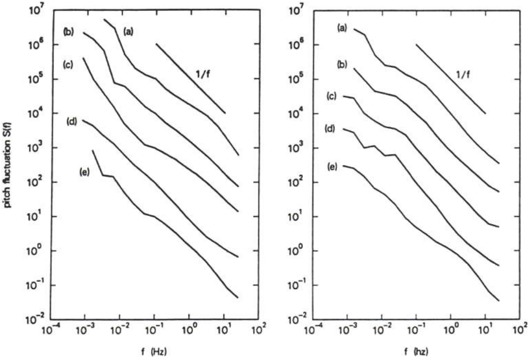


Figure: **Left:** a) Babenzele Pygmies b) Japanese traditional c) Indian classical. d) Russian folklore e) USA blues **Right:** a) Medieval music b) Beethoven's 3. symphony c) Debussy piano d) Strauss, Ein Heldenleben e) The Beatles, Stage Pepper

[3] R. Voss and J. Clarke, "'$1/f$ noise' in music and speech," Nature, vol. 258, pp. 317–318, 1975

[4] K. Hsü, *Applications of fractals and chaos*, Springer-Verlag, pp. 21–39, 1993

2. Fractal geometry of rhythm[5]

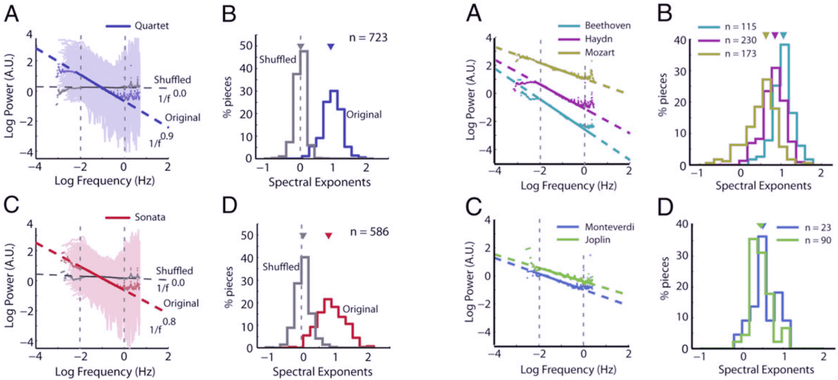


Figure: $1/f^D$ rhythm spectra are ubiquitous across genres. Analysis of 558 compositions spanning over a period of four centuries.

[5] D. Levitina, P. Chordiab, V. Menonc, "Musical rhythm spectra from Bach to Joplin obey a 1/f power law," PNAS, vol. 109, no. 10, pp. 3716–3720, 2012

2. Fractal geometry of rhythm[6]

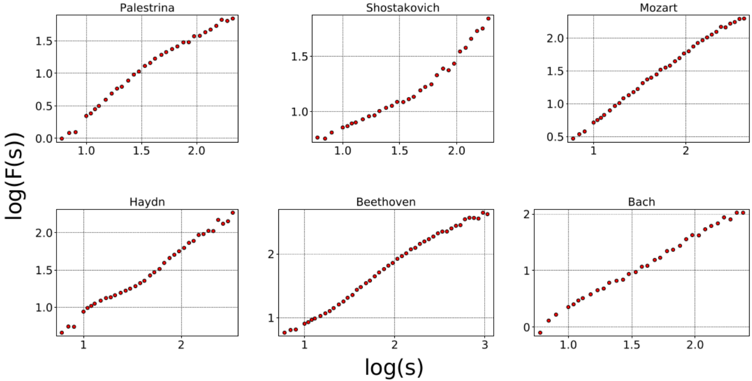


Figure: Musical pieces may have different statistical behaviours within different ranges i.e. different correlations at different time scales (multi-fractality).

[6] A. González-Espinoza, H. Larralde, G. Martińez-Mekler, M. Müller, "Multiple scaling behavior and nonlinear traits in music scores," Physics and Society, 2017

2. Fractal geometry of rhythm[6]

Composer

Profile					
Palestrina	15	0	5	0	1
Bach	30	2	24	7	0
Haydn	25	2	14	2	5
Mozart	22	1	11	0	2
Beethoven	17	7	35	3	1
Dvorak	4	8	10	2	1
Shostakovich	15	14	17	2	0

Figure: A crossover with two regimes is possible. The musical pieces have different statistical behaviours within different ranges i.e. different correlations at different time scales (multifractality).

[6] A. González-Espinoza, H. Larralde, G. Martińez-Mekler, M. Müller, "Multiple scaling behavior and nonlinear traits in music scores," Physics and Society, 2017

2. Fractal geom. of loudness fluctuation[7]

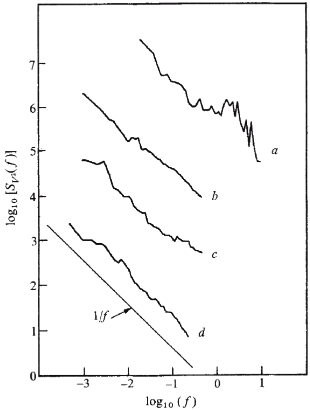


Figure: Loudness fluctuation spectra for a) Scott Joplin Piano Rags, b) classical radio station, c) rock station, d) news and talk station.

[7] R. V. Voss and J. Clarke, "'$1/f$ noise' in music and speech," Nature, vol. 258, pp. 317–318, 1975.

2. Mathematical properties of music

- Components of human composed music (melody, rhythm, harmony, loudness) have pre-fractal geometry (statistical fractals). They are fractal-like objects.

- *Fractal* dimension D of human composed music may have values in the range $[0.5, 2]$.[8]

- Fractal music is generated by directly exploiting the fractal-like nature of human composed music.

[8] M. Bigerelle, A. Iost, "Fractal dimension and classification of music," Chaos, Solitons and Fractals, vol. 11, pp. 2179–2192, 2000.

