Magnetic Pendulum in Three Magnetic Potentials

Introduction

This visual and interactive teaching aid is used to demonstrate the concept of *chaos* and chaotic properties of a nonlinear systems during the lectures of courses EMR0060 and YFX1520.

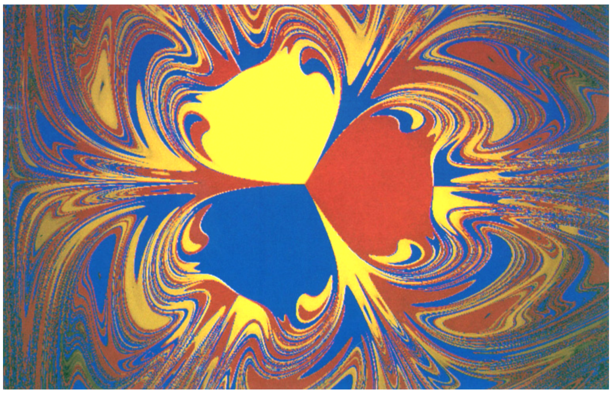
Equation of motion

The top view projection of pendulum position (x, y) is described by the following nonlinear equations of motion:

$$\begin{cases} \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + R\dfrac{\mathrm{d}x}{\mathrm{d}t} - \displaystyle\sum_{i=1}^{3} \dfrac{x_i - x}{\left(\sqrt{(x_i - x)^2 + (y_i - y)^2 + D^2}\right)^3} + Cx = 0, \\ \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + R\dfrac{\mathrm{d}y}{\mathrm{d}t} - \displaystyle\sum_{i=1}^{3} \dfrac{y_i - y}{\left(\sqrt{(x_i - x)^2 + (y_i - y)^2 + D^2}\right)^3} + Cy = 0, \end{cases} \tag{1}$$

where R is proportional to the air resistance and overall attenuation, C is proportional to the effects of gravity, the i-th magnet is positioned at (x_i, y_i) where $i = 1, 2, 3$, and D is the distance between the pendulum at rest position $(0, 0)$ and the plane of magnets.

Additionally, we assume that the pendulum length is *long* compared to the spacing between the magnets. Thus, we may assume for simplicity that the pendulum moves about on the xy-plane rather than on a sphere with a large radius.



H. Peitgen, *et al*, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag, 2004, pp. 707–414.

Figure 1: Basins of attraction of the three magnets shown with the red, blue and yellow colours.

The basin of attraction of system (1) shown in Fig. 1 is a fractal with an infinite complexity. The prediction of the end state of the pendulum for most initial conditions is impossible in practice especially if the integration time exceeds several multiples of Lyapunov time.

Further information for interested students

If students have any further questions or show a genuine interest in general topics of chaotic dynamics, you may direct them to the Author (e-mail: `dima@ioc.ee`, or URL: `https://www.ioc.ee/~dima/`).