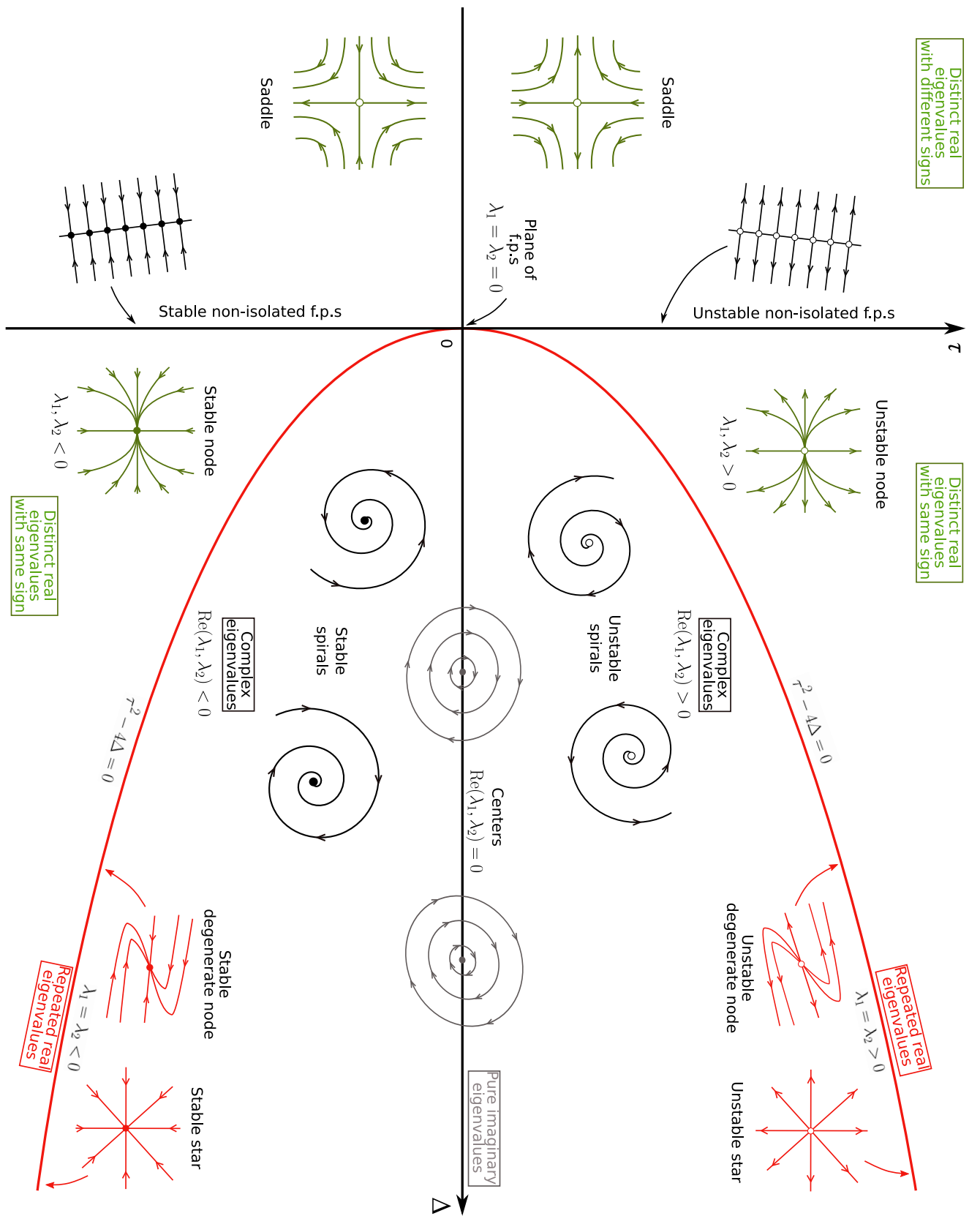


The classification of fixed points of linear homogeneous 2-D systems $\dot{x} = Ax$ where trace $\tau = \lambda_1 + \lambda_2$ and determinant $\Delta = \lambda_1\lambda_2$ are determined by 2×2 system matrix A .



The presented classification can also be summarised in a concise flowchart:

- ▶ if $\Delta < 0$:
Isolated fixed point
CASE 1: **Saddle point**¹
- ▶ if $\Delta = 0$:
Non-isolated fixed points
 - if $\tau < 0$:
CASE 5a: **Line of stable fixed points**²
 - if $\tau = 0$:
CASE 5b: **Plane of fixed points**³
 - if $\tau > 0$:
CASE 5a: **Line of unstable fixed points**⁴
- ▶ if $\Delta > 0$:
Isolated fixed point
 - if $\tau < -\sqrt{4\Delta}$:
CASE 2a: **Stable node**⁵
 - if $\tau = -\sqrt{4\Delta}$:
 - if there is one uniquely determined eigenvector (the other is non-unique):
CASE 4a: **Stable degenerate node**⁶
 - if there are no uniquely determined eigenvectors (both are non-unique):
CASE 4b: **Stable star**⁷
 - if $-\sqrt{4\Delta} < \tau < 0$:
CASE 2b: **Stable spiral**⁸
 - if $\tau = 0$:
CASE 3: **Centre**⁹
 - if $0 < \tau < \sqrt{4\Delta}$:
CASE 2b: **Unstable spiral**¹⁰
 - if $\tau = \sqrt{4\Delta}$:
 - if there is one uniquely determined eigenvector (the other is non-unique):
CASE 4a: **Unstable degenerate node**¹¹
 - if there are no uniquely determined eigenvectors (both are non-unique):
CASE 4b: **Unstable star**¹²
 - if $\sqrt{4\Delta} < \tau$:
CASE 2a: **Unstable node**¹³

General notes:

- ▶ For 2-D linear systems, the above predictions are always accurate.
- ▶ For 2-D nonlinear systems, when the above are used as predictions of nonlinear dynamics:
 - The descriptions are always correct for cases 1, 5, 8, 10, and 13 but can be inaccurate for cases 2, 3, 4, 6, 7, 9, 11, and 12.
 - Ambiguous cases 6, 7, 11, and 12 at least have their stability correctly determined.
 - If the system is conservative, a prediction of case 9 is accurate.