3. Fractal music composition

Selection of the frequency range. For example chromatic (equal-tempered) scale:

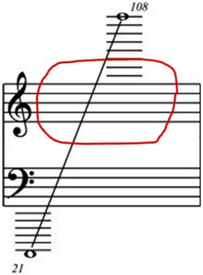
- Frequency step (octave, tones)

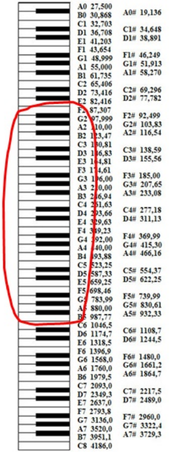
  $\Delta f = f_{n+1} - f_n,$

  $f(i) = \sqrt[12]{2^i}$

- Normalise all fractals to the selected range

$$\begin{cases} f_0 = 27.5 \text{ [Hz]} \\ f_{n+1} = \sqrt[12]{2}\, f_n \end{cases} \quad (6)$$





The chromatic scale is a system of tuning in which the frequency interval between every pair of adjacent notes has the same ratio.

3. Fractal music composition

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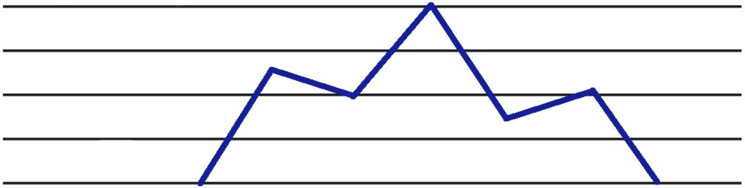

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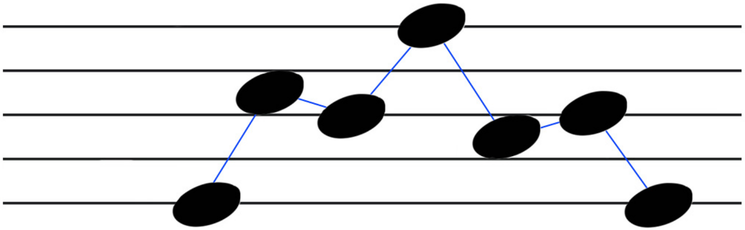

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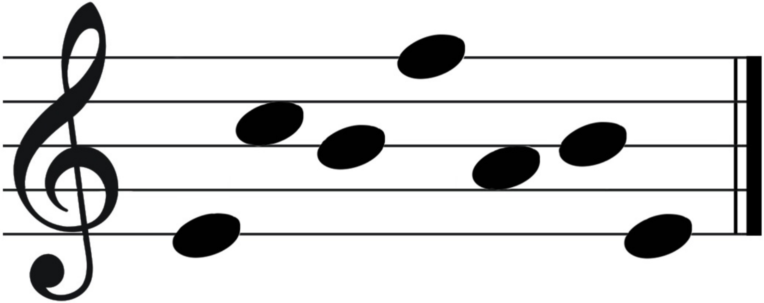

3. Fractal music composition

Selection of music instrument timbre or timbres.

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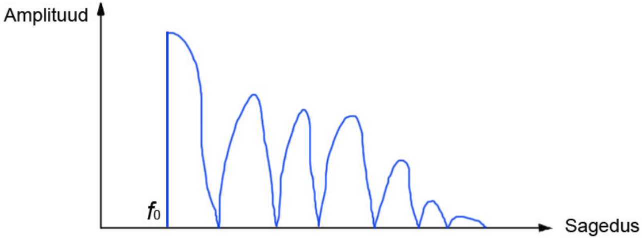

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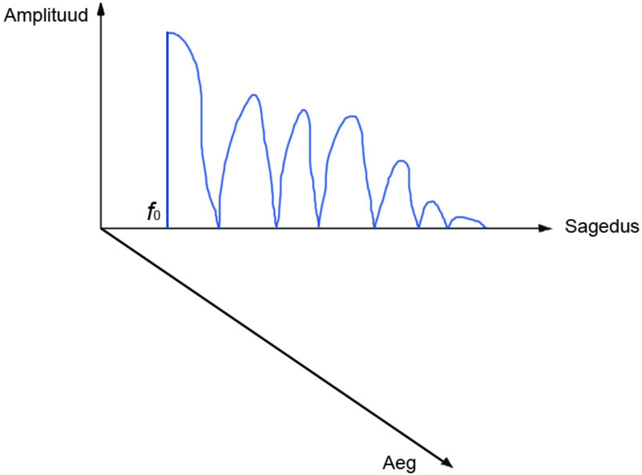

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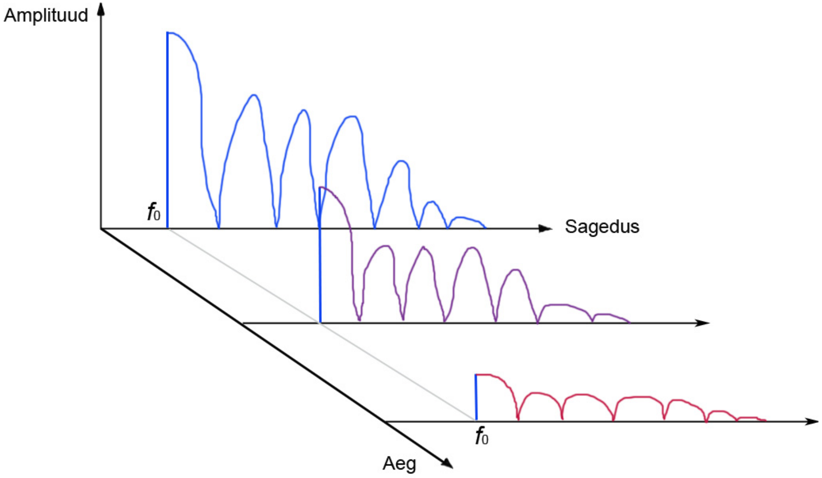


4. Musical fractals

Graphical fractals (the L-systems).

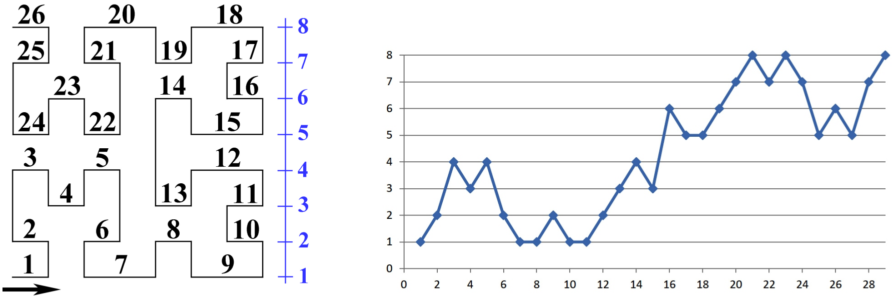


Figure: The plane filling Hilbert curve and its 1-D interpretation.

