Index of .nb files

The files (totalling 80) are used during the lectures to demonstrate the nonlinear phenomena discussed. The .nb files can be opened with Wolfram Mathematica software.

Lecture 1

Link	File name	Description
nb#1	numerics_1.nb	Numerical integration of dynamical system describing a magnetic pendulum's motion. A pendulum in which a metal ball (or a magnet) is attached to its end and which is oscillating over a plane where a set of attractive magnets are present (2–6 magnets).
nb#2	numerics_1b.nb	Analytic solution to a nonlinear ODE. Numerical solution and phase portrait of 1-D flow $\dot{x} = \sin x$.
nb#3	numerics_1c.nb	Numerical solution and phase portrait of the 1-D logistic equation.

Lecture 2

Link	File name	Description
nb#1	numerics_2.nb	Bifurcation diagrams and 1-D systems: saddle node bifurcation, transcritical bifurcation, pitchfork bifurcation (supercritical), pitchfork bifurcation (subcritical).

Lecture 3

Link	File name	Description
nb#1	numerics_3.nb	1-D approximation corresponding to the over-damped bead on a hoop dynamics: the 1-D phase portrait and a family of numerical solutions corresponding to a set of initial conditions.
nb#2	numerics_3b.nb	Over-damped bead on a hoop dynamics: graphical determination of the fixed points and the bifurcation diagram (first-order model).
nb#3	numerics_3c.nb	Numerical solution and phase portrait of the over-damped bead on a hoop problem (second-order model).

Lecture 4

Link	File name	Description
nb#1	numerics_4.nb	Vector field plotting. Integrated solution and phase portrait of the damped harmonic oscillator.
nb#2	numerics_4b.nb	Calculation of matrix determinant, trace, eigenvalues and eigenvectors using computer. Classification of fixed points in 2-D linear homogeneous systems.

Lecture 5

Link	File name	Description
nb#1	numerics_5.nb	Effect of nonlinearity on a linear centre (a borderline case). Numerical solution and phase portrait of the same problem.
nb#2	numerics_5b.nb	Competitive cohabitation of sheep and rabbits (the Lotka-Volterra predator–pray model). Linear analysis of fixed points and system manifolds. Numerical solution and phase portrait.
nb#3	numerics_5c.nb	The Lotka-Volterra predator–pray model for sharks and fish (cyclic behaviour). Numerical solution and phase portrait.
nb#4	numerics_5d.nb	Interactive comparison of dynamics of the Liénard equation and its linearisation.
nb#5	numerics_5e.nb	Particle in a double well potential, conservative system and homoclinic orbits. Linear analysis of fixed points, numerical solution and phase portrait.

Lecture 6

Link	File name	Description
nb#1	numerics_6.nb	Mathematical pendulum and heteroclinic orbit. Integrated numerical solution.
nb#2	numerics_6b.nb	The Dulac's criterion and limit-cycles. A numerical example: integrated solution and phase portrait.
nb#3	numerics_6c.nb	The Dulac's criterion and limit-cycles. Another numerical example: integrated solution and phase portrait.

nb#4	numerics_6d.nb	The Poincaré-Bendixson theorem. Phase portrait of a nonlinear system given in polar coordinates.
nb#5	numerics_6e.nb	Glycolysis phase portrait and null-clines. Numerical solution of the glycolysis model.

Lecture 7

Link	File name	Description
nb#1	numerics_7.nb	Examples of bifurcations in 2-D systems: saddle-node bifurcation, transcritical bifurcation, supercritical and subcritical pitchfork bifurcations. Interactive code.
nb#2	numerics_7b.nb	Linear analysis of the system from the previous .nb file.
nb#3	numerics_7c.nb	Examples of bifurcations in 2-D systems: the supercritical Hopf bifurcation. Integrated solution and bifurcation diagram.
nb#4	numerics_7d.nb	Examples of bifurcations in 2-D systems: the subcritical Hopf bifurcation. Integrated solution and bifurcation diagram.
nb#5	numerics_7e.nb	Examples of bifurcations in 2-D systems: a saddle-node coalescence of limit-cycles. Bifurcation diagram.
nb#6	numerics_7f.nb	Examples of bifurcations in 2-D systems: SNIPER (saddle-node infinite period bifurcation). Integrated solution and phase portrait.
nb#7	numerics_7g.nb	Examples of bifurcations in 2-D systems: a homoclinic bifurcation or saddle-loop bifurcation (solitons). Integrated solution and phase portrait.

