Revision questions

Following are the questions and topics to consider while preparing for the final exam. Similar non-verbatim questions will be included in the exam.

Lecture 1

1. What is dynamics?
2. Name a dynamical system.
3. Define nonlinearity.
4. Determine if the following equations/systems are linear or nonlinear:

$$\dot{x} = \sin x, \tag{1}$$

$$\dot{x} = \ln x, \tag{2}$$

$$\begin{cases} \dot{x} = y, \\ \dot{y} = xy, \end{cases} \tag{3}$$

$$\ddot{x} + \dot{x} + x = 0. \tag{4}$$

5. What is ordinary homogeneous differential equation?
6. Define 1-D dynamical system. Name a 1-D problem.
7. What is phase space?
8. What is phase portrait?
9. Sketch 1-D phase portrait of the following systems:

$$\dot{x} = 3\cos x, \tag{5}$$

$$\dot{x} = 0.5x^2 - 1, \tag{6}$$

$$\dot{x} = x^3. \tag{7}$$

10. What is a fixed point?
11. How to a find fixed point of a differential equations?
12. Find the fixed point or points of the following system:

$$\ddot{x} + \dot{x} + x = 0. \tag{8}$$

13. Explain fixed point stability.
14. What is linear analysis of a system?

Lecture 2

15. What does linearisation of a nonlinear system imply?
16. Linearise the following 1-D system

$$\dot{x} = x^3 - x \tag{9}$$

17. What is bifurcation?
18. What is bifurcation diagram?
19. What is saddle-node bifurcation?
20. What is transcritical bifurcation?
21. What is pitchfork bifurcation?
22. What is supercritical pitchfork bifurcation?
23. What is subcritical pitchfork bifurcation?
24. What is normal form in the context of bifurcations?
25. Are oscillation possible in 1-D systems?
26. Why are oscillations impossible in 1-D systems?
27. What does uniqueness of solutions imply in the context of phase space trajectories?

Lecture 3

28. What is symmetry-broken solution?
29. What is dimensional analysis of an equation of motion?
30. What is dimensionless form of an equation?
31. What is normalised form of an equation?
32. How many initial conditions does first-order ODE have?
33. How many initial conditions does second-order ODE have?
34. Explain the notion of different time scales of a dynamical system.
35. Derive the dimensionless form of the following equation of motion:

$$m\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + b\frac{\mathrm{d}u}{\mathrm{d}t} + ku = 0, \tag{10}$$

where u is the displacement, m is the mass and t is the time. Additionally, determine the dimensions of damping coefficient b and stiffness k.

36. Derive the dimensionless form of the following nonhomogeneous equation of motion:

$$m\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + b\frac{\mathrm{d}u}{\mathrm{d}t} + ku = F_0\cos\omega_0 t, \tag{11}$$

where u is the displacement, m is the mass, F_0 and ω_0 are the driving force parameters, and t is the time. Additionally, determine the dimensions of damping coefficient b, stiffness k, driving force F_0 and driving force frequency ω_0.

Lecture 4

37. How to plot a 2-D phase portrait of a system?
38. What are 2-D homogeneous linear systems?
39. What are non-homogeneous systems?
40. Classification of fixed points in 2-D systems.
41. Sketch a saddle node fixed point.
42. Sketch a stable node fixed point.
43. Sketch an unstable node fixed point.
44. Sketch a stable spiral (fixed point).
45. Sketch an unstable spiral (fixed point).
46. Sketch a center (fixed point).
47. Sketch a stable non-isolated fixed point.
48. Sketch an unstable non-isolated fixed point.
49. What are 2-D homogeneous nonlinear systems?
50. What does it mean that a fixed point is Lyapunov stable?
51. Give an example of Lyapunov stable fixed point.

Lecture 5

52. Provide an example of nonlinear 2-D system.
53. Explain linearisation of 2-D systems about fixed points.
54. Can all nonlinear systems be linearised with the aim of identifying their fixed point type?
55. Linearise the following system

$$\begin{cases} \dot{x} = 4x - 4xy, \\ \dot{y} = -9y + 18xy. \end{cases} \tag{12}$$

56. Without taking derivatives, linearise the following systems:

$$\begin{cases} \dot{x} = -y + xy, \\ \dot{y} = x, \end{cases} \tag{13}$$

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = x + y^2. \end{cases} \tag{14}$$

57. Define the Jacobian matrix of a system.
58. Sketch a homoclinic orbit.
59. Define conservative dynamical system.

Lecture 6

60. Expand on the connection between 2-D conservative systems and centres.
61. Sketch a heteroclinic orbit.
62. What is limit-cycle?
63. Sketch a stable limit-cycle.
64. Sketch an unstable limit-cycle.
65. Sketch a half-stable (stable from outside) limit-cycle.
66. Sketch a half-stable (stable from inside) limit-cycle.
67. Define and sketch a null-cline.
68. What is the Dulac's criterion?
69. State the Poincaré-Bendixson theorem.
70. Does the Poincaré-Bendixson theorem apply to 3-D systems?
71. Can chaos occur in 2-D systems?