Logistic map

$$y_{n+1} = ry_n(1 - y_n), \qquad (7)$$

where $r \in (0, 4]$ is the control parameter.

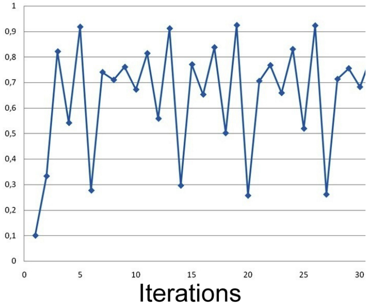


Figure: Logistic map, where $y_0 = 0.1$ and $r = 3.71$.

Audio example:
Logistic map
Duration 15 s

4. $1/f$ noise generator (pink noise)

$1/f$ noise

$$y_{n+1} = my_n + k\sqrt{1 - m^2}, \qquad (8)$$

where parameter $m \in [0, 1]$ and k is a random number. When generating music k values are calculated using Logistic map with $r = 4$.

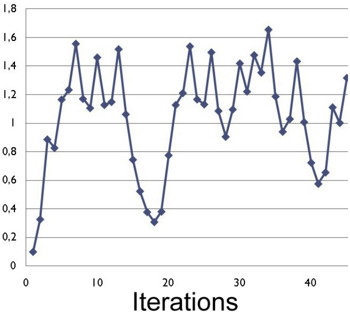


Figure: $1/f$ noise, where $x_0 = 0.1$ and $m = 0.7$.

Audio example:
Pink noise generator
Duration 15 s

4. Lorenz fractal

Note: Do not confuse with the Lorenz system (a strange attractor).

Lorenz fractal

$$y_{n+1} = a\left(3y_n - 4y_n^3\right), \qquad (9)$$

where parameter $a \in [0, 1]$. For music generation a is selected within the range $[0.65, 1]$.

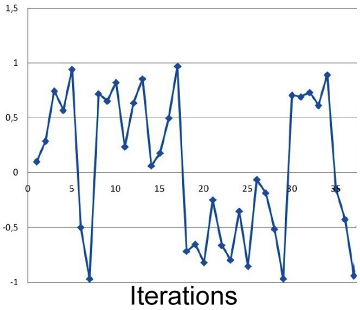


Figure: Lorenz fractal, where $a = 0.97$ and $y_0 = 0.1$.

Audio example:
Lorenz fractal
Duration 15 s

4. Hénon fractal

Hénon fractal

$$\begin{cases} x_{n+1} = 1 + y_n - a x_n^2, \\ y_{n+1} = b x_n, \end{cases} \quad (10)$$

for music generation select $a = 1.4$ and $0 < b < 0.3$.

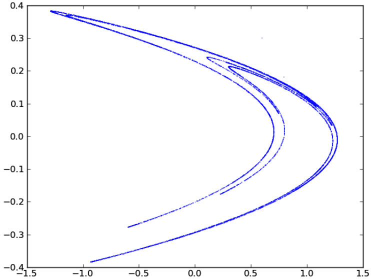


Figure: Hénon fractal, where $a = 1.4$, $b = 0.3$, $x_0 = y_0 = 1$ (10^4 iterates).

Audio example:
Hénon fractal
Duration 15 s

4. Hopalong fractal

Hopalong fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sqrt{|bx_n - c|} \\ y_{n+1} = a - x_n, \end{cases} \tag{11}$$

where a, b and c are the control parameters.

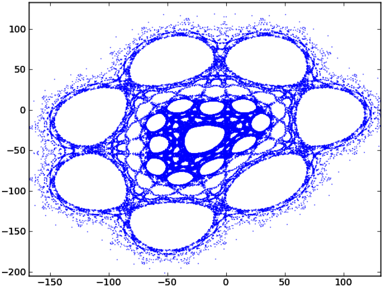


Figure: Hopalong fractal, where $a = -55$, $b = 17$, $c = -21$, $x_0 = y_0 = 0$ ($5 \cdot 10^4$ iterates).

Audio example:
Hopalong fractal
Duration 15 s

4. Gingerbreadman map

Gingerbreadman map

$$\begin{cases} x_{n+1} = 1 - y_n + |x_n|, \\ y_{n+1} = x_n, \end{cases} \qquad (12)$$

where $x_0 = -0.1$ and $y_0 = 0$.

Audio example:
Gingerbreadman map
Duration 15 s

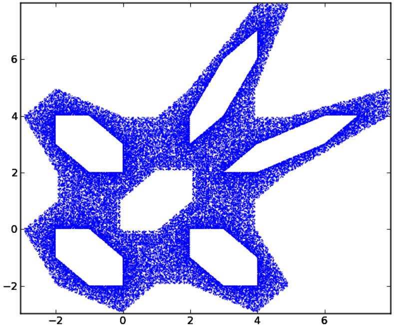


Figure: Gingerbreadman map, where $x_0 = -0.1$, $y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. L-systems (Lindenmayer systems)

A type of formal grammar created by Aristid Lindenmayer. The grammar is used for modelling various biological processes.

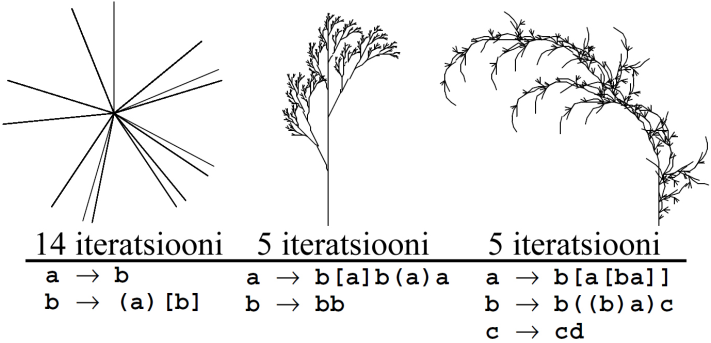


Figure: L-systems and the resulting fractal *plants*.

