# Lectures on nonlinear dynamics

| Lecture no. | **Topics covered** (during a typical semester) |
|---|---|
| Lecture 1 | *Practical course navigation*, introduction, magnetic pendulum, history, dynamics, nonlinearity, ordinary homogeneous differential equation, 1-D problem, phase space, phase portrait, fixed point and its stability. |
| Lecture 2 | 1-D problems, linear analysis, bifurcation, bifurcation diagram, impossibility of oscillations in 1-D systems. |
| Lecture 3 | Over-damped bead on a rotating hoop, dimensional analysis and dimensionless form of equations of motion, introduction to 2-D systems, uniqueness of solution and phase space trajectories. |
| Lecture 4 | 2-D homogeneous linear systems, classification of fixed points in 2-D systems, the Lyapunov stability, basin of attraction. |
| Lecture 5 | 2-D homogeneous nonlinear systems, linearisation of 2-D systems about its fixed points, stability and type of nonlinear fixed points, system's Jacobian matrix, homoclinic orbit, stable and unstable manifolds, conservative systems. |
| Lecture 6 | 2-D conservative systems and centres, closed orbits and limit-cycles, null-cline, heteroclinic orbit, the Dulac's criterion, the Poincaré-Bendixson theorem. |
| Lecture 7 | Classification of bifurcations in 2-D, bifurcations of fixed points, the Hopf bifurcation, the supercritical and subcritical Hopf bifurcations, bifurcations of closed orbits, hysteresis on the level of cycles, examples of dynamical instabilities. |
| Lecture 8 | Quasi-periodicity, 3-D and higher order systems, 3-D limit-cycle, introduction to chaos (deterministic chaos, chaos theory), chaotic water wheel, the Lorenz attractor, *coursework requirements.* |
| Lecture 9 | Attractor and strange attractor, chaos, analysis of the Lorenz attractor, transient and intermittent chaos, the Lyapunov exponents, the Kolmogorov entropy, predictability horizon, examples of chaos, cobweb diagram and recurrence map or recurrence relation (1-D map). |
| Lecture 10 | 1-D maps, the Lorenz map, the logistic map, sine map, period doubling bifurcation, tangent bifurcation, transient and intermittent chaos in maps, orbit diagram (or the Feigenbaum diagram), the Feigenbaum constants, universals of unimodal maps, universal route to chaos. |
| Lecture 11 | Feigenbaum's analysis of period doubling, superstability of fixed points and period-p points, renormalisation, universal limiting function, discrete-time dynamics analysis methods, the Poincaré section, the Poincaré map, the Lorenz section, attractor reconstruction. |
| Lecture 12 | Fractal microstructure of strange attractors, introduction to fractal geometry, fractal dimension, similarity and box dimensions, the Cantor set, the von Koch curve, 2-D maps, the Hénon map. |
| Lecture 13 | Stretching–folding–re-injection dynamics in cobweb diagrams, basin of attraction (maps), sensitive dependence on initial conditions (maps), linearisation of 2-D maps, classification of fixed points in 2-D maps, linear analysis of the Hénon map, video feedback effect. |
| Lecture 14 | Fractals and fractal geometry, coastline paradox, spectral characteristics of dynamical systems, 1-D complex valued maps, the Mandelbrot set, the Fatou and Julia sets, the Mandelbrot set and nonlinear dynamical systems, introduction to applications of fractal geometry and chaos: synchronisation in nature. |
| Lecture 15 | Concluding remarks and summary, overview of the course, *exam.* |

# Logical structure of the course

Logical structure refers to the way information in the course is organised. Logical structure indicates how the course is built, as opposed to what the course is teaching/presenting.

