# Magnetic Pendulum in Three Magnetic Potentials

## Introduction

This teaching aid is used to demonstrate the concept of *chaos* in a nonlinear system. You can learn about chaos by taking the course Nonlinear dynamics (YFX1560).

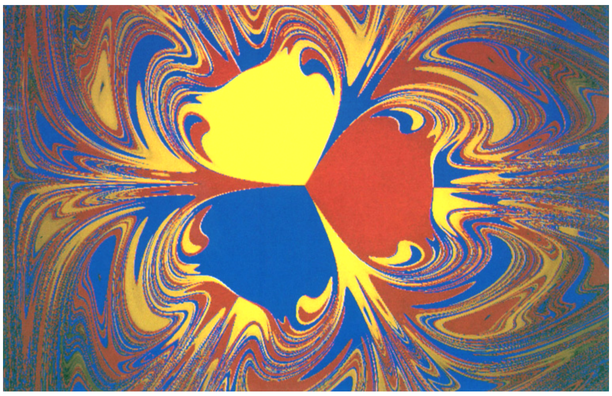
## Equations of motion

The top view projection of the pendulum position $(x, y)$ is described by the following nonlinear equations of motion:

$$\begin{cases} \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + R\dfrac{\mathrm{d}x}{\mathrm{d}t} - \displaystyle\sum_{i=1}^{3} \dfrac{x_i - x}{\left(\sqrt{(x_i - x)^2 + (y_i - y)^2 + D^2}\right)^3} + Cx = 0, \\ \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + R\dfrac{\mathrm{d}y}{\mathrm{d}t} - \displaystyle\sum_{i=1}^{3} \dfrac{y_i - y}{\left(\sqrt{(x_i - x)^2 + (y_i - y)^2 + D^2}\right)^3} + Cy = 0, \end{cases} \tag{1}$$

where $R$ is proportional to the air resistance and overall attenuation, $C$ is proportional to the effects of gravity, the $i$-th magnet is positioned at $(x_i, y_i)$ where $i = 1, 2, 3$, and $D$ is the distance between the pendulum at rest position $(0, 0)$ and the plane of magnets.

Additionally, we assume that the pendulum length is *long* compared to the spacing between the magnets. Thus, we may assume for simplicity that the pendulum moves about on the $xy$-plane rather than on a sphere with a large radius.



H. Peitgen, *et al*, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag, 2004, pp. 707–414.

Figure 1: Basins of attraction of the magnets shown with the red, blue and yellow colours.

The basin of attraction of system (1) shown in Fig. 1 is a fractal with an infinite complexity. The prediction of the end state of the pendulum for most initial conditions is impossible in practice especially if the integration time exceeds a several multiples of a time period called the Lyapunov time.

## Further information for interested students

If students have any further questions or show a genuine interest in general topics of chaotic dynamics, you may direct them to the Author (e-mail: `dmitri.kartofelev@taltech.ee`, or URL: `https://www.tud.ttu.ee/web/dmitri.kartofelev/`).