4. L-systems

The categories of L-systems:

- Context free (0L) and context sensitive (iL: 0L, 1L, 2L, etc.)
- Deterministic (DL) and stochastic
- Multiplying (PL) and non-multiplying
- Tabular L-systems (TL)
- Parametric L-systems

4. L-systems

Simple example of an L-system.

Rule: P1: $a \rightarrow ab$

P2: $b \rightarrow a$

Axiom: b

$$
\begin{aligned}
n = 0: &\quad b \\
n = 1: &\quad a \\
n = 2: &\quad ab \\
n = 3: &\quad aba \\
n = 4: &\quad abaab \\
n = 5: &\quad \underline{abaababa}
\end{aligned}
$$

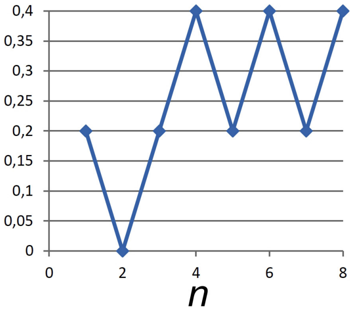


Figure: Interpretation: $a = 0.2$ and $b = -0.2$.

Audio example:
Fractal L-system
Duration 15 s

4. Morse-Thue sequence

The formation of the Morse-Thue sequence (4-bit case):

Decimal	Binary	Sum of binary digits
1	0001	1
2	0010	1
3	0011	2
4	0100	1
5	0101	2
6	0110	2
7	0111	3
8	1000	1
9	1001	2

4. Morse-Thue sequence

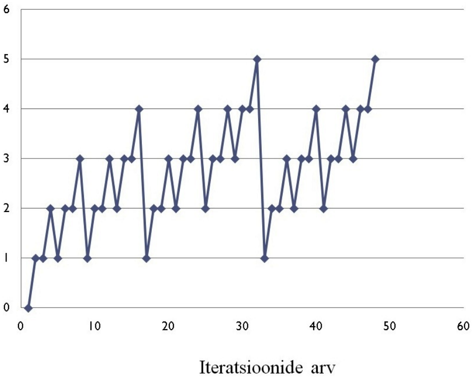


Figure: The Morse-Thue sequence.

Audio example:
Morse-Thue sequence
Duration 15 s

4. Hailstone numbers or $3n + 1$ numbers

Rule for generating the sequence:

1. Select integer greater than 1.
2. If the number is even divide by 2.
3. It the number is odd multiply by 3 and add 1.
4. Stop or repeat.

Hailstone numbers with initial condition 7.

7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, stop or repeat $\circlearrowleft$

4. Hailstone numbers or $3n + 1$ numbers

Hailstone number in the case of the initial condition $y_0 = 27$.

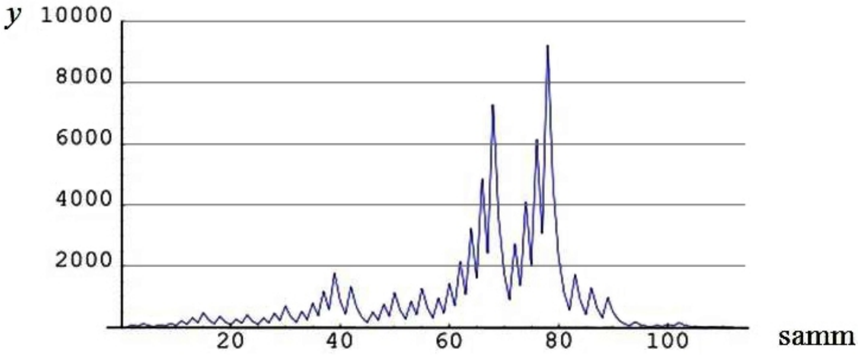


Figure: Hailstone numbers, where $y_0 = 27$. Showing 111 iterates before reaching 1.

4. Popcorn fractal

$$\begin{cases} x_{n+1} = x_n - h\sin(y_n + \tan 3y_n) \\ y_{n+1} = y_n - h\sin(x_n + \tan 3x_n) \end{cases} \tag{13}$$

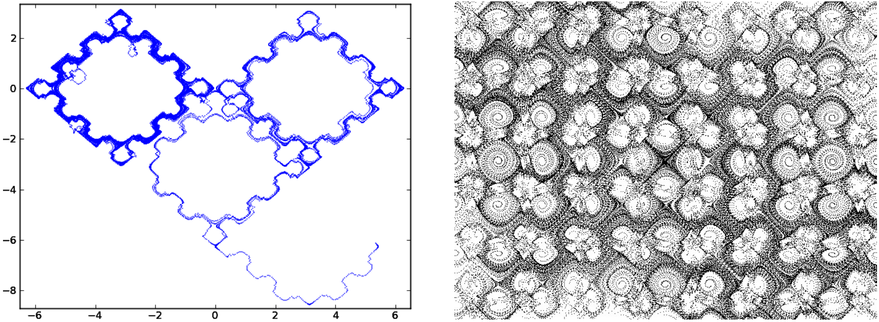


Figure: Popcorn fractal, where $h = 0.05$, $x_0 = -0.1$, $y_0 = 0$ ($5 \cdot 10^4$ iterates).

4. Quadrup-Two fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sin(\ln|b(x_n - c)|) \tan^{-1}|c(x_n - b)|^2 \\ y_{n+1} = a - x_n \end{cases} \quad (14)$$

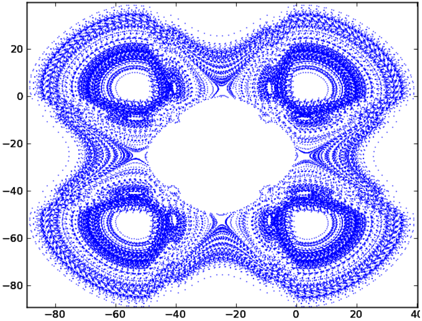


Figure: Quadrup-Two fractal, where $a = -50$, $b = -1$, $c = -41$, $x_0 = -1$, $y_0 = 1$ ($5 \cdot 10^4$ iterates).

