COURSEWORK ASSIGNMENTS:

Nonlinear Dynamics

YFX1560

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Analysis of Dynamical Systems

Variant 1

Part 1: Liénard type equation

Analyse 2-D system.

$$\ddot{x} + \mu \left(x^2 - 1\right) \dot{x} + \tanh(x) = 0,$$

where μ is the constants and it can be shown that for $\mu > 0$ only one periodic solution exists.

Part 2: Rössler attractor[1]

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = b + z(x - c), \end{cases}$$

where a, b ja c are constants.

Parameter	Version **1.1**	Version **1.2**
a	0.2	0.1
b	0.2	0.1
c	5.7	14.0

[1]Some aspects of the dynamics of this system are discussed during the lectures.

Analysis of Dynamical Systems

Variant 2

Part 1: Bacterial respiration by Fairén and Velarde

Analyse 2-D system.

$$\begin{cases} \dot{x} = B - x - \dfrac{xy}{1+Qx^2}, \\ \dot{y} = A - \dfrac{xy}{1+Qx^2}, \end{cases}$$

where constants A, B and Q are positive.

Parameter	Version **2.1**	Version **2.2**
A	2.0	2.0
B	3.0	3.0
Q	6.5	3.5

Part 2: Lorenz attractor[1]

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz, \end{cases}$$

where σ, r, and b are constants.

Parameter	Value
σ	10
b	$\frac{8}{3} = 2.(6)$
r	28

[1] Some aspects of the dynamics of this system are discussed during the lectures.

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Variant 3

Part 1: Brusselator

Analyse 2-D system.

$$\begin{cases} \dot{x} = a - x - bx + x^2 y, \\ \dot{y} = bx - x^2 y, \end{cases}$$

where a and $b > 0$ are constants.

Parameter	Version **3.1**	Version **3.2**
a	0.4	1.0
b	1.2	1.7

Part 2: Newton–Leipnik chaotic system

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -ax + y + 10yz, \\ \dot{y} = -x - 0.4y + 5xz, \\ \dot{z} = bz - 5xy, \end{cases}$$

where a, $b > 0$ and $a = 0.4$ and $b = 0.175$.

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Variant 4

Part 1: Ueda oscillator

Analyse 2-D system.

$$\ddot{x} + k\dot{x} + x^3 = B\cos(\omega t),$$

where k, B, and ω are constants.

Parameter	Version **4.1**	Version **4.2**
k	0.05	0.05
B	7.5	12
ω	1.0	1.317

Part 2: Thomas' cyclically symmetric attractor

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = \sin(y) - bx, \\ \dot{y} = \sin(z) - by, \\ \dot{z} = \sin(x) - bz, \end{cases}$$

where b is a constant and corresponds to how dissipative the system is, and acts as a bifurcation parameter. Select $b < 0.208186$ and $b \neq 0$.

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Variant 5

Part 1: Duffing oscillator[1]

Analyse 2-D system.

$$\ddot{x} + \delta\dot{x} - \beta x + \alpha x^3 = f\cos(\omega t),$$

where α, β, δ, ω, and f are constants.

Parameter	Version **5.1**	Version **5.2**
α	100	1
β	1	1
δ	1	0.15
ω	3.679	1.12
f	2.4	0.3

Part 2: Sprott A, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -x + yz, \\ \dot{z} = 1 - y^2. \end{cases}$$

[1]Some aspects of the dynamics of this system are discussed during the lectures.

Analysis of Dynamical Systems

Variant 6

Part 1: Chemical reaction (chlorine dioxide–iodine–malonic acid reaction)

Analyse 2-D system.

$$\begin{cases} \dot{x} = a - x - \dfrac{4xy}{1+x^2}, \\ \dot{y} = bx\left(1 - \dfrac{y}{1+x^2}\right), \end{cases}$$

where a and b are constants.

Parameter	Version **6.1**	Version **6.2**
a	10	10
b	4	2

Part 2: Lorenz-84 model

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -y^2 - z^2 - ax + aF, \\ \dot{y} = xy - bxz - y + G, \\ \dot{z} = bxy + xz - z, \end{cases}$$

where a, b, F and G are constants.