Lecture 8

Link	File name	Description
nb#1	numerics_8.nb	Trajectories on the surface of a torus: interactive 3-D plot. Periodic and quasi-periodic trajectories on the surface of a torus. Trefoil (for $p = 3$, $q = 2$) and cinquefoil knots (for $p = 5$ and $q = 2$).
nb#2	numerics_8b.nb	Quasi-periodic oscillators. Quasi-periodic decoupled system: periodic solution, quasi-periodic solution. The Fourier and power spectra of the solutions.
nb#3	numerics_8c.nb	Quasi-periodic coupled oscillations: the Fourier and power spectra of the solutions.
nb#4	numerics_8d.nb	Quasi-periodic coupled oscillations: interactive code.
nb#5	numerics_8e.nb	An example of a chaotic system: the Lorenz mill. Power spectra of the time-series solutions of the Lorenz mill system. Flow trajectory plotting in 3-D.
nb#6	numerics_8f.nb	An example of a chaotic system: the Lorenz attractor. Numerical integration of the Lorenz attractor (interactive 3-D plot, interactive 2-D (x-z projection) plot).
nb#7	numerics_8g.nb	A remark on 3-D phase portrait plotting. 3-D phase portrait and flow visualisation of the Lorenz attractor.
nb#8	Analysis_tool.nb	**Coursework analysis tool:** Mathematica notebook for analysing 2-D and 3-D systems. Use of the tool does not require coding skills.

Lecture 9

Link	File name	Description
nb#1	numerics_9.nb	Dissipative chaotic flow of the Lorenz attractor visualisation via particle tracking. Calculation of divergence of 3-D and higher dimensional flows.
nb#2	numerics_9b.nb	The Lyapunov exponent of 3-D chaotic systems, visual demonstration.
nb#3	numerics_9c.nb	Demonstration of the effect of calculation accuracy and precision on the numerical solution and the predictability horizon of a chaotic system. Transient and intermittent chaos.
nb#4	numerics_9d.nb	Numerical solutions of chaotic systems: the Lorenz attractor, the Rössler attractor, the Chen attractor, the modified Chen attractor, the Lu attractor, the Pan-Xu-Zhou attractor, the Bouali attractor, and the algebraically simplest chaotic dissipative flow by J. C. Sprott.
nb#5	numerics_9e.nb	Numerical solutions of chaotic systems: double mathematical pendulum with one and two sets of initial conditions. Phase portrait of the double pendulum system.
nb#6	numerics_9f.nb	Numerical solutions of chaotic systems: the gravitational three body problem (planar). Interactive code with several dynamic visualisation styles.
nb#7	numerics_9g.nb	Introduction to cobweb diagramming. Interactive cobweb diagrams of 1-D maps $x_{n+1} = rx_n$ and $x_{n+1} = r(x_n - x_n^2)$ where r is the control parameter.
nb#8	numerics_9h.nb	Interactive cobweb diagram and iterates of the Lorenz map.

nb#9	numerics_9i.nb	The Lorenz map: quantitatively accurate Lorenz mapping.

Lecture 10

Link	File name	Description
nb#1	numerics_10.nb	Cobweb diagram and iterates of the (generalised) Lorenz map. Lyapunov exponent $\lambda(r)$ of the logistic map.
nb#2	numerics_10b.nb	Cobweb diagram of the logistic map. Orbit diagram of the logistic map. Period doubling bifurcation.
nb#3	numerics_10c.nb	The logistic map: cobweb diagram, orbit diagram and map iterates, the Lyapunov exponent.
nb#4	numerics_10d.nb	Period-3 window and intermittency in the logistic map.
nb#5	numerics_10e.nb	The sine map iterates, cobweb and orbit diagrams where $x_n \in [0, 1]$.
nb#6	numerics_10f.nb	The sine map iterates, cobweb and orbit diagrams where $x_n \in [-1, 1]$.

Lecture 11

Link	File name	Description
nb#1	numerics_11.nb	Comparison of superstable and stable fixed points in the logistic map.
nb#2	numerics_11b.nb	Calculation of universal limiting function g and value of the Feigenbaum constant α in the case of the logistic map using power series expansion: one and four term approximations.
nb#3	numerics_11c.nb	The periodically forced damped Duffing oscillator: numerical solutions and a phase portrait. The 2π-periodic Poincaré section of the Duffing oscillator (static).
nb#4	numerics_11d.nb	The periodically forced damped Duffing oscillator and a dynamic animation of the Poincaré section.
nb#5	numerics_11e.nb	Numerical solution of the Rössler attractor. The Poincaré section of the Rössler attractor.
nb#6	numerics_11f.nb	Periodic orbits in the Rössler attractor for varied c value.
nb#7	numerics_11g.nb	Higher-dimensional attractor reconstruction from 1-D time-domain signals. Examples: a sine wave, a quasi-periodic signal, and all three variables $x(t)$, $y(t)$ and $z(t)$ of the Lorenz attractor calculated for the *usual* chaotic parameter values.