4. Mira fractal

$$\begin{cases} x_{n+1} = by_n + ax_n + \dfrac{2(1-a)x_n^2}{1+x_n^2} \\ y_{n+1} = -x_n + ax_{n+1} + \dfrac{2(1-a)x_{n+1}^2}{1+x_{n+1}^2} \end{cases} \tag{15}$$

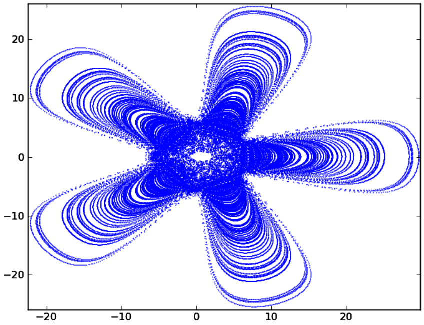


Figure: Mira fractal: $a = 0.31$, $b = 1$, $x_0 = 12$, $y_0 = 0$ ($5 \cdot 10^4$ iterates).

4. Tree growth rings fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n |\sin x_n \cos b + c - x_n \sin(a+b+c)| \\ y_{n+1} = a - x_n \end{cases} \tag{16}$$

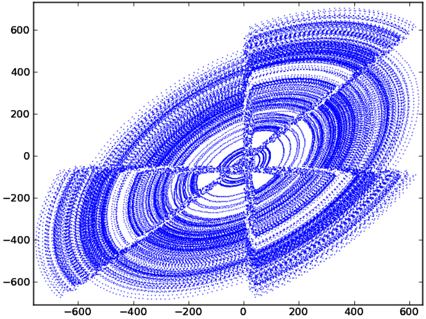


Figure: Tree growth rings fractal, where $a = -50$, $b = -1$, $c = -45$, $x_0 = y_0 = 1$ $(5 \cdot 10^4$ iterates).

4. Martin's fractal

$$\begin{cases} x_{n+1} = y_n - \sin x_n \\ y_{n+1} = a - x_n \end{cases} \quad (17)$$

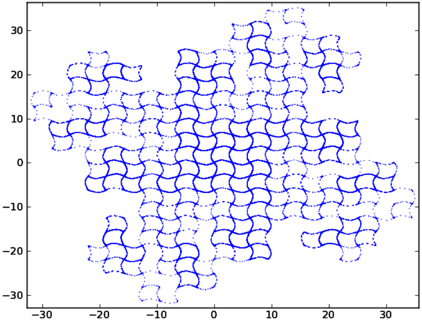


Figure: Martin's fractal, where $a = 0.31$, $x_0 = y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. Hopalong 2 fractal

$$\begin{cases} x_{n+1} = y_n + \operatorname{sgn} x_n |b x_n - c| \\ y_{n+1} = a - x_n \end{cases} \tag{18}$$

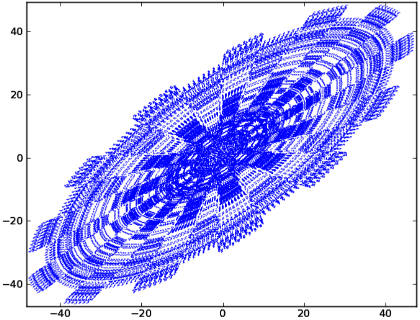


Figure: Hopalong 2 fractal, where $a = 0.6$, $b = 1.5$, $c = -2.5$, $x_0 = 1$, $y_0 = 1$ $(5 \cdot 10^4$ iterates).

4. Hopalong 3 fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sqrt{|x_n \sin a - \cos a|} \\ y_{n+1} = a - x_n \end{cases} \tag{19}$$

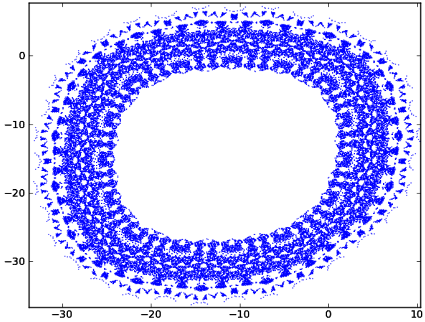


Figure: Hopalong 3 fractal, where $a = -26$, $x_0 = y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. Hopalong 4 fractal

$$\begin{cases} x_{n+1} = y_n + \operatorname{sgn} x_n \sqrt{|bx_n - c|} \\ y_{n+1} = a - x_n \end{cases} \tag{20}$$

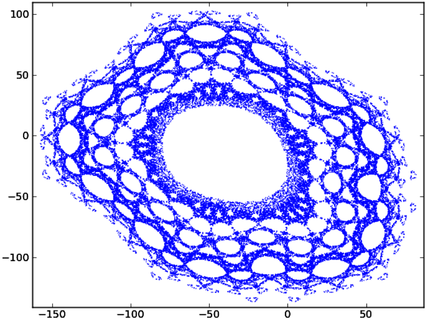


Figure: Hopalong 4 fractal, where $a = -55$, $b = 17$, $c = -21$, $x_0 = 0$, $y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. Hopalong 5 fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \sqrt{|bx_n^2 - c|} \\ y_{n+1} = a - x_n \end{cases} \tag{21}$$

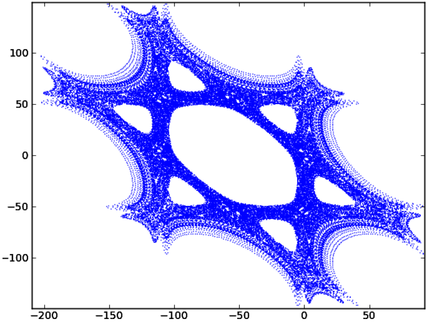


Figure: Hopalong 5 fractal, where $a = -55$, $b = -1$, $c = -41$, $x_0 = 0$, $y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. Hopalong 6 fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \cos |bx_n - c| \\ y_{n+1} = a - x_n \end{cases} \tag{22}$$

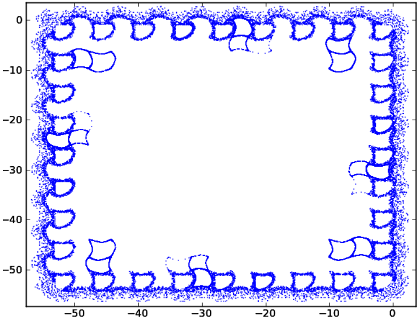


Figure: Hopalong 6 fractal, where $a = -54$, $b = -1$, $c = -42$, $x_0 = 0$, $y_0 = 0$ $(5 \cdot 10^4$ iterates).

