Index of suggested refereed articles

Suggested reading list for the student taking YFX1520 course. All linked papers are in the public domain or have Open Access and are used for educational proposes only.

Lecture 3

Link	File name	Citation and abstract
Read#1	reading.pdf	Steven H. Strogatz, *Nonlinear Dynamics and Chaos With Applications to Physics, Biology, Chemistry, and Engineering*, Perseus Books Publishing, L.L.C., pp. 73–79, (1994). * * * Ludwig *et al.* (1978) proposed and analyzed an elegant model of the interaction between budworms and the forest. They simplified the problem by exploiting a separation of time scales: the budworm population evolves on a fast time scale, whereas the trees grow and die on a slow time scale. Thus, as far as the budworm dynamics are concerned, the forest variables may be treated as constants. At the end of the analysis, we will allow the forest variables to drift very slowly—this drift ultimately triggers an outbreak.

Lecture 6

Link	File name	Citation and abstract
Paper#1	paper0.pdf	Evgeni E. Sel'kov, "Self-oscillations in glycolysis 1. A simple kinetic model," *European Journal of Biochemistry*, **4**(1), pp. 79–86, (1968) doi:10.1111/j.1432-1033.1968.tb00175.x * * * The paper describes a simple kinetic model of an open monosubstrate enzyme reaction with substrate inhibition and product activation. A comparison between the model and the phosphofructokinase reaction shows a close resemblance between their dynamical properties. This makes it possible to explain qualitatively most experimental data on single-frequency oscillations in glycolysis. A mathematical analysis of the model has shown the following. 1. In the model, at a definite relationship between the parameters, self-oscillations arise. 2. The condition of self-excitation is satisfied more readily with a lower source rate, larger product sink rate constants, lower product-enzyme affinity and higher enzyme activity. 3. Self-oscillations exist only in a certain range of values of the parameter determining the degree of substrate inhibition. This range increases with decreasing source rate. Too strong or, conversely, too weak substrate inhibition leads to damped oscillations. 4. The period of self-oscillations depends on the degree of substrate inhibition, the source rate, the sink rate constant, the enzyme activity, the affinity of the substrate and the product for the enzyme; it decreases with an increase in these values. 5. With an increase in the relative sink rate constant the steady state amplitude of self-oscillations initially increases until a definite maximum is reached and then drops to zero. 6. A self-oscillatory state in the phosphofructokinase reaction exists only when the maximum rate of this reaction is essentially higher than the source rate, and lower than the maximum rate of the reactions controlling the sink of the products. 7. An experimental investigation of self-oscillations in the phosphofructokinase reaction may be considerably simplified by using a reconstituted system consisting of a small number of reactions with an irreversible sink of the products and artificial substrate supply. In this case the above relationship (section 6) should hold.

Lecture 8

Link	File name	Citation and abstract
Paper#1	paper1.pdf	Edward N. Lorenz, "Deterministic nonperiodic flow," *Journal of the Atmospheric Sciences*, **20**(2), pp. 130–141, (1963). doi:10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2 * * * Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions. A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic. The feasibility of very-long-range weather prediction is examined in the light of these results.