Parameter	Value
a	0.25
b	4.0
F	8.0
G	1.0

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Variant 7

Part 1: Glycolysis[1]

Analyse 2-D system.

$$\begin{cases} \dot{x} = -x + ay + x^2 y, \\ \dot{y} = b - ay - x^2 y, \end{cases}$$

where a and b are constants.

Parameter	Value
a	0.08
b	0.6

Part 2: Simplest dissipative flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\dddot{x} + A\ddot{x} - \dot{x}^2 + x = 0,$$

where constant $A = 2.017$.

[1]Some aspects of the dynamics of this system are discussed during the lectures.

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Variant 8

Part 1: Van der Pol oscillator[1]

Analyse 2-D system.

$$\ddot{x} - b\left(1 - x^2\right)\dot{x} + x = 0,$$

where b is a constants.

Parameter	Version **8.1**	Version **8.2**
b	5	1

Part 2: Sprott B, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x - y, \\ \dot{z} = 1 - xy. \end{cases}$$

[1]Some aspects of the dynamics of this system are discussed during the lectures.

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Variant 9

Part 1: Forced Van der Pol oscillator

Analyse 2-D system.

$$\ddot{x} - b\left(1 - x^2\right)\dot{x} + x = F\cos(\omega t),$$

where b, F, and ω are constants.

Parameter	Version **9.1**	Version **9.2**
b	5	1
F	4	2
ω	3.717	6.171

Part 2: Sprott C, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x - y, \\ \dot{z} = 1 - x^2. \end{cases}$$

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Variant 10

Part 1: Morse equation

Analyse 2-D system.

$$\ddot{x} + \alpha\dot{x} + \beta\left(1 - \mathrm{e}^{-x}\right)\mathrm{e}^{-x} = F\cos(\omega t),$$

where α, β, F, and ω are constants.

Parameter	Value
α	0.8
β	8
F	2.5
ω	4.171

Part 2: Sprott E, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x^2 - y, \\ \dot{z} = 1 - 4x. \end{cases}$$

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Variant 11

Part 1: Nerve impulse action potential (Bonhoeffer-Van der Pol)

Analyse 2-D system.

$$\begin{cases} \dot{x} = x - \dfrac{x^3}{3} - y + F\cos(\omega t), \\ \dot{y} = c(x + a - by), \end{cases}$$

where a, b, c, F, and ω are constants.

Parameter	Value
a	0.7
b	0.8
c	0.1
F	0.6
ω	1

Part 2: Sprott G, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = 0.4x + z, \\ \dot{y} = xz - y, \\ \dot{z} = -x + y. \end{cases}$$

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Variant 12

Part 1: Lotka-Volterra equations[1] (predator–prey model)

Analyse 2-D system.

$$\begin{cases} \dot{x} = ax - xy, \\ \dot{y} = xy - by, \end{cases}$$

where a and b are constants.

Parameter	Value
a	2
b	1

Part 2: Sprott I, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -0.2y, \\ \dot{y} = x + z, \\ \dot{z} = x + y^2 - z. \end{cases}$$

[1]Some aspects of the dynamics of this system are discussed during the lectures.

Analysis of Dynamical Systems

Variant 13

Part 1: Duffing-Van der Pol oscillator

Analyse 2-D system.

$$\ddot{x} - \alpha\left(1 - x^2\right)\dot{x} - \omega_0^2 x + \beta x^3 = F\cos(\omega t),$$

where α, β ω_0, ω, and F are constants.

Parameter	Version **13.1**	Version **13.1**
α	2.3	2.3
β	1.0	1.0
ω_0	1.1	1.1
F	3.0	3.0
ω	1.73	6.73

Part 2: Sprott K, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = xy - z, \\ \dot{y} = x - y, \\ \dot{z} = x + 0.3z. \end{cases}$$

Analysis of Dynamical Systems

Variant 14

Part 1: Velocity dependent forced oscillation

Analyse 2-D system.

$$\left(1 + \lambda x^2\right) \ddot{x} - \lambda x \dot{x}^2 + \alpha \dot{x} + \omega_0^2 x = F \sin(\omega t),$$

where λ, α, ω_0, ω, and F are constants.