4. Hopalong 7 fractal

$$\begin{cases} x_{n+1} = y_n - \operatorname{sgn} x_n \log |bx_n - c| \\ y_{n+1} = a - x_n \end{cases} \tag{23}$$

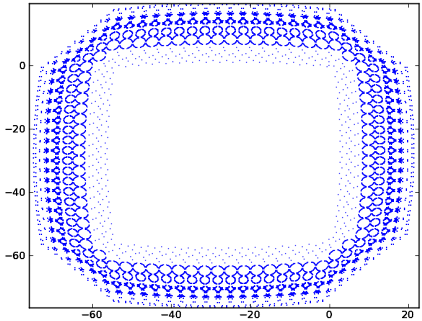


Figure: Hopalong 7 fractal, where $a = -55$, $b = -1$, $c = -31$, $x_0 = 0$, $y_0 = 0$ ($5 \cdot 10^4$ iterates).

4. Which fractals to use?

Use fractals whose fractal dimension has values in the range $[0.5, 2]$.[9]

Note 1: Fractals can sound non-musical (scary, unpleasant, etc.).

Note 2: Some human composed musical compositions do not have a clearly defined scale invariance.

Note 3: Some human composed musical compositions have multifractal features.

[9] M. Bigerelle, A. Iost, "Fractal dimension and classification of music," Chaos, Solitons and Fractals, vol. 11, pp. 2179–2192, 2000.

5. Properties of musical fractals

Variation within a fractal:

- Sensitive dependants on the control parameters.
- Sensitive dependants on the initial conditions.

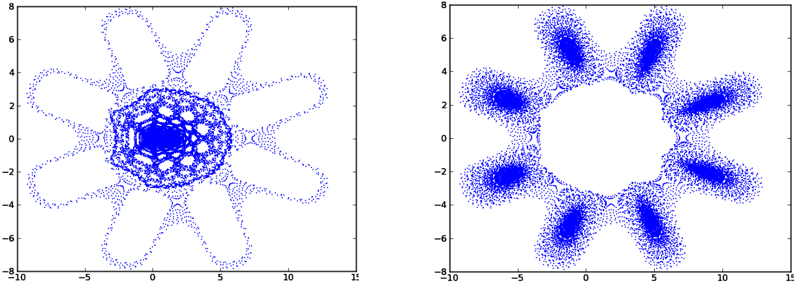
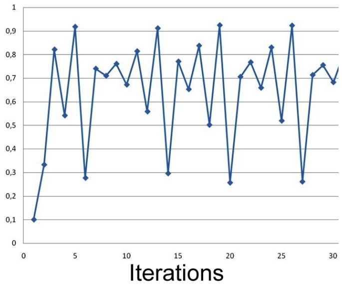


Figure: Mira fractal, where $a = 0.7$, $b = 0.9998$, $x_0 = 9$, $y_0 = 0$ ($5 \cdot 10^4$ iterates), the difference in the initial conditions $(\Delta x_0, \Delta y_0) = (0, 10^{-4})$.

Variation between fractals:

- The global dynamics of fractals are different.
- Statistical properties.



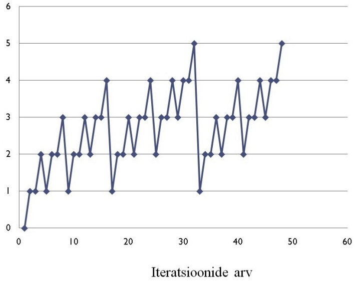
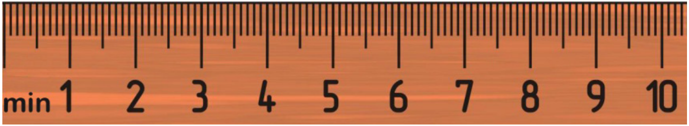


Figure: Logistic map, where $y_0 = 0.1$, $r = 3.71$ compared to the Morse-Thue sequence shown above on Slide 35.

5. Composition of musical scores

Transitions between different *parts* of the composition is advisable to link together using the golden ratio or the Fibonacci sequence members.



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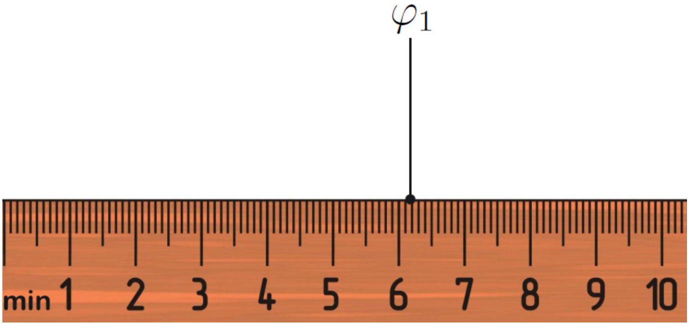

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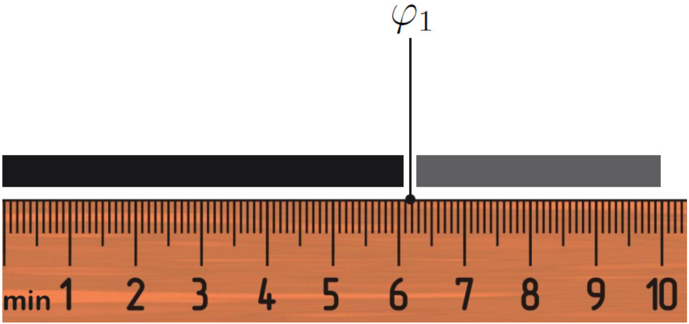

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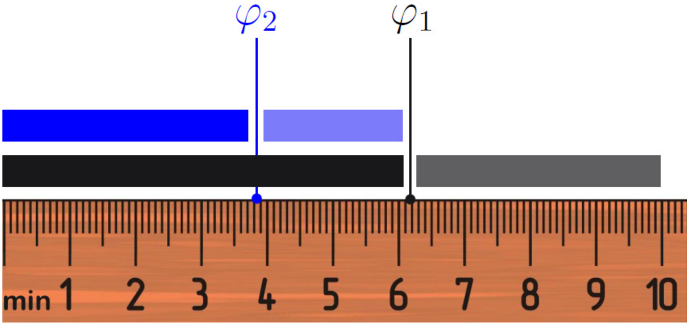

