

Index of .nb files

The files (totalling 80) are used during the lectures to demonstrate the nonlinear phenomena discussed. The .nb files can be opened with Wolfram Mathematica software. Use, modify and distribute under CC BY 4.0 licence.

Lecture 1

Link	File name	Description
nb#1	numerics_1.nb	Numerical integration of dynamical system describing a magnetic pendulum's motion. A pendulum in which a metal ball (or a magnet) is attached to its end and which is oscillating over a plane where a set of attractive magnets are present (2–6 magnets).
nb#2	numerics_1b.nb	Analytic solution to a nonlinear ODE. Numerical solution and phase portrait of 1-D flow $\dot{x} = \sin x$.
nb#3	numerics_1c.nb	Numerical solution and phase portrait of the 1-D logistic equation.

Lecture 2

Link	File name	Description
nb#1	numerics_2.nb	Bifurcation diagrams and 1-D systems: saddle node bifurcation, transcritical bifurcation, pitchfork bifurcation (supercritical), pitchfork bifurcation (subcritical).

Lecture 3

Link	File name	Description
nb#1	numerics_3.nb	1-D approximation corresponding to the over-damped bead on a hoop dynamics: the 1-D phase portrait and a family of numerical solutions corresponding to a set of initial conditions.
nb#2	numerics_3b.nb	Over-damped bead on a hoop dynamics: graphical determination of the fixed points and the bifurcation diagram (first-order model).
nb#3	numerics_3c.nb	Numerical solution and phase portrait of the over-damped bead on a hoop problem (second-order model).

Lecture 4

Link	File name	Description
nb#1	numerics_4.nb	Vector field plotting. Integrated solution and phase portrait of the damped harmonic oscillator.
nb#2	numerics_4b.nb	Calculation of matrix determinant, trace, eigenvalues and eigenvectors using computer. Classification of fixed points in 2-D linear homogeneous systems.

Lecture 5

Link	File name	Description
nb#1	numerics_5.nb	Effect of nonlinearity on a linear centre (a borderline case). Numerical solution and phase portrait of the same problem.
nb#2	numerics_5b.nb	Competitive cohabitation of sheep and rabbits (the Lotka-Volterra predator–prey model). Linear analysis of fixed points and system manifolds. Numerical solution and phase portrait.
nb#3	numerics_5c.nb	The Lotka-Volterra predator–prey model for sharks and fish (cyclic behaviour). Numerical solution and phase portrait.
nb#4	numerics_5d.nb	Interactive comparison of dynamics of the Liénard equation and its linearisation.
nb#5	numerics_5e.nb	Particle in a double well potential, conservative system and homoclinic orbits. Linear analysis of fixed points, numerical solution and phase portrait.

Lecture 6

Link	File name	Description
nb#1	numerics_6.nb	Mathematical pendulum and heteroclinic orbit. Integrated numerical solution.
nb#2	numerics_6b.nb	The Dulac's criterion and limit-cycles. A numerical example: integrated solution and phase portrait.
nb#3	numerics_6c.nb	The Dulac's criterion and limit-cycles. Another numerical example: integrated solution and phase portrait.

nb#4	numerics_6d.nb	The Poincaré-Bendixson theorem. Phase portrait of a nonlinear system given in polar coordinates.
nb#5	numerics_6e.nb	Glycolysis phase portrait and null-clines. Numerical solution of the glycolysis model.

Lecture 7

Link	File name	Description
nb#1	numerics_7.nb	Examples of bifurcations in 2-D systems: saddle-node bifurcation, transcritical bifurcation, supercritical and subcritical pitchfork bifurcations. Interactive code.
nb#2	numerics_7b.nb	Linear analysis of the system from the previous .nb file.
nb#3	numerics_7c.nb	Examples of bifurcations in 2-D systems: the supercritical Hopf bifurcation. Integrated solution and bifurcation diagram.
nb#4	numerics_7d.nb	Examples of bifurcations in 2-D systems: the subcritical Hopf bifurcation. Integrated solution and bifurcation diagram.
nb#5	numerics_7e.nb	Examples of bifurcations in 2-D systems: a saddle-node coalescence of limit-cycles. Bifurcation diagram.
nb#6	numerics_7f.nb	Examples of bifurcations in 2-D systems: SNIPER (saddle-node infinite period bifurcation). Integrated solution and phase portrait.
nb#7	numerics_7g.nb	Examples of bifurcations in 2-D systems: a homoclinic bifurcation or saddle-loop bifurcation (solitons). Integrated solution and phase portrait.