Lecture 9

Link	File name	Citation and abstract
Paper#1	paper1b.pdf	Jacques Laskar, "Large-scale chaos in the solar system," *Astronomy and Astrophysics*, **287**(1), pp. L9–L12, (1994). Stable URL: http://adsabs.harvard.edu * * * Numerous integrations of the solar system have been conducted, with very close initial conditions, totaling an integration time exceeding 100 Gyr. The motion of the large planets is always very regular. The chaotic zone explored by Venus and the Earth is moderate in size. The chaotic zone accessible to Mars is large and can lead to eccentricities greater than 0.2. The chaotic diffusion of Mercury is so large that its eccentricity can potentially reach values very close to 1, and ejection of this planet out of the solar system resulting from close encounter with Venus is possible in less than 3.5 Gyr.
Paper#2	paper1c.pdf	Wayne B. Hayes, Anton V. Malykh, Christopher M. Danforth, "The interplay of chaos between the terrestrial and giant planets," *Monthly Notices of the Royal Astronomical Society*, **407**(3), pp. 1859–1865, (2010). doi:10.1111/j.1365-2966.2010.17027.x * * * We report on some simple experiments on the nature of chaos in our planetary system. We make the following interesting observations. First, we look at the system of Sun + four Jovian planets as an isolated five-body system interacting only via Newtonian gravity. We find that if we measure the Lyapunov time of this system across thousands of initial conditions all within observational uncertainty, then the value of the Lyapunov time seems relatively smooth across some regions of initial condition space, while in other regions it fluctuates wildly on scales as small as we can reliably measure using numerical methods. This probably indicates a fractal structure of Lyapunov exponents measured across initial condition space. Then, we add the four inner terrestrial planets and several post-Newtonian corrections such as general relativity into the model. In this more realistic Sun + eight-planet system, we find that the above structure of chaos for the outer planets becomes uniformly chaotic for almost all planets and almost all initial conditions, with a Lyapunov time-scale of about 5–20 Myr. This seems to indicate that the addition of the inner planets adds more chaos to the system. Finally, we show that if we instead remove the outer planets and look at the isolated five-body system of the Sun + four terrestrial planets, then the terrestrial planets alone show no evidence of chaos at all, over a large range of initial conditions inside the observational error volume. We thus conclude that the uniformity of chaos in the outer planets comes not from the inner planets themselves, but from the interplay between the outer and inner ones. Interestingly, however, there exist rare and isolated initial conditions for which one individual outer planetary orbit may appear integrable over a 200-Myr time-scale, while all the other planets simultaneously appear chaotic.

Lecture 10

Link	File name	Citation and abstract
Paper#1	paper2.pdf	Robert M. May, "Simple mathematical models with very complicated dynamics," *Nature*, **261**, pp. 459–467, (1976). doi:10.1038/261459a0 * * * First-order difference equations arise in many contexts in the biological, economic and social sciences. Such equations, even though simple and deterministic, can exhibit a surprising array of dynamical behaviour, from stable points, to a bifurcating hierarchy of stable cycles, to apparently random fluctuations. There are consequently many fascinating problems, some concerned with delicate mathematical aspects of the fine structure of the trajectories, and some concerned with the practical implications and applications. This is an interpretive review of them.
Paper#2	paper3.pdf	Mitchell J. Feigenbaum, "Quantitative universality for a class of nonlinear transformations," *Journal of Statistical Physics*, **19**(1), pp. 25–52, (1978). doi:10.1007/BF01020332 * * * A large class of recursion relations $x_{n+1} = \lambda f(x_n)$ exhibiting infinite bifurcation is shown to possess a rich quantitative structure essentially independent of the recursion function. The functions considered all have a unique differentiable maximum $\bar{x}$. With $f(\bar{x}) - f(x) \sim \lvert x-\bar{x}\rvert^z$ (for $\lvert x-\bar{x}\rvert$ sufficiently small), $z > 1$, the universal details depend only upon z. In particular, the local structure of high-order stability sets is shown to approach universality, rescaling in successive bifurcations, asymptotically by the ratio α ($\alpha = 2.5029078750957...$ for $z = 2$). This structure is determined by a universal function $g^*(x)$, where the 2^nth iterate of f, $f^{(n)}$, converges locally to $\alpha^{-n}g^*(\alpha^n x)$ for large n. For the class of f's considered, there exists a λ_n such that a 2^n-point stable limit cycle including $\bar{x}$ exists; $\lambda_\infty - \lambda_n \sim \delta^{-n}$ ($\delta = 4.669201609103...$ for $z = 2$). The numbers α and δ have been computationally determined for a range of z through their definitions, for a variety of f's for each z. We present a recursive mechanism that explains these results by determining g^* as the fixed-point (function) of a transformation on the class of f's. At present our treatment is heuristic. In a sequel, an exact theory is formulated and specific problems of rigor isolated.
Paper#3	paper4.pdf	Mitchell J. Feigenbaum, "Universal behavior in nonlinear systems," *Physica D: Nonlinear Phenomena*, **7**(1–3), pp. 16–39, (1983). doi:10.1016/0167-2789(83)90112-4 * * * A semipopular account of the universal scaling theory for the period doubling route to chaos is presented.