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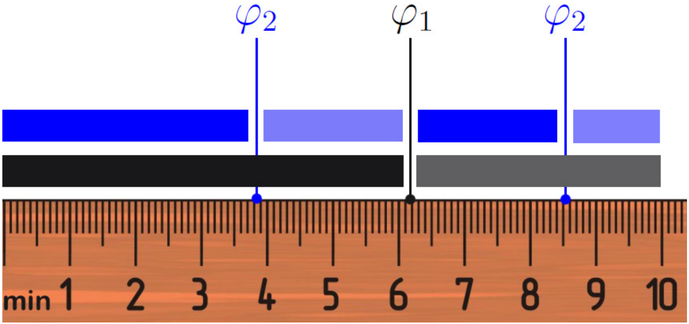

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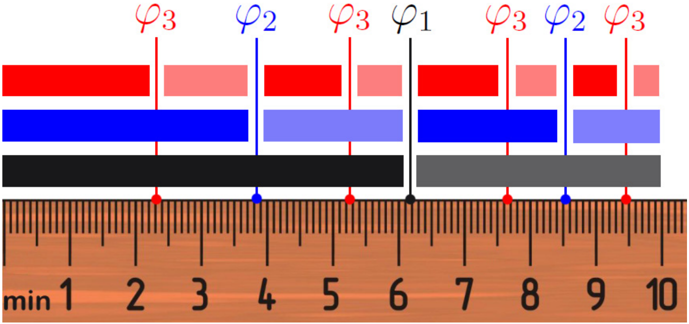

5. Musical examples

By applying the aforementioned suggestions, it is possible to create music that is similar to human composed music.

Audio example:
Example composition no. 1
Duration 40 s

Audio example:
Example composition no. 2
Duration 40 s

Audio example:
Example composition no. 3
Duration 40 s

"The engine might compose elaborate and scientific pieces of music of any degree of complexity or extent." – Ada Lovelace

6. How to explain fractal music?

[9] K. J. Hsü, 1983.
[10] Y. Yu, R. Romero, and T. S. Lee, 2005.
[11] M. A. Schmuckler, D. L. Gilden, 1993.
[12] T. Musha, 1997.

6. How to explain fractal music?

- Natural phenomena, for example the Richardson effect.[9]

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- Human speech and text – the Zipf law.
- Human and animal kinematics (example: clapping of hands).[12]
- Biological processes (fluctuations of heart rhythm, fluctuations of eye pupils while focusing, organ α-rhythms).[11]

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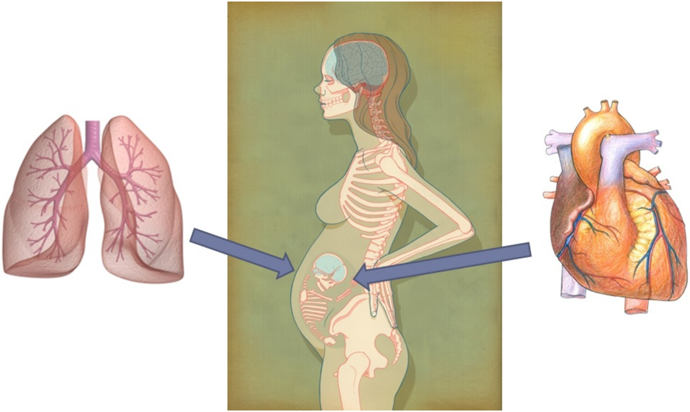
[11] M. A. Schmuckler, D. L. Gilden, 1993.
[12] T. Musha, 1997.

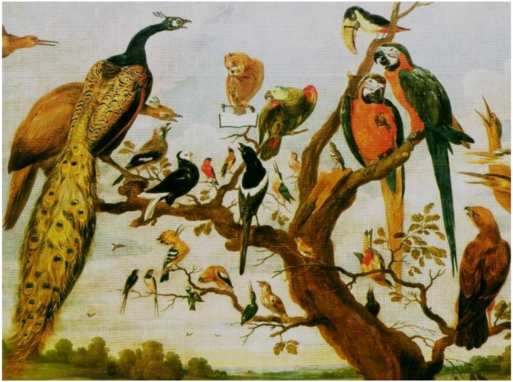
6. How to explain fractal music?

Periodic, quasi-periodic and chaotic processes in the human body and the nature at large. The human evolution and fetus brain development.



6. How to explain fractal music?

Musicality (also synchronism) can be found in numerous species. Example: birds[13] and insects.



Songs are use to attract a mate. Evolutionary selection pressure.

[13] T. C. Roeske, D. Kelty-Stephen, S. Wallot, Scientific Reports 8:4570, 2018

6. How to explain fractal music?

Self-similarity appears in social behaviour, in dynamics of economical and financial time-series (to some extent), networks, climate, physics, etc.

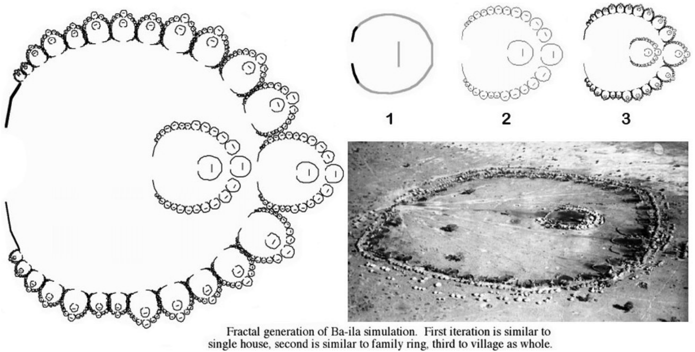


Figure: Ba-Ila village in the Southern Zambia, Africa.

6. How to explain fractal music?

Human brain on fractal stimuli.

Self-similar (in frequency domain) Shepard tone's *fractal* dimension $D > 1$.

Audio example:
Shepard tone
Duration 40 s

[14]R. N. Shepard, "Circularity in judgments of relative pitch," J. Acoust. Soc. Am., vol. 36 no. 12 pp. 2346–2353, 1964.

[15]Manfred R. Schroeder, *Fractals, Chaos, Power Laws*, (W.H. Freeman, ed.), New York, 1991.

6. How to explain fractal music?

Human brain on fractal stimuli.