Lecture 12

Link	File name	Description
nb#1	numerics_12.nb	Dynamics of the Rössler attractor: an animation of particle tracking in a 3-D flow.
nb#2	numerics_12b.nb	The Cantor set fractal generator.
nb#3	numerics_12c.nb	The von Koch curve and the Koch Snowflake fractal generator.
nb#4	numerics_12d.nb	The Hénon map: fixed points and map iterates, chaotic and periodic (period-2) solutions. Local fractal microstructure of the Hénon map.
nb#5	numerics_12e.nb	The Hénon map: mapping and iterating a rectangle.
nb#6	numerics_12f.nb	The Hénon map: calculation and plotting orbit diagrams, featuring $x(a)$, $x(b)$, $y(a)$, $y(b)$ orbit diagrams.
nb#7	numerics_12g.nb	The Hénon map: alternative ways of calculating the map iterates.
nb#8	numerics_12h.nb	Random examples of linear and nonlinear 2-D maps with nonlinear system matrices: calculation and plotting of map iterates.
nb#9	numerics_12i.nb	The Gingerbreadman map calculated for initial conditions $x_0 = -0.3$, $y_0 = 0$ showing $4 \cdot 10^4$ iterates.

Lecture 13

Link	File name	Description
nb#1	numerics_13.nb	Dynamic animation of the iterates of a 2-D map given in the following form: $$\begin{aligned} r_{n+1} &= r_n^2, \\ \theta_{n+1} &= \theta_n - \sin\theta_n, \end{aligned} \qquad (1)$$ where r is the distance in radial direction and θ is the angle. The notion of a basin of attraction.
nb#2	numerics_13b.nb	Iterates of linear 2-D maps and their phase portraits.

nb#3	numerics_13c.nb	Examples of linear dynamics of 2-D maps. Classification of fixed points of linear maps: node, saddle, degenerate node, star, rotation.
nb#4	numerics_13d.nb	The Hénon map dynamics: mapping and iterating a rectangle with the map; period-2 cycle trajectory in xy-plane; orbit diagram.
nb#5	numerics_13e.nb	Linear analysis of the Hénon map. Stability of its fixed points and their local dynamics.
nb#6	numerics_13f.nb	An example of discrete-time chaotic attractor: chaotic 2-D map given in polar coordinates.

Lecture 14

Link	File name	Description
nb#1	numerics_14.nb	Interactive fractal tree generator.
nb#2	numerics_14b.nb	Comparison of power spectra of periodic, quasi-periodic, and chaotic signals. The file is featuring the examples used previously throughout the course.
nb#3	numerics_14c.nb	Complex valued 1-D maps: Generation of the Mandelbrot and corresponding Fatou sets, the Multibrot sets, generation of the Mandelbrot-like complex sets with the corresponding Fatou sets using arbitrary polynomials and rational functions.
nb#4	numerics_14d.nb	The Hilbert curve: a space filling curve with fractal dimension $d = 2.0$.
nb#5	numerics_14e.nb	The Mandelbrot set for $\mathrm{Im}(z) = 0$ and $\mathrm{Im}(c) = 0$: cobweb diagram, orbit diagram and map iterates.
nb#6	numerics_14f.nb	The Mandelbrot set for $\mathrm{Im}(z) = 0$ and $\mathrm{Im}(c) = 0$: cobweb diagram, the set itself and its iterates.
nb#7	numerics_14g.nb	Interactive program that plots and measures the length of the von Koch snowflake proto-fractal using different measurement resolutions.
nb#8	numerics_14h.nb	Interactive program that plots and measures the length of the Estonian coastline using different measurement resolutions.

Lecture 15

Link	File name	Description
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Coursework

Link	File name	Description
nb	Analysis_tool.nb	Analysis tool for performing linear analysis of 2-D nonlinear systems, plotting and visualisation of 2-D and 3-D nonlinear systems (Wolfram Mathematica notebook).
ipynb	TBA	Analysis tool for performing linear analysis of 2-D nonlinear systems, plotting and visualisation of 2-D and 3-D nonlinear systems (Jupyter notebook).

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