- 1-D systems (homogeneous ODEs)
- 2-D systems
  - Classification of fixed points (linear systems)
  - Classification of bifurcations
- *Quasi-periodisity.*
- 3-D systems and higher order systems
  - Strange attractors and chaos
  - Fractal geometry
  - Fractal microstructure of strange attractors
- *Poincaré map*
- 1-D maps and period doubling
- 2-D maps
  - Classification of fixed points (linear maps)
- Higher dimensional maps and complex valued maps

# Syllabus of nonlinear dynamics course

Nonlinearity and nonlinear world. The sources of nonlinearities due to physics and geometry. Nonlinear mathematical models. Basic theory of ODEs and practical numerical integration. Attractors, bifurcations. Mathematically determined chaos. Orbit and the Feigenbaum diagrams, the Lorenz section, the Poincaré section. Fractality, fractal structures. Recurrence maps and feedback loops. The Mandelbrot set, and the Fatou and Julia sets their connection to nonlinear dynamical systems. Fractal dimensions. Universal route to chaos via period doubling cascade. Identification of chaotic processes. Analytical and numerical methods, Lyapunov exponent. Entropy. Horizon of predictability. Examples from physics, mechanics, biology and ecology. Applications of chaos theory and fractal geometry.

# Mittelineaarne dünaamika, kursuse tutvustus

Mittelineaarne maailmapilt. Mittelineaarsuse füüsikaline ja geomeetriline tagapõhi. Mittelineaarsed matemaatilised mudelid. Atraktorid. Bifurkatsioonid. Matemaatiliselt determineeritud kaos. Orbiidi ja Feigenbaumi diagarmmid, Lorenz lõige, Poincaré lõige. Fraktaalsed struktuurid. Kujutised. Mandelbroti hulk ning Fatou ja Julia hulgad nende seos mittelinearsete süsteemidega. Fraktaalsed dimensioonid. Kaootiliste protsesside identifitseerimine. Ljapunovi eksponent. Entroopia. Ennustatavuse horisont. Näited füüsikast, mehaanikast, bioloogiast ja ökoloogiast. Kaose teooria ja fraktaalse geomeetria rakendused.

# Course grading criteria

Nonlinear Dynamics, YFX1520
ECTS credits: 6.0

| Learning outcome | Assessment method | Assessment criterion |
|---|---|---|
| Student knows the behaviour and methods of analysis of nonlinear systems. Student recognises and can analyse chaotic regimes. Student can explain the fundamentals and ohter concepts used in nonlinear dynamics: classical models, types of fixed points, bifurcations, emergence of chaos, methods of analysis, fractality and fractal geometry. | **1. Lecture attendance** | Student must attend at least 56.3962445% of lectures/exercise lessons. |
| | **2. Coursework** | Analysis of nonlinear dynamical systems (two-part coursework) is performed using the analysis methods presented during the lectures. Student understands the dynamics of given problem/ problems. |
| | **3. Exam** | Student knows the fundamentals, other notions and concepts of nonlinear dynamics. Student is able to practically apply the learned knowledge. |
| | **Criterion for passing the course:** Nonlinear dynamics is a course that ends with an exam. All general rules and rights dictated and provided by TalTech to the students apply. Successfully completed coursework is prerequisite for taking the final exam.<br>The final grade is formed as follows:<br>**5 (excellent)** — student demonstrates excellent knowledge: He/she solves without mistakes typical problems of the course; He/she knows perfectly the concepts and relations of the subject and required reasoning and proofs (91%—100% from the capacity of the course);<br>**4 (very good)** — student demonstrates very good knowledge: He/she solves typical problems of the course making small number of mistakes; He/she knows perfectly the concepts and relations of the subject and required reasoning and proofs (81%—90% from the capacity of the course);<br>**3 (good)** — student demonstrates good knowledge: He/she solves typical problems of the course with small number of mistakes; He/she knows the concepts and relations of the subject and required reasoning and proofs but makes certain number of mistakes (71%—80% from the capacity of the course);<br>**2 (satisfactory)** — student demonstrates satisfactory knowledge: He/she solves typical problems of the course making mistakes; He/she knows the concepts and relations of the subject but makes in required reasoning and proofs many mistakes (61%—70% from the capacity of the course);<br>**1 (poor)** — student demonstrates scarce knowledge: He/she solves typical problems of the course with many mistakes; He/she knows superficially the concepts and relations of the subject and makes in required reasoning and proofs great number of mistakes (51%—60% from the capacity of the course). | |