Lecture 8

Link	File name	Description
nb#1	numerics_8.nb	Trajectories on the surface of a torus: interactive 3-D plot. Periodic and quasi-periodic trajectories on the surface of a torus. Trefoil (for $p = 3$, $q = 2$) and cinquefoil knots (for $p = 5$ and $q = 2$).
nb#2	numerics_8b.nb	Quasi-periodic oscillators. Quasi-periodic decoupled system: periodic solution, quasi-periodic solution. The Fourier and power spectra of the solutions.
nb#3	numerics_8c.nb	Quasi-periodic coupled oscillations: the Fourier and power spectra of the solutions.
nb#4	numerics_8d.nb	Quasi-periodic coupled oscillations: interactive code.
nb#5	numerics_8e.nb	An example of a chaotic system: the Lorenz mill. Power spectra of the time-series solutions of the Lorenz mill system. Flow trajectory plotting in 3-D.
nb#6	numerics_8f.nb	An example of a chaotic system: the Lorenz attractor. Numerical integration of the Lorenz attractor (interactive 3-D plot, interactive 2-D (x - z projection) plot).
nb#7	numerics_8g.nb	A remark on 3-D phase portrait plotting. 3-D phase portrait and flow visualisation of the Lorenz attractor.
nb#8	Analysis_tool.nb	Coursework analysis tool: Mathematica notebook for analysing 2-D and 3-D systems. Use of the tool does not require coding skills.

Lecture 9

Link	File name	Description
nb#1	numerics_9.nb	Dissipative chaotic flow of the Lorenz attractor visualisation via particle tracking. Calculation of divergence of 3-D and higher dimensional flows.
nb#2	numerics_9b.nb	The Lyapunov exponent of 3-D chaotic systems, visual demonstration.
nb#3	numerics_9c.nb	Demonstration of the effect of calculation accuracy and precision on the numerical solution and the predictability horizon of a chaotic system. Transient and intermittent chaos.
nb#4	numerics_9d.nb	Numerical solutions of chaotic systems: the Lorenz attractor, the Rössler attractor, the Chen attractor, the modified Chen attractor, the Lu attractor, the Pan-Xu-Zhou attractor, the Bouali attractor, and the algebraically simplest chaotic dissipative flow by J. C. Sprott.
nb#5	numerics_9e.nb	Numerical solutions of chaotic systems: double mathematical pendulum with one and two sets of initial conditions. Phase portrait of the double pendulum system.
nb#6	numerics_9f.nb	Numerical solutions of chaotic systems: the gravitational three body problem (planar). Interactive code with several dynamic visualisation styles.
nb#7	numerics_9g.nb	Introduction to cobweb diagramming. Interactive cobweb diagrams of 1-D maps $x_{n+1} = rx_n$ and $x_{n+1} = r(x_n - x_n^2)$ where r is the control parameter.
nb#8	numerics_9h.nb	Interactive cobweb diagram and iterates of the Lorenz map.

nb#9	numerics_9i.nb	The Lorenz map: quantitatively accurate Lorenz mapping.
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Lecture 10

Link	File name	Description
nb#1	numerics_10.nb	Cobweb diagram and iterates of the (generalised) Lorenz map. Lyapunov exponent $\lambda(r)$ of the logistic map.
nb#2	numerics_10b.nb	Cobweb diagram of the logistic map. Orbit diagram of the logistic map. Period doubling bifurcation.
nb#3	numerics_10c.nb	The logistic map: cobweb diagram, orbit diagram and map iterates, the Lyapunov exponent.
nb#4	numerics_10d.nb	Period-3 window and intermittency in the logistic map.
nb#5	numerics_10e.nb	The sine map iterates, cobweb and orbit diagrams where $x_n \in [0, 1]$.
nb#6	numerics_10f.nb	The sine map iterates, cobweb and orbit diagrams where $x_n \in [-1, 1]$.

Lecture 11

Link	File name	Description
nb#1	numerics_11.nb	Comparison of superstable and stable fixed points in the logistic map.
nb#2	numerics_11b.nb	Calculation of universal limiting function g and value of the Feigenbaum constant α in the case of the logistic map using power series expansion: one and four term approximations.
nb#3	numerics_11c.nb	The periodically forced damped Duffing oscillator: numerical solutions and a phase portrait. The 2π -periodic Poincaré section of the Duffing oscillator (static).
nb#4	numerics_11d.nb	The periodically forced damped Duffing oscillator and a dynamic animation of the Poincaré section.
nb#5	numerics_11e.nb	Numerical solution of the Rössler attractor. The Poincaré section of the Rössler attractor.
nb#6	numerics_11f.nb	Periodic orbits in the Rössler attractor for varied c value.
nb#7	numerics_11g.nb	Higher-dimensional attractor reconstruction from 1-D time-domain signals. Examples: sine wave, quasi-periodic signal, $y(t)$ of the Lorenz attractor.