Lecture 11

Link	File name	Citation and abstract
Paper#1	paper5.pdf	Mitchell J. Feigenbaum, "The universal metric properties of nonlinear transformations," *Journal of Statistical Physics*, **21**(6), pp. 669–706, (1979). doi:10.1007/BF01107909 * * * The role of functional equations to describe the exact local structure of highly bifurcated attractors of $x_{n+1} = \lambda f(x_n)$ independent of a specific f is formally developed. A hierarchy of universal functions $g_r(x)$ exist, each descriptive of the same local structure but at levels of a cluster of 2^r points. The hierarchy obeys $g_{r-1}(x) = -\alpha g_r(g_r(x/\alpha))$, with $g = \lim_{r\to\infty} g_r$ existing and obeying $g(x) = -\alpha g(g(x/\alpha))$, an equation whose solution determines both g and α. For r asymptotic $$g_r \sim g - \delta^{-r}h \qquad (*)$$ where $\delta > 1$ and h are determined as the associated eigenvalue and eigenvector of the operator $\mathscr{L}$: $$\mathscr{L}[\phi] = -\alpha[\phi(g(x/\alpha)) + g'(g(x/\alpha))\phi(-x/\alpha)]$$

		We conjecture that $\mathscr{L}$ possesses a unique eigenvalue in excess of 1, and show that this δ is the λ-convergence rate. The form (*) is then continued to all λ rather than just discrete λ_r and bifurcation values Λ_r and dynamics at such λ is determined. These results hold for the high bifurcations of any fundamental cycle. We proceed to analyze the approach to the asymptotic regime and show, granted $\mathscr{L}$'s spectral conjecture, the stability of the g_r limit of highly iterated λf's, thus establishing our theory in a local sense. We show in the course of this that highly iterated λf's are conjugate to g_r's, thereby providing some elementary approximation schemes for obtaining λ_r for a chosen f.

Lecture 12

Link	**File name**	**Citation and abstract**
Paper#1	paper6.pdf	Michel Hénon, "A two-dimensional mapping with a strange attractor," *Communications in Mathematical Physics*, **50**(1), pp. 69–77, (1976). doi:10.1007/BF01608556 * * * Lorenz (1963) has investigated a system of three first-order differential equations, whose solutions tend toward a "strange attractor". We show that the same properties can be observed in a simple mapping of the plane defined by: $x_{i+1} = y_i + 1 - ax_i^2$, $y_{i+1} = bx_i$. Numerical experiments are carried out for $a = 1.4$, $b = 0.3$. Depending on the initial point (x_0, y_0), the sequence of points obtained by iteration of the mapping either diverges to infinity or tends to a strange attractor, which appears to be the product of a one-dimensional manifold by a Cantor set.

Lecture 14

Link	**File name**	**Citation and abstract**
Paper#1	paper7.pdf	Benoit Mandelbrot, "How long is the coast of Britain? Statistical self-similarity and fractional dimension," *Science, New Series*, **156**(3775), pp. 636–638, (1967). Stable URL: www.jstor.org/stable/1721427 * * * Geographical curves are so involved in their detail that their lengths are often infinite or, rather, undefinable. However, many are statistically "self-similar," meaning that each portion can be considered a reduced-scale image of the whole. In that case, the degree of complication can be described by a quantity D that has many properties of a "dimension," though it is fractional; that is, it exceeds the value unity associated with the ordinary, rectifiable, curves.