Lecture 7

72. Classification of bifurcations in 2-D.
73. What is the Hopf bifurcation?
74. What is the supercritical Hopf bifurcation?
75. What is the subcritical Hopf bifurcation?
76. What are global bifurcations of closed orbits?
77. Name some global bifurcations of closed limit-cycles.
78. What is a saddle-node coalescence (or bifurcation) of limit-cycles?
79. What is hysteresis on the level of cycles?
80. Name dangers associated with the *Hopf* bifurcation.
81. What is a saddle-node infinite period bifurcation?
82. What is a (saddle-loop or) homoclinic bifurcation?
83. Name examples of dynamical instabilities.

Lecture 8

84. What is quasi-periodicity?
85. Can quasi-periodic system generate a chaotic solution? Why?
86. Do limit-cycles exist in 3-D phase spaces? Sketch an example.
87. What are 3-D and higher order systems?
88. What is chaos in the context of dynamical systems (deterministic chaos, chaos the-

ory)?
89. Name properties of chaotic systems.
90. What does it mean that a chaotic system has a SRB measure (the Sinai-Ruelle-Bowen measure)?
91. What is chaotic water wheel?
92. What is the Lorenz attractor?

Lecture 9

93. Define attractor.
94. Define strange attractor.
95. What is the difference between a strange attractor and an attractor?
96. Name properties of the Lorenz attractor.
97. What are the Lyapunov exponents?
98. What is the Lyapunov exponent?
99. What determines the number of Lyapunov exponents?
100. What is the Kolmogorov entropy?
101. What is predictability horizon?
102. What is the Lyapunov time?
103. Can a long-term solution to a chaotic system be predicted? Explain.
104. List some examples of chaos in nature.
105. What is final-state sensitivity?
106. What is chaos?
107. What is intermittent chaos?
108. What is transient chaos?
109. What is crisis?
110. What is strange non-chaotic attractor?

Lecture 10

111. What is cobweb diagram?
112. What is recurrence map or recurrence relation?
113. What is 1-D map?
114. How to find fixed points of 1-D maps?
115. What is the Lorenz map?
116. What is the logistic map?
117. What is sine map?
118. What is period doubling?
119. What is period doubling bifurcation?
120. What is tangent bifurcation?
121. Do odd number periods (period-p orbits) exist in chaotic systems?
122. Do even number periods (period-p orbits) exist in chaotic systems?
123. Can maps produce transient chaos?
124. Can maps produce intermittency?
125. Can maps produce intermittent chaos?
126. What is orbit diagram (or the Feigenbaum diagram)?
127. What are the Feigenbaum constants?

Lecture 11

128. What are the values of the Feigenbaum constants?
129. What are the Feigenbaum constants (more in-depth answer)?
130. Define superstable fixed point of a map.
131. Define superstable period-p point (or period-p cycle) of a map.
132. What are the universals of unimodal maps?
133. What is the universal route to chaos?
134. Idea behind renormalisation?
135. What are the universal limiting functions in the context of maps?
136. Name discrete-time dynamics analysis methods.
137. What is the Poincaré section?
138. What is the Poincaré map (return map)?
139. What is the Lorenz section?

Lecture 12

140. How is it possible for two trajectories with almost equal initial conditions to deviate exponentially and remain attracted to a strange attractor (remain in the basin of attraction)?
141. Give an example of a dynamics that features global stability and local instability.
142. Explain fractal microstructure of strange attractors.
143. Define fractal.
144. What is pre-fractal?
145. Construct a simple fractal (general idea).
146. What is self-similarity?
147. What is scale-invariance?
148. Are all fractals self-similar?
149. What is fractal geometry?
150. What is fractal dimension?
151. What are similarity and box counting dimensions?
152. What is a power law?
153. What is the Cantor set?
154. What is the von Koch curve?
155. What is a 2-D map?
156. How to find fixed points of 2-D maps (period-1 point)?
157. What is the Hénon map, its significance?

Lecture 13

158. Is it possible to see stretching–folding–re-injection dynamics in cobweb plots?
159. What does linearisation of a nonlinear 2-D map imply?
160. Define sensitive dependence on initial conditions in maps.
161. Define basin of attraction of a map.
162. Sketch a saddle fixed point.
163. Sketch a stable node (sink) fixed point.
164. Sketch an unstable node (source) fixed point.
165. What are improper oscillations of map iterates?
166. What is the cause of improper oscillation of map iterates in terms of eigenvalues?
167. What is the video feedback effect?

Lecture 14

168. Define fractal (technical definition).
169. Define pre-fractal.
170. Explain the coastline paradox.
171. Can a coastline be described with Euclidean geometry?
172. What determines spectral characteristics of dynamical systems?
173. What is a 1-D complex valued map?
174. What are the Mandelbrot set and the Fatou sets?
175. What is the Julia set?
176. Assuming $z = x + \mathrm{i}y$, $c = r + \mathrm{i}s$, and $z, c \in \mathbb{C}$, show that map in the form
$$\begin{cases} x_{n+1} = x_n^2 - y_n^2 + r, \\ y_{n+1} = 2x_n y_n + s, \end{cases} \tag{15}$$
is the real counterpart of the Mandelbrot set.
177. What is the physical meaning of the Mandelbrot set?
178. What is the physical meaning of the Fatou sets?
179. What is the generalised Mandelbrot set also known as the Multibrot set?
180. Name an example of self-similar phenomena in nature.

NB! Positively graded coursework is a prerequisite for taking the exam.