Lecture 12

Link	File name	Description
nb#1	numerics_12.nb	Dynamics of the Rössler attractor: an animation of particle tracking in a 3-D flow.
nb#2	numerics_12b.nb	The Cantor set fractal generator.
nb#3	numerics_12c.nb	The von Koch curve and the Koch Snowflake fractal generator.
nb#4	numerics_12d.nb	The Hénon map: fixed points and map iterates, chaotic and periodic (period-2) solutions. Local fractal microstructure of the Hénon map.
nb#5	numerics_12e.nb	The Hénon map: mapping and iterating a rectangle.
nb#6	numerics_12f.nb	The Hénon map: calculation and plotting orbit diagrams, featuring $x(a)$, $x(b)$, $y(a)$, $y(b)$ orbit diagrams.
nb#7	numerics_12g.nb	The Hénon map: alternative ways of calculating the map iterates.
nb#8	numerics_12h.nb	Random examples of linear and nonlinear 2-D maps with nonlinear system matrices: calculation and plotting of map iterates.
nb#9	numerics_12i.nb	The Gingerbreadman map calculated for initial conditions $x_0 = -0.3$, $y_0 = 0$ showing $4 \cdot 10^4$ iterates.

Lecture 13

Link	File name	Description
nb#1	numerics_13.nb	Dynamic animation of the iterates of a 2-D map given in the following form: $\begin{aligned} r_{n+1} &= r_n^2, \\ \theta_{n+1} &= \theta_n - \sin \theta_n, \end{aligned} \tag{1}$ <p>where r is the distance in radial direction and θ is the angle. The notion of a basin of attraction.</p>
nb#2	numerics_13b.nb	Iterates of linear 2-D maps and their phase portraits.

nb#3	numerics_13c.nb	Examples of linear dynamics of 2-D maps. Classification of fixed points of linear maps: node, saddle, degenerate node, star, rotation.
nb#4	numerics_13d.nb	The Hénon map dynamics: mapping and iterating a rectangle with the map; period-2 cycle trajectory in xy -plane; orbit diagram.
nb#5	numerics_13e.nb	Linear analysis of the Hénon map. Stability of its fixed points and their local dynamics.
nb#6	numerics_13f.nb	An example of discrete-time chaotic attractor: chaotic 2-D map given in polar coordinates.

Lecture 14

Link	File name	Description
nb#1	numerics_14.nb	Interactive fractal tree generator.
nb#2	numerics_14b.nb	Comparison of power spectra of periodic, quasi-periodic, and chaotic signals. The file is featuring the examples used previously throughout the course.
nb#3	numerics_14c.nb	Complex valued 1-D maps: Generation of the Mandelbrot and corresponding Fatou sets, the Multibrot sets, generation of the Mandelbrot-like complex sets with the corresponding Fatou sets using arbitrary polynomials and rational functions.
nb#4	numerics_14d.nb	The Hilbert curve: a space filling curve with fractal dimension $d = 2.0$.
nb#5	numerics_14e.nb	The Mandelbrot set for $\text{Im}(z) = 0$ and $\text{Im}(c) = 0$: cobweb diagram, orbit diagram and map iterates.
nb#6	numerics_14f.nb	The Mandelbrot set for $\text{Im}(z) = 0$ and $\text{Im}(c) = 0$: cobweb diagram, the set itself and its iterates.
nb#7	numerics_14g.nb	Interactive program that plots and measures the length of the von Koch snowflake proto-fractal using different measurement resolutions.
nb#8	numerics_14h.nb	Interactive program that plots and measures the length of the Estonian coastline using different measurement resolutions.

Lecture 15

Link	File name	Description
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Coursework

Link	File name	Description
nb	Analysis_tool.nb	Analysis tool for performing linear analysis of 2-D nonlinear systems, plotting and visualisation of 2-D and 3-D nonlinear systems (Wolfram Mathematica notebook).
ipynb	TBA	Analysis tool for performing linear analysis of 2-D nonlinear systems, plotting and visualisation of 2-D and 3-D nonlinear systems (Jupyter notebook).

Acknowledgements

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