Parameter	Version **14.1**	Version **14.2**
λ	2.4	1.4
α	1.1	4.1
ω_0	5.0	4.3
F	2.7	6.1
ω	3.78	2.98

Part 2: Sprott M, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -z, \\ \dot{y} = -x^2 - y, \\ \dot{z} = 1.7 + 1.7x + y. \end{cases}$$

Analysis of Dynamical Systems

Variant 15

Part 1: Particle in a double well potential with linear damping

Analyse 2-D system.

$$\ddot{x} + \gamma\dot{x} - \frac{1}{2}\left(1 - x^2\right)x = 0,$$

where γ is the coefficient of damping and $\gamma = 0.1$.

Part 2: Modified Chen attractor

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - xz + cy + m, \\ \dot{z} = xy - bz, \end{cases}$$

where the constants have the following values: $a = 35$, $b = 3$, $c = 28$, $m = 23.1$.

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Variant 16

Part 1: Bonhoeffer-Van der Pol oscillator

Analyse 2-D system.

$$\begin{cases} \dot{x} = x - \dfrac{x^3}{3} - y + A, \\ \dot{y} = c(x + a - by), \end{cases}$$

where A, a, b, and c are constants.

Parameter	Version **16.1**	Version **16.2**
a	0.7	0.7
b	0.8	0.8
c	0.1	0.1
A	0.6	0.3

Part 2: Sprott O, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = y, \\ \dot{y} = x - z, \\ \dot{z} = x + xz + 2.7y. \end{cases}$$

Analysis of Dynamical Systems

Variant 17

Part 1: Nameless system #1

Analyse 2-D system

$$\begin{cases} \dot{x} = (x+2)\sqrt{2x^2+1} - \arctan(y-2), \\ \dot{y} = \sin(x+2) + \mathrm{e}^{3y-6} - 1, \end{cases}$$

where the fixed point is $(x^*, y^*) = (-2, 2)$.

Part 2: Sprott Q, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -z, \\ \dot{y} = x - y, \\ \dot{z} = 3.1x + y^2 + 0.5z. \end{cases}$$

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Variant 18

Part 1: Nameless system #2

Analyse 2-D system

$$\begin{cases} \dot{x} = (x+1)^2\cos(2x) - 4\ln(y-3), \\ \dot{y} = \sin(x+1) + \dfrac{2(y-4)^2}{y+1}, \end{cases}$$

where the fixed point is $(x^*, y^*) = (-1, 4)$.

Part 2: Sprott S, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -x - 4y, \\ \dot{y} = x + z^2, \\ \dot{z} = 1 + x. \end{cases}$$

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Variant 19

Part 1: Nameless system #3

Analyse 2-D system

$$\begin{cases} \dot{x} = x - \sin(3(x-1)) - (y-1)^2 \tan(y) - 1, \\ \dot{y} = 2\sin^2(x-1) + y^2 - y^6, \end{cases}$$

where the fixed point is $(x^*, y^*) = (1, 1)$.

Part 2: Sprott H, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = -y + z^2, \\ \dot{y} = x + 0.5y, \\ \dot{z} = x - z. \end{cases}$$

Analysis of Dynamical Systems

Variant 20

Part 1: Liénard equation

Analyse 2-D system.

$$\ddot{x} - \left(\mu - x^2\right)\dot{x} + x = 0,$$

where μ is a constant.

Parameter	Version **20.1**	Version **20.2**
μ	-0.33	1.0

Part 2: Chen attractor

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - xz + cy, \\ \dot{z} = xy - bz, \end{cases}$$

where the constants have values $a = 35$, $b = 3$, $c = 28$.

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Variant 21

Part 1: Nameless system #4

Analyse 2-D system

$$\begin{cases} \dot{x} = -x - y + x\left(x^2 + 2y^2\right), \\ \dot{y} = x - y + y\left(x^2 + 2y^2\right). \end{cases}$$

Part 2: Sprott L, chaotic flow

Determine whether the following 3-D system represents a strange attractor or not.

$$\begin{cases} \dot{x} = y + 3.9z, \\ \dot{y} = 0.9x^2 - y, \\ \dot{z} = 1 - x. \end{cases}$$

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