Self-similar (in frequency domain) Shepard tone's *fractal* dimension $D > 1$.

Audio example:
Shepard tone
Duration 40 s

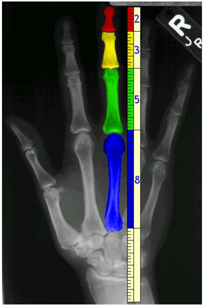
Possible hypothesis: brain at large scale and auditory system are trying to register and/or process the tone as a 1-D signal.[14] [15]

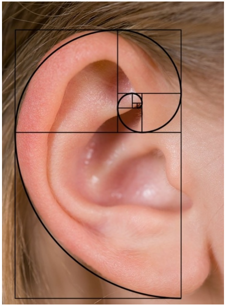
[14] R. N. Shepard, "Circularity in judgments of relative pitch," J. Acoust. Soc. Am., vol. 36 no. 12 pp. 2346–2353, 1964.

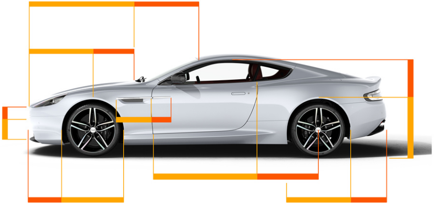
[15] Manfred R. Schroeder, *Fractals, Chaos, Power Laws*, (W.H. Freeman, ed.), New York, 1991.

6. How to explain fractal music?

Golden ratio and the Fibonacci sequence in nature and applications.







6. Conclusions

- Human composed music has pre-fractal geometry.
- Human musical compositions are fractal-like objects.
- Simplest techniques for generating fractal melody and music were introduced.
- Three musical fractals were played.

Q & A

References

- B. B. Mandelbrot, “The Fractal Geometry of Nature,” W. H. Freeman and Company, New York, 1982.
- K. J. Hsü and A. Hsü, “Self-similarity of the '1/f noise' called music,” Proc. Natl. Acad. Sci., vol. 88, pp. 3507–3509, 1991.
- K. J. Hsü and A. Hsü, “Fractal geometry of music,” Proc. Natl. Acad. Sci., vol. 87, pp. 938–941, 1990.
- K. J. Hsü, “Applications of fractals and chaos,” Springer-Verlag, pp. 21–39, 1993.
- D. J. Levitina, P. Chordiab, and V. Menonc, “Musical rhythm spectra from Bach to Joplin obey a 1/f power law,” PNAS, vol. 109, no. 10, pp. 3716–3720, 2012.
- M. Bigerelle and A. Iost, “Fractal dimension and classification of music,” Chaos, Solitons and Fractals, vol. 11, pp. 2179–2192, 2000.
- N. Nettheim, “On the spectral analysis of melody,” Journal of New Music Research, vol. 21, pp. 135–148, 1992.
- R. V. Voss and J. Clarke, “'1/f noise' in music and speech,” Nature, vol. 258, pp. 317–318, 1975.
- M. A. Kaliakatsos-Papakostas, A. Floros, and M. N. Vrahatis, “Music Synthesis Based on Nonlinear Dynamics,” in proceedings of Bridges 2012: Mathematics, Music, Art, Architecture, Culture, July 25–29, Baltimore, USA, pp. 467–470, 2012.
- A. E. Coca, G. O. Tost, and L. Zhao, “Characterizing chaotic melodies in automatic music composition,” Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 20, no. 3, pp. 033125-1–033125-12, 2010.
- A. Alpern (1995), “Techniques for algorithmic composition of music,” Hampshire College.

References

- K. J. Hsü, "Actualistic Catastrophism: Address of the retiring President of the International Association of Sedimentologists," Sedimentology, vol. 30, no. 1, pp. 3–9, 1983.
- Y. Yu, R. Romero, and T. S. Lee, "Preference of sensory neural coding for 1/f signals," Phys. Rev. Lett., vol. 94, pp. 108103-2–108103-4, 2005.
- J. Jeong, M. K. Joung, S. Y. Kim "Quantification of emotion by nonlinear analysis of the chaotic dynamics of electroencephalograms during perception of 1/f music," Biol. Cybern., vol. 78, pp. 217–225, 1998.
- Grout, Donald Jay & Claude V. Palisca (1996), "A History of Western Music," 5th ed. W. W. Norton and Company: New York
- P. Prusinkiewicz "Score Generation with L-systems," *Proc. Intl. Computer Music Conf '86*, pp. 455–457, 1986.
- Stelios Manousakis (2006,) "Musical L-systems" *MSc thesis sonology*, The Royal Conservatory, The Hague
- John A. Maurer IV (1999), "A Brief History of Algorithmic Composition"
- A.J. Crilly, Rae A. Earnshaw, Huw Jones (Editors) (1993), "Applications of Fractals and Chaos: The Shape of Things," *Springer*
- P. C. Ivanov, "From 1/f noise to multifractal cascades in heartbeat dynamics," Chaous vol. 11, no. 3, pp. 641–652, 2001.
- T. Musha, "1/f fluctuations in biological systems," in Proceedings - 19th International Conference - IEEE/EMBS Oct. 30–Nov. 2, 1997 Chicago, IL. USA, pp. 2692–2697
- M. A. Schmuckler, D. L. Gilden, "Auditory Perception of Fractal Contours," Journal of Experimental Psychology: Human Perception and Performance, vol. 19, no. 3. pp. 641–660, 1993.

References

- "Evolutionary and Biologically Inspired Music, Sound, Art and Design," Proc. 5th International Conference, EvoMUSART 2016, Porto, Portugal, March 30–April 1, 2016 ,Colin Johnson, Vic Ciesielski, João Correia, Penousal Machado (Eds.)
- A. González-Espinoza, H. Larralde, G. Martińez-Mekler, M. Müller, "Multiple scaling behavior and nonlinear traits in music scores," Physics and Society (submitted), 2017
- M. S. Islas, "Words as block of notes and Zipf law in music using visibility algorithm," Physics and Society (submitted), 2017
- T. C. Roeske, D. Kelty-Stephen, S. Wallot, "Multifractal analysis reveals music-like dynamic structure in songbird rhythms," Scientific Reports 8:4570, 2018

Revision questions

- Name some applications of fractal geometry and chaos theory.
- What is fractal music composition?
- Why does fractal music sound